A Comprehensive Mathematical Model for Analysis of WR-Resolvers under Stator Short Circuit Fault

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1. INTRODUCTION

High-efficiency electrical machines are widely used in electric vehicles and other industrial applications [1]-[2]. Most of the high-efficiency machines are inverter-driven ones that their improved performance is depended on their accurate position estimation. In this regards, position sensors are employed. The most common position sensors are optical encoders and resolvers. Although optical encoders offer higher accuracy and lower cost, they have undesirable performance in harsh environments where there are wide temperature variation, high level of noise, and vibration [3]-[4]. Therefore, in such applications engineers prefer to use resolvers that have a robust structure due to their similarity to electrical machines. In fact, resolvers are two-phase synchronous generators with high-frequency excitation voltage. They have an excitation winding on the rotor and two-phase signal windings on the stator. Since the excitation winding is fed by an AC voltage, a rotary transformer (RT) can be used for non-contact transferring of the voltage [4]. In this regard, the primary coil of the RT is wound on a primary, stationary core of RT and fed using a high-frequency voltage. Then, the induced voltage on the secondary coil that is located on the rotor is used for supplying the rotor winding. Despite the advantages of RT in non-contact transferring the voltages, its usage adds some difficulties such as excessive dimensions of the sensor, increased phase-shift error, and accuracy deterioration as a result of RT’s leakage flux linking the resolver’s windings [5]. Therefore, some researches propose to omit the individual core of RT by grooving a new slot perpendicular to the main slots of the sensor [5]-[7].

Some other researchers try to omit the rotor winding and consequently their proposed resolvers
don't need any RT. They proposed variable reluctance (VR) resolvers [8]-[15]. VR resolvers have no-winding on the rotor and the excitation winding is moved to the stator side. They worked based on the sinusoidal variation of air-gap's reluctance. So, the resolvers are divided into two groups: sinusoidal air-gap length [8]-[10], and sinusoidal area [10]-[14] resolvers. Although VR resolvers have simpler structure and lower price in comparison to WR resolvers, they have some difficulties in 2-pole applications [15]. Therefore, in 2-pole applications, WR resolvers are often preferred to VR types and in the focus of this paper is on WR resolvers. Although there are different configurations of WR resolvers include cylindrical resolvers [16], disk type ones [3]-[8] and linear resolvers [17]-[18], the most commercial WR resolvers are cylindrical ones. Furthermore, none of the mentioned researches referred to electrical faults. However, there is high risk of short circuit fault in resolvers due to their thin wires. It worth noting that short circuit fault in other electrical machines is considered in literature [19]. However, the only manuscripts in this field are [20]-[23]. Among them [20]-[23] have been done on VR resolvers using finite element analysis. The only manuscript that discusses the short circuit fault of wound rotor resolvers is [23]. In [23] a mathematical model is proposed for performance evaluation of the WR resolvers under short circuit fault in stator windings. It is supposed that there is only one turn to turn fault in one of the stator windings. While in this paper that is the extended version of [23], the mathematical model and the equivalent circuit are obtained to consider multiple faults, simultaneously.

It should be mentioned that diagnosing turn-to-turn fault at the very beginning of its development is necessary to avoid the propagation of the turn-to-turn SC fault to the whole coil/winding(s) and undesirable performance of the motion control drive. In this regard, an accurate but computationally fast model is required. Although time stepping finite element method is the most reliable and accurate method for modeling of electromagnetic sensors, it is time-consuming and it is not recommended for use in online fault diagnosing/canceling programs. Therefore, in this paper, an analytical model based on d-q axes theory is proposed to study the performance of the WR resolvers. The success of the model is evaluated by experimental measurements.

2. CONFIGURATION OF THE SENSOR

The studied resolver is a two-pole, cylindrical, WR resolver. Both the stator and the rotor have made from a laminated ferromagnetic material. The stator has 12 slots and the rotor has 16 slots. The schematic of the studied resolver’s core along with its rotary transformer is shown in Fig. 1.

It should be mentioned that in WR resolvers usually a perpendicular winding is wound in the rotor slots along with the excitation winding. It is a short circuit winding, called damper winding [13]. For the healthy resolvers with conventional distributed or on-tooth variable turn windings, using the damper winding has no significant effect on the performance and only helps to improve the accuracy of the sensor under mechanical faults. While, for the resolvers equipped with the fractional slot or constant turn on-tooth winding, even in healthy condition using damper winding helps to suppress the sub-harmonics and improve the accuracy.

Figure 1: The schematic of the studied brushless resolver: (a) The stator and the primary of RT, (b) The rotor and the secondary of RT.

3. ANALYTICAL MODEL

The schematic of the stator and rotor windings is shown in Fig. 2. As mentioned earlier, the stator has a two-phase perpendicular winding that is connected to the resolver to digital converter (RDC), called sine and cosine windings. Since the input impedance of the RDC is significantly high, the current in the signal windings of the resolver is negligible [4]. Also, the rotor has a two-phase perpendicular winding: the excitation winding that is fed by the secondary coil of RT and a damper winding. Some turn to turn short circuit faults are assumed on the winding SCAn, SCB1, SCB2, ..., SCBn.  

Figure 2: The schematic of the resolver's windings.
The voltage equations of the windings can be written as:

\[
\begin{bmatrix}
    v_a \\
    v_b
\end{bmatrix} = \begin{bmatrix}
    R_s & 0 \\
    0 & R_r
\end{bmatrix} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
    \lambda_a \\
    \lambda_b
\end{bmatrix}
\]

(1)

\[
\begin{bmatrix}
    v_a \\
    v_b
\end{bmatrix} = \begin{bmatrix}
    R_s & 0 \\
    0 & R_r
\end{bmatrix} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
    \lambda_a \\
    \lambda_b
\end{bmatrix}
\]

(2)

\[V_{sckf} = 0 = R_{sckf}i_{sckf} + \frac{d}{dt}f, f = A, B \text{ and } k = 1 \text{ to } n\]

(3)

where \(R_s\) denotes the stator resistance, \(R_r\) rotor resistance, \(R_{sck}\) short circuit turns’ resistance, \(i\) current, \(v\) voltage, and \(\lambda\) flux linkage. Subscripts a, and b denotes phases a, and b, respectively. The superscripts s, r, and sc refer to stator, rotor, and short-circuit, respectively. The flux linkages, in the terms of the winding inductances and currents, for \(k=1, 2, \ldots, n\), can be written as:

\[
\begin{bmatrix}
    \lambda_a \\
    \lambda_b
\end{bmatrix} = L_{ss} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + L_{sr} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \sum_{k=1}^{n} \left( k_{sckf} L_{ms} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + k_{sckf} L_{ms} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right)
\]

(4)

\[
\begin{bmatrix}
    \lambda_a \\
    \lambda_b
\end{bmatrix} = \begin{bmatrix}
    L_{ss} & L_{sr} \\
    L_{sr} & L_{rr}
\end{bmatrix} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \sum_{k=1}^{n} \left( L_{sckf} i_{sckf} + L_{rckf} i_{rckf} \right)
\]

(5)

\[
\lambda_{sckf} = L_{sckf} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \lambda_{sckf} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \sum_{j=1}^{n} L_{sckf} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix}
\]

(6)

\[
\lambda_{sckf} = L_{sckf} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \lambda_{sckf} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix} + \sum_{j=1}^{n} L_{sckf} \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix}
\]

(7)

where the submatrices of the stator-to-stator, and rotor-to-rotor, winding inductances are:

\[
L_{ss} = \begin{bmatrix}
    L_{sas} & L_{asb} \\
    L_{bas} & L_{bbs}
\end{bmatrix} = \begin{bmatrix}
    L_{is} + L_{ms} & 0 \\
    0 & L_{is} + L_{ms}
\end{bmatrix}
\]

(8)

\[
L_{sr} = \begin{bmatrix}
    L_{sar} & L_{asr} \\
    L_{brs} & L_{brs}
\end{bmatrix} = \begin{bmatrix}
    L_{lr} + L_{mr} & 0 \\
    0 & L_{lr} + L_{mr}
\end{bmatrix}
\]

(9)

and those of mutual inductances, for \(k=1, 2, \ldots, n\), are defined as:
Then, the equations can be written in stationary reference frame. Furthermore, the rotor and the short circuit quantities should be referred to the stator side. Finally, the stator and rotor voltage equations can be written as:

\[ \begin{bmatrix} v_d^s \\ v_q^s \end{bmatrix} = R_s \begin{bmatrix} i_d^s \\ i_q^s \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_d^s \\ \lambda_q^s \end{bmatrix} \]  
(23)

\[ \begin{bmatrix} v_d^r \\ v_q^r \end{bmatrix} = \left( \frac{N_s}{N_r} \right)^2 R_r \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_d^r \\ \lambda_q^r \end{bmatrix} + \omega \begin{bmatrix} 0 \\ 1 \end{bmatrix} \lambda_d^r 
\]  
(24)

where

\[ \begin{bmatrix} \lambda_d^s \\ \lambda_q^s \end{bmatrix} = L_{ls} \begin{bmatrix} i_d^s \\ i_q^s \end{bmatrix} + L_{ms} \begin{bmatrix} i_d^s \\ i_q^s \end{bmatrix} + \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} \]  
(25)

\[ \begin{bmatrix} \lambda_d^r \\ \lambda_q^r \end{bmatrix} = \left( \frac{N_s}{N_r} \right)^2 L_{lr} \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} + L_{ms} \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} + \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} \]  
(26)

\[ \sum_{k=1}^{n} \left( \lambda_d^{s,k} \begin{bmatrix} i_d^s \\ i_q^s \end{bmatrix} + \lambda_q^{s,k} \begin{bmatrix} i_d^s \\ i_q^s \end{bmatrix} \right) = \lambda_d^{s,k} \begin{bmatrix} i_d^s \\ i_q^s \end{bmatrix} + \lambda_q^{s,k} \begin{bmatrix} i_d^s \\ i_q^s \end{bmatrix} \]  
(27)

\[ \begin{bmatrix} \lambda_d^{r,k} \\ \lambda_q^{r,k} \end{bmatrix} = \left( \frac{N_s}{N_r} \right)^2 \lambda_d^{r,k} \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} + \lambda_q^{r,k} \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} + \lambda_m^{r,k} \begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} \]  
(28)

For the voltage equation of short-circuit winding, considering \( f = A \), \( B \), and \( k = 1, 2, ..., n \), Eq. (3) can be rewritten as:

\[ 0 = \begin{bmatrix} \cos(\theta_{scfk}) \\ \sin(\theta_{scfk}) \end{bmatrix} \left( R_{scfk} \frac{dv}{dt} + \frac{d\lambda_{scfk}}{dt} \right) \]  
(29)

Considering the transformation matrix of:

\[ \begin{bmatrix} f_d^{scfk} \\ f_q^{scfk} \end{bmatrix} = \begin{bmatrix} \cos(\theta_{scfk}) \\ \sin(\theta_{scfk}) \end{bmatrix} f_{scfk} \]  
(30)

where \( f \) can be voltage, current, or flux linkage, the voltage of (27), after referring to the stator side can be written as:

\[ 0 = R_{scAk} \begin{bmatrix} i_d^{scAk} \\ i_q^{scAk} \end{bmatrix} + \frac{d}{dt} k_{scAk} \begin{bmatrix} \lambda_d^{scAk} \\ \lambda_q^{scAk} \end{bmatrix} \]  
(31)

\[ 0 = R_{scBk} \begin{bmatrix} 0 \\ i_q^{scBk} \end{bmatrix} + \frac{d}{dt} k_{scBk} \begin{bmatrix} \lambda_d^{scBk} \\ 0 \end{bmatrix} \]  
(32)

Those equations can be used to draw an equivalent circuit for the resolver. In this regards, substituting (31) and (32) into (29) and (30), will result:

\[ R_s \begin{bmatrix} i_d^{scAk} \\ 0 \end{bmatrix} = -k_{scAk} \frac{d}{dt} \begin{bmatrix} \lambda_d^{scAk} \\ \lambda_q^{scAk} \end{bmatrix} \]  
(33)

\[ R_s \begin{bmatrix} 0 \\ i_q^{scBk} \end{bmatrix} = -k_{scBk} \frac{d}{dt} \begin{bmatrix} \lambda_d^{scBk} \\ 0 \end{bmatrix} \]  
(34)

Furthermore, since the stator currents are negligible, the voltage drop on the stator windings’ resistance and leakage inductance is almost zero. Therefore, substituting (25) into (22), will result in:

\[ \frac{d}{dt} \begin{bmatrix} \lambda_d^s \\ \lambda_q^s \end{bmatrix} = \begin{bmatrix} v_d^s \\ v_q^s \end{bmatrix} \]  
(35)

Then, substituting (35), into (33) and (34):

\[ i_d^{scAk} = -k_{scAk} \frac{d}{dt} v_d^s \]  
(36)

Then, the d-q equivalent circuit of the resolver under short circuit fault can be given as shown in Figs. 3-a, and 3-b. After simplifying the parallel branches, the simplified d-q circuit of Figs. 3-c, and 3-d is obtained.

The mentioned equations are employed in MATLAB/SIMULINK to analyze the performance of the sensor. The output voltages of SIMULINK are used to calculate the position error of the resolver. In this regard, Hilbert transform is used to obtain the envelope of the voltages [4]. Then, inverse tangent method is used for calculating the position. Comparing that position with the reference position leads to position error of the sensor. Fig. 4 shows the block diagram of the simulation.

The analog voltages of the health resolver obtained from the SIMULINK are presented in Fig. 5-a. After using Hilbert transform the envelope of the signals is obtained [4]. The envelope of the sine voltage versus that of cosine voltage is presented in Fig. 5-b. Then, harmonic content of the envelopes can be calculated. Total harmonic distortion (THD) of envelopes, maximum position error (MPE) and the average of absolute position error (AAPE) for the health resolver are 0.0531%, 0.2465”, and 0.0129”, respectively. It should be mentioned that the computation time required for one mechanical rotation of the rotor, using Intel® Core™ i7-6500U CPU@2.50 GHz, is about one second. Therefore, the proposed analytical model can be employed in the on-line fault detection algorithms or design and optimization of the sensor.
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The simulations are divided into two groups. In the first group, four independent short circuit faults are simulated. In the first simulation, called SC-1, the fault is considered on the cosine coil with maximum number of turns, where 1/16 of the turn numbers are shorted. In SC-2, 1/14 of the turns of cosine winding are shorted. In SC-3, and SC-4, 1/16 and 1/8 of the turn of sine winding are assumed to be short circuit, respectively. The second group that is referred to simultaneous faults consisted of three simulations. In SC-5, 1/16 of the turn numbers of both sine and cosine coils with maximum number of turns is considered.

In the second simulation, SC-6, two short circuit faults (1/12 turn number on the coils with 130 turn number) on sine winding is considered. The last simulation is referred to two faults on the sine windings (1/16 turn short circuit on the cosine tooth of 150 turn number, and 1/8 turn on the coil with 75 turn number).

Harmonic contents of the sine and cosine voltages' envelope for the health resolver along with those for different independent fault conditions are presented in Figs. 6-a, and 6-b, respectively. Those results for the simultaneous faults are given in Figs. 6-c, and 6-d, respectively. It can be seen that the harmonic levels are increased under short circuit fault condition.
Figure 6: The harmonic content of the voltages’ envelope: (a) for the sine voltage (independent faults), (b) for the cosine voltage (independent faults), (c) for the sine voltage (simultaneous faults), and (d) for the cosine voltage (simultaneous faults).

Figs. 7-a through 7-c show the variation of envelopes’ THD, MPE, and AAPE under the different scenarios of short circuit faults using the proposed analytical model, respectively. Considering the results of THD and the AAPE, it seems that simultaneous short circuit faults have more destructive effect on the performance of the resolver. However, it should be mentioned that the most reliable index for performance evaluation of resolvers is AAPE. Because THD is not sensitive to the harmonic orders while position error is. Finally, it can be concluded that the performance of the sensor is significantly deteriorated under different short circuit faults. Therefore, it is required to diagnose the fault occurrence before its propagation to whole winding or undesirable performance of the drive system.

Figure 7: The performance of the studied resolver under different fault conditions: (a) THD of envelopes, (b) MPE, and (c) AAPE.

4. EXPERIMENTAL EVALUATION

To evaluate the results of the proposed analytical model, an experimental test is carried out. The experimental test circuit is shown in Fig. 8-a. It can be seen that a DC motor is used to rotate the resolver with 300 rpm. The 4 kHz excitation voltage is built using a digitally synthesized function generator with the resolution of 0.1 Hz and the output signals are captured and saved using a digital oscilloscope. The output voltages are presented in Fig. 8-b. Those voltages are imported to the MATLAB to use Hilbert transform for calculating position error. Comparing the results of the experimental test with those of simulations indicates an acceptable agreement. However, the experimental position error is higher than that of simulation. To explain this difference, some points should be considered. The effect of rotary transformer is not considered in the simulations. So, for more agreement of results, it is required to model...
the rotary transformer as well as the resolver. Furthermore, the non-linear behavior of the magnetizing core of the resolver is not taken into account. Although the operating point of resolvers is far from the saturation region [4], it may be stuck in the non-linear area of the beginning of the magnetizing curve. The other reason for difference between the results is using inaccurate parameters in d-q model. So, authors plan to conduct some more research on parameter identification of the resolver and improve the model by taking the RT and the non-linear magnetization curve of the ferromagnetic core into account.

Figure 8: The experimental evaluation: (a) The test circuit, (b) The measured voltages.

5. CONCLUSION

Although the current of signal windings of the resolver is almost negligible, since their employed wire is very thin, there is high possibility of short circuit fault in signal windings. Therefore, in this paper a comprehensive mathematical model based on d-q axes theory was proposed to evaluate the performance of cylindrical, wound rotor resolvers under short circuit fault. Different amount/number of faults, even turn to turn short circuit faults, could be simulated using the proposed model, independently and simultaneously. Furthermore, the proposed model was fast enough to be integrated into an on-line fault detection algorithm. The simulations of the proposed model were done in MATLAB-SIMULINK and the Hilbert transform was used to decouple the excitation voltage. Then, the envelope of the AM voltages was used for calculating the position error. The results of the model were evaluated by comparing them with those of experimental measurement.

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REFERENCES

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BIographies

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