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Research paper

Finite-Time Consensus Control of Euler-Lagrange Multi-agent Systems in the Presence of Stochastic Disturbances and Actuator Faults

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Abstract

Background and Objectives: This article discusses a finite-time fault-tolerant consensus control for stochastic Euler-Lagrange multi-agent systems.

Methods: First, the finite-time consensus controller of Euler-Lagrange multi-agent systems with stochastic disturbances is presented. Then, the proposed controller is extended as a fault-tolerant controller in the presence of faults in the actuators. In these two cases, the sliding-mode distributed consensus controllers are designed.

Results: The results section is the most important part of the abstract and nothing should compromise its range and quality. This is because readers who peruse an abstract do so to learn about the findings of the study. The results section should therefore be the longest part of the abstract and should contain as much detail about the findings as the journal word count permits.

Conclusion: The proposed theorems in this paper guarantee that the consensus tracking errors are bounded in probability and after a finite-time remain in a desired area close to the origin in the mean-square senses. The obtained theorems were applied to consensus control of the robotic manipulators to indicate the performance of the proposed controllers.

Introduction

This Recently, multi-agent systems (MASs) have gained plenty of attention among scientists and engineers; this attention is because of their applications in numerous fields like robotics [1], power systems [2], wireless sensor networks [3], hybrid systems [4] and so on. Consensus control is a basic problem in the MASs, whose goal is the convergence of the agents' states or agents' outputs on a specific variable. In MASs, the consensus problem is considered in two approaches: behavior-based approach [5] and leader-follower approach [6]. In a behavior-based approach, based on the distributed information of the agents, the agents achieve an agreement on special value [5]. Consensus control, when a leader exists, is divided into leader following and virtual leader, in which, the follower agents must follow the trajectories produced by the leader or virtual agents [6]. Most of the works available in the literature regarding MASs in the past decade, have considered the agents' model as a linear equation [7], while in the reality, agents have nonlinear dynamics and stochastic terms. Because of the benefits of the sliding mode method in the nonlinear control field [8], the consensus control of nonlinear dynamics with Lipschitz conditions by using the sliding mode approach was investigated in [9]. The Euler-Lagrange equation is a practical nonlinear model, which is employed for modelling most nonlinear mechanical and robotic systems. In [10] and [11], the consensus control for Euler-Lagrange MASs in leader following and behavior-based approaches was investigated. On the other hand, many applications of Euler-Lagrange MASs in real environments involve stochastic factors such as stochastic noise [12] and stochastic environments [13]. Although systems are subject to disturbances for a variety of different reasons, they have been mostly modeled deterministically due to their unknown nature, or in order to have simpler calculations [14]. While in systems like vibrations of robotic arms [15] or airplanes flying in the presence of wind [16], the system should be modeled with stochastic differential equations, due to the nature of the disturbance occurred in them. The control of Euler-Lagrange MASs along with stochastic disturbances is a novel topic not thoroughly investigated in the literature. In [17] and [18], the distributed consensus control for Euler-Lagrange MASs with the stochastic disturbances was proposed by a prescribed performance control and pinning control approach, respectively.

In many researches in the literature performed in the field of consensus control, convergence with an infinite settling time has been demonstrated, which means that an infinite time is required for the agents to reach the desired arrangement or consensus. However, in a lot of practical applications, the consensus goals of the system are required to occur in finite time. Finite-time control of systems not only increases the convergence speed, but also increases the capability of eliminating the disturbance effects, as well as the robustness of the system against uncertainties. Thus, researchers try to design finite-time controllers for MASs, in order to ensure the occurrence of the system's goals in finite-time [19]. In [20] and [21], the finite-time consensus control was investigated for deterministic Euler-Lagrange MASs. Also in [22], the finite-time formation control for second-order stochastic nonlinear MASs was proposed. In contrast to single systems, high numbers of faults happen in the sensors and actuators of MASs, which not only reduce the performance of the faulty agents, in some cases they make the whole system unstable. Therefore, fast identification and reduction of the faults' effects have great importance in MASs. Efforts to overcome this problem in the literature are generally divided into two groups. In the first group, the faults are identified using adaptive control methods, and the input control is designed based on the amount of the fault, which requires complicated calculations [23]-[25]. In the second approach, a fixed robust input control is designed according to the faults that are probable to happen in the system, which usually result in applying inputs bigger than the needs of the system [26], [27]. On the other hand, the use of both of the following approaches in stochastic systems, provides greater challenges due to the nature of the systems. In [28] and [29], fault tolerant control of stochastic systems using adaptive and robust control are proposed, but the system stability in finite-time is not guaranteed. In this paper, we introduce a finite-time leader-following fault-tolerant consensus tracking control of Euler-Lagrange MASs with stochastic disturbances. In contrast to fault-tolerant methods which are designed for deterministic disturbances, we consider stochastic disturbances, which yield a more realistic model. Moreover in this study, a finite-time stochastic control approach is presented, which provides better control performance, containing a rapidly disappearing transient response. The main novel contribution of this study contains the finite-time fault-tolerant control for Euler-Lagrange MASs with the stochastic disturbances in a mean-square sense. This implies that the mean-square of tracking errors after a finite-time remain in a desired bound close to the origin. Moreover, the proposed controller is extended as a fault-tolerant controller, which guarantees the stability of the system in the presence of actuator faults. This article is structured as follows: Section 2 provides the technical work preparation. Two finite-time controllers both without actuator faults and in the presence of actuator faults are provided in Section 3. The simulation of consensus control for robotic manipulators with stochastic disturbances is presented in Section 4 and the conclusions are made in Section 5.

Technical Work Preparation

A. Graph theory

In multi-agent systems, graph theory is employed to specify the agents and their relations. A graph with N node is defined as $\mathcal{G} = (V, E, A)$, in which $V = \{1, \ldots, N\}$ denotes the set of agents, $E \subset V \times V$ is the edge-set , and $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ representing the adjacency matrix with $a_{ii} = 0$. Moreover, $a_{ij} > 0$ means that node j can send the information to node i; otherwise $a_{ij} = 0$. The graph Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ can be defined as [18]:

$$l_{ij} = -a_{ij},\ j \neq i,$$
$$l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}. \quad (1)$$

The communication between the leader and the followers are stated in a matrix $B = diag\{b_1, \ldots, b_N\}$. If the i-th follower receives information from the leader, $b_i = 1$; otherwise $b_i = 0$.

B. Problem formulation

The model of each Euler-Lagrange agent in the presence of stochastic disturbance is considered as [30]:

$$M_i(q_i)\ddot{q}_i + C(q_i,\dot{q}_i)\dot{q}_i + G(q_i) = \tau_i(t) - J_i^T(q_i)\xi(t), \quad (2)$$

where q_i, $\dot{q}_i$, $\ddot{q}_i \in \mathbb{R}^m$ for $i \in \{1, \ldots, N\}$ are the position, angular velocity, and accretion vectors, respectively and $\tau_i(t) \in \mathbb{R}^m$ is the input torque. Moreover, $M_i(q_i) \in \mathbb{R}^{m\times m}$, $C(q_i,\dot{q}_i) \in \mathbb{R}^{m\times m}$, and $G(q_i) \in \mathbb{R}^m$ denote mas matrix, centrifugal, Coriolis terms, and gravity terms, respectively. Furthermore, $J_i^T(q_i)$ is the nonsingular Jacobin matrix and $\xi(t)$ is a standard white noise, which exerted by the stochastic environment. Since the white noise in not derivable, using $\xi(t) = \frac{dw}{dt}$, Eq. (2) could be rewritten as follows:

$$dq_i(t) = p_i(t)dt,$$

$$dp_i(t) = \left(M_i^{-1}(q_i)\,\tau_i(t) + h_i(q_i,p_i)\right)dt + k_i(q_i)dw(t), \quad (3)$$

where $h_i(q_i,p_i) = M_i^{-1}(q_i)[C(q_i,p_i)p_i + G(q_i)]$, $k_i(q_i) = -M_i^{-1}(q_i)J_i(q_i)$, and $w(t)$ is an independent standard Brownian motion.

Moreover, the following equations are used as the leader's dynamics:

$$dq_o(t) = p_o(t),$$

$$dp_o(t) = \left(M_o^{-1}(q_o)\,\tau_o(t) + h_o(q_o,p_o)\right)dt, \quad (4)$$

where $h_o(q_o,p_o) = M_o^{-1}(q_o)[C(q_o,p_o)p_o + G(q_o)]$.

Define the consensus errors as subsequent [18]:

$$\varepsilon_{1i}(t) = \sum_{j=1}^{N} a_{ij}\left[q_i(t) - q_j(t)\right] + b_i[q_i(t) - q_o(t)]\,, \quad (5)$$

$$\varepsilon_{2i}(t) = \sum_{j=1}^{N} a_{ij}\left[p_i(t) - p_j(t)\right] + b_i[p_i(t) - p_o(t)], \quad (6)$$

Using Kronecker product and Eqs. (5) and (6) , we have [18]:

$$\varepsilon_1(t) = [(L+B)\otimes I_m]\cdot\tilde{q}(t), \quad (7)$$

$$\varepsilon_2(t) = [(L+B)\otimes I_m]\cdot\tilde{p}(t), \quad (8)$$

where $\varepsilon_1(t) \triangleq [\varepsilon_{11}^T(t), \ldots, \varepsilon_{1N}^T(t)]^T$, $\varepsilon_2(t) \triangleq [\varepsilon_{21}^T(t), \ldots, \varepsilon_{2N}^T(t)]^T$, $\tilde{q}(t) = q(t) - \mathbf{1}\otimes q_o(t)$, $\tilde{p}(t) = p(t) - \mathbf{1}\otimes p_o(t)$, $q(t) = [q_1^T, \ldots, q_N^T]^T$, $p(t) = [p_1^T, \ldots, p_N^T]^T$, $\mathbf{1} = [1, 1, \ldots, 1]_{N\times 1}^T$.

The Ito derivatives of Eqs. (7) and (8) gives:

$$d\varepsilon_1(t) = \varepsilon_2(t)dt,$$

$$d\varepsilon_2(t) = [(L+B)\otimes I_m\cdot\left(H - \mathbf{1}\otimes h(p_o,q_o,t) - M^{-1}(q)\,\tau(t) - \mathbf{1}\otimes M_o^{-1}(q_o)\,\tau_o(t)\right)]dt + [(L+B)\otimes I_m\cdot K]dw, \quad (9)$$

where $H = [h(x_1,v_1,t)^T, \ldots, h(x_N,v_N,t)^T]^T$, $K = [k(x_1,v_1,t)^T, \ldots, k(x_N,v_N,t)^T]^T$, $\tau(t) = [\tau_1{}^T, \ldots, \tau_N{}^T]^T$, and $M^{-1}(q) = diag\,[M_1^{-1}(q_1), \ldots, M_N^{-1}(q_N)]$.

Assumption 1. [23]: The undirected graph of the MAS is connected.

Assumption 2. [9]**:** Functions h is Lipchitz. Consequently, there exist positive scalers ρ_1 and ρ_2 such that:

$$\|H(p,q,t) - 1\otimes h(p_o,q_o,t)\| \le \rho_1\|p - 1\otimes p_o\| + \rho_2\|q - 1\otimes q_o\|, \quad (10)$$

where $i \in \{1, \ldots, N\}$.

Assumption 3. [18]**:** The stochastic term is bounded as follows:

$$\|K(q,t)\|_2 \le \rho_3, \quad (11)$$

where ρ_3 is non-negative constant.

Definition 1. [18]**:** Let $V(x(t),t)$ as the Lyapunov function for the stochastic nonlinear system:

$$\mathrm{d}\varepsilon(t) = \mathrm{f}(\varepsilon(t))\mathrm{d}t + \mathrm{g}(\varepsilon(t))\mathrm{d}w. \quad (12)$$

the infinitesimal generator is presented as:

$$LV(\varepsilon(t),t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}f(\varepsilon(t),t) + \frac{1}{2}Tr\left\{\left(g(\varepsilon(t),t)\right)^T\frac{\partial^2 V}{\partial x^2}\left(g(\varepsilon(t),t)\right)\right\}, \quad (13)$$

Definition 2. [18]**:** Iflim$_{r^*\to\infty}$ sup$_{t>0}$ $P\left(\|\varepsilon(t)\| > r^*\right) = 0$, a stochastic process $\varepsilon(t)$ is bounded in probability.

Definition 3. [31]**:** If there exist scalars $\epsilon > 0$ and $T^*(\epsilon, \varepsilon_0)$ for the solution $\varepsilon(t)$ of the system (12), such that:

$$E[\|\varepsilon(t)\|^2] < \epsilon, \qquad \forall t > T^* + t_0, \quad (14)$$

then $\varepsilon(t)$ will be finite-time stable in the mean-square sense.

Lemma 1. [18] (Young's Inequality): For any vectors $x, y \in \mathbb{R}^n$, the inequality $x^T y \le (b^c/c)\|x\|^c + (1/db^d)\|y\|^d$ is correct, where $b > 0$, $c > 1$, $d > 1$, and $(c-1)(d-1) = 1$.

Lemma 2. [31]: If for the solution of stochastic system (12), $\varepsilon(t)$, there exists a Lyapunov function $V(\varepsilon)$, a positive scalers c_1, c_2, and class $\mathbb{K}_\infty$-functions $\overline{\alpha_1}$, $\overline{\alpha_2}$ such that:

$$\begin{cases} \overline{\alpha_1}(\varepsilon) \le V(\varepsilon) \le \overline{\alpha_2}(\varepsilon) \\ LV(\varepsilon) \le -c_1 V^{c_3}(\varepsilon) + c_2{}' \end{cases} \quad (15)$$

then, there exist a positive scalers T^* and ε, such that $E\|\varepsilon(t)\|^2 < \epsilon$ for $t > T^*$, which are calculated from the following relationships:

$$\epsilon = 4\left(\frac{c_2}{(1-c_4)c_1}\right)^{\frac{1}{4c_3}}, \quad (16)$$

$$T^* = \frac{1}{(1-c_3)c_1c_4}[V^{1-c_3}(\varepsilon(0)) - \left(\frac{c_2}{(1-c_4)c_1}\right)^{\frac{1-c_3}{c_3}}], \quad (17)$$

Results and Discussion

The finite-time consensus tracking control of the stochastic Euler-Lagrange MASs, both without actuator faults and with actuator faults are presented within the following two subsections.

A. Finite-time consensus control

Our aim is to use a sliding-mode approach to design a finite-time tracking control law for the leader-following consensus control of the MASs.

Consider a sliding surface for each agent as:

$$s_i = \varepsilon_{2i} + \mu\varepsilon_{1i}, \qquad i = 1, \dots, N, \tag{18}$$

where μ is a positive scalar. The compacted from of Eq. (18) leads to:

$$s = \varepsilon_2 + \mu\varepsilon_1, \tag{19}$$

where $s = [s_1^T, \dots, s_N^T]^T$.

The following is derived by calculating the derivation of Eq. (19) and substituting Eq. (9):

$$\begin{aligned} ds &= d\varepsilon_2 + \mu d\varepsilon_1 \\ &= \big[\mu\varepsilon_2 + (L+B)\otimes I_m \cdot \big(H - \mathbf{1}\otimes h(p_o,q_o,t) - M^{-1}(q)\,\tau(t) - \mathbf{1}\otimes M_o^{-1}(q_o)\,\tau_o(t)\big)\big]\,dt \\ &\quad + [(L+B)\otimes I_m \cdot K]dw, \end{aligned} \tag{20}$$

The following theorem offers a control law that guarantees the finite-time mean-square stability of stochastic Euler-Lagrange MASs.

Theorem 1: Consider the Euler-Lagrange MAS multi-agent system defined with the followers and leader in (3), (4) under the Assumptions 1-3, and the input control:

$$\tau = M(q)\cdot[(L+B)^{-1}\otimes I_m]\cdot \big[b\otimes M_o^{-1}(q_o)\,\tau_o(t) - \mu\varepsilon_2 - sgn(s)\big(n_1\big(\rho_1\|\varepsilon_1\| + \rho_2\|\varepsilon_2\|\big) + n_0 + 1\big)\big], \tag{21}$$

where $b = [b_1, \dots, b_N]^T$, $n_1 = \|(L+B)\|\|(L+B)^{-1}\|$, and n_0 is a desired positive constant. Then the leader following consensus control is achieved. Moreover, the consensus tracking errors given in Eqs. (7) and (8) are guaranteed to be bounded in probability and after a finite-time remain in a desired area close to the origin in the mean-square senses.

Proof. By applying infinitesimal generator on the following Lyapunov candidate:

$$V = \tfrac{1}{4}(s^T s)^2, \tag{22}$$

and replacing Eq. (9) into Eq. (22) follows that:

$$\begin{aligned} LV &= s^T s s^T[(L+B)\otimes I_m \cdot \big(H - \mathbf{1}\otimes h(q_o,p_o,t) + M^{-1}(q)\,\tau(t) - \mathbf{1}\otimes M_o^{-1}(q_o)\,\tau_o(t)\big) + \mu\varepsilon_2] + \\ &\quad \tfrac{1}{2}Tr[\big((L+B)\otimes I_m\cdot K\big)^T(2ss^T + s^T sI)\big((L+B)\otimes I_m \cdot K\big)]. \end{aligned} \tag{23}$$

Applying Assumption 2 in Eq. (10) on the first term of the inequality (23) gives:

$$\begin{aligned} &\|(L+B)\otimes I_m \cdot \big(H - \mathbf{1}\otimes h(q_o,p_o,t)\big)\| \\ &\le \|(L+B)\|(\rho_1\|q_i - q_0\| + \rho_2\|p_i - p_0\|) \\ &\le \|(L+B)\|(\rho_1\|\tilde{q}\| + \rho_2\|\tilde{p}\|) \\ &\le n_1\big(\rho_1\|\varepsilon_1\| + \rho_2\|\varepsilon_2\|\big). \end{aligned} \tag{24}$$

Using Lemma 1 and Assumption 3, one can write:

$$\begin{aligned} &\tfrac{1}{2}Tr\left[\big((L+B)\otimes I_m\cdot K\big)^T(2ss^T + s^T sI)\big((L+B)\otimes I_m \cdot K\big)\right] \le \tfrac{3}{2}\|s\|^2\big\|\big((L+B)\otimes I_m\cdot K\big)\big\|_F^2 \\ &\le \tfrac{3}{2}\|s\|^2 l_r\|L+B\|^2\rho_3^2 \\ &\le \|s\|^3 + \tfrac{1}{2}l_r^3\,\|L+B\|^6\rho_3^6, \end{aligned} \tag{25}$$

where $l_r = \mathrm{rank}\big((L+B)\otimes I_m\cdot K\big)$ is a positive constant, which is selected according to the equivalence of norm-2 and Frobenius norm.

Substituting inequalities (24) and (25) and into Eq. (23) gives:

$$\begin{aligned} LV &\le \|s\|^3[n_1\big(\rho_1\|\varepsilon_1\| + \rho_2\|\varepsilon_2\|\big) + 1] + \\ &\quad s^T ss^T\big[(L+B)\otimes I_m\cdot M^{-1}(q)\tau + \mu\varepsilon_2 - b\otimes M_o^{-1}(q_o)\,\tau_o(t)\big] + \tfrac{1}{2}l_r^3\,\|L+B\|^6\rho_3^6. \end{aligned} \tag{26}$$

Replacing τ given in Eq. (21) into (26) yields:

$$LV \le -n_0 V^{\frac{3}{4}} + \tfrac{1}{2}l_r^3\,\|L+B\|^6\rho_3^6. \tag{27}$$

From Eq. (27) and Lemma 2, it is guaranteed that the sliding manifold s after the finite-time will converge to the following small bound:

$$E[\|s(t)\|^2] < \epsilon_1, \qquad \forall t > T_1^* + t_0, \tag{28}$$

where:

$$T_1^* = \frac{4}{n_0 c_4}\left[V^{\frac{1}{4}}\big(s(0)\big) - \left(\frac{l_r^3\,\|L+B\|^6\rho_3^6}{2(1-c_4)n_0}\right)^{\frac{1}{3}}\right], \tag{29}$$

$$\epsilon_1 = 4\left(\frac{\frac{1}{2}l_r^3\,\|L+B\|^6\rho_3^6}{(1-c_4)n_0}\right)^{\frac{1}{3}}. \tag{30}$$

According to Eqs. (9) and (19) , one has:

$$d\varepsilon_1(t) = (-\mu\varepsilon_1(t) + s(t))dt, \tag{31}$$

The infinitesimal generator of Lyapunov function $V_\varepsilon = \frac{1}{4}({\varepsilon_1}^T\varepsilon_1)^2$ gives:

$$\begin{aligned} LV_\varepsilon(t) &= \big(-\mu\,\varepsilon_1^T\varepsilon_1(t) + \varepsilon_1^T s(t)\big) \\ &\le \big(-\mu + \tfrac{1}{2}\big)\|\varepsilon_1(t)\|^2 + \tfrac{1}{2}\|s(t)\|^2. \end{aligned} \tag{32}$$

Inside the boundary layer (28), Eq. (32) can be written as:

$$E[LV_\varepsilon] = -\big(\mu - \tfrac{1}{2}\big)E[V_\varepsilon^{\frac{1}{2}}] + \tfrac{1}{2}\epsilon_1. \tag{33}$$

From Eq. (33) and Lemma 2, for $\mu > \frac{1}{2}$, it is guaranteed that:

$$E\left[\|\varepsilon_1(t)\|^2\right] < \epsilon_2, \qquad \forall t > T_2^* + t_0, \tag{34}$$

where:

$$T_2^* = \frac{2}{(\mu-\frac{1}{2})c_4}\left[V^{\frac{1}{2}}\left(\varepsilon_1(0)\right) - \frac{\frac{1}{2}\epsilon_1}{(1-c_4)(\mu-\frac{1}{2})}\right] + T_1^*, \qquad (35)$$

$$\epsilon_2 = 4\left(\frac{\frac{1}{2}\epsilon_1}{(1-c_4)(\mu-\frac{1}{2})}\right)^{\frac{1}{2}}, \qquad (36)$$

which means the mean-square stability in finite-time.

Moreover, the tracking errors are bounded in probability, because for any positive constant δ, one can write:

$$E\left(\|\varepsilon_1(t)\|^2\right) = \int \|\varepsilon_1(t)\|^2 P(dw) \geq$$

$$\int_{\|\varepsilon_1(t)\|>\delta}^{\infty} \|\varepsilon_1(t)\|^2 P(dw) \geq \delta^2 P(\|\varepsilon_1(t)\| > \delta). \qquad (37)$$

Considering Definition 2 and substituting (34) into (37), leads to:

$$\lim_{\delta\to\infty} \sup_{t>0} P\left(\|\varepsilon_1(t)\| > \delta\right)$$

$$\leq \lim_{\delta\to\infty} \sup_{t>0} \frac{E\left(\|\varepsilon_1(t)\|^2\right)}{\delta^2} \leq \lim_{\delta\to\infty} \sup_{t>0} \frac{\epsilon_2}{\delta^2} = 0. \qquad (38)$$

Furthermore, by the similarity way it can be proved that $\varepsilon_2(t)$ is bounded in probability, too.

Remark 1. In Eq. (21) the compacted form of the input controller is proposed. The input control of the i-th agent is rewritten as:

$$\tau_i(t) = [\left(l_{ii}(t) + b_i(t)\right)^{-1} \cdot (-\sum_{j=1,j\neq i}^{N} l_{ij}(t)u_j(t) - \mu\varepsilon_{2i} - sgn(s_i)n_1(\rho_1\|\varepsilon_{1i}\| + \rho_2\|\varepsilon_{2i}\|) + 1 + n_0 + \tau_o(t))]. \qquad (39)$$

B. Fault-tolerant consensus control

In this subsection, a finite-time fault-tolerant consensus controller of stochastic Euler-Lagrange MASs is presented.

The described dynamic equation (3) under actuator faults can be rewritten as:

$$dq_i(t) = p_i(t)dt,$$

$$dp_i(t) = \left(M_i^{-1}(q_i)\left(I - \alpha_i(t)\right)\tau_i(t) + d_i(t) + h_i(q_i,\dot{q}_i)\right)dt + k_i(q_i)dw(t), \qquad (40)$$

where $d_i(t) \in \mathbb{R}^m$ and $\alpha_i(t) = diag\,[\alpha_{i1}(t), \ldots, \alpha_{im}(t)] \in \mathbb{R}^m$ with $0 \leq \alpha_{im} \leq \alpha_{max} < 1$ for $i \in \{1, \ldots, N\}$ denote the actuator bias fault and effectiveness fault, respectively. The virtual leader's dynamic is the same as one given in Eq. (4).

Moreover, the tracking errors in Eq. (9) will be changed as:

$$d\varepsilon_1(t) = \varepsilon_2(t)dt,$$

$$d\varepsilon_2(t) = [(L+B)\otimes I_m \cdot (H - 1\otimes h(p_o,q_o,t) - M^{-1}(q)\,(I_{mN} - \alpha)\tau(t) + d(t) - 1\otimes M_o^{-1}(q_o)\,\tau_o(t))]dt$$
$$+[(L+B)\otimes I_m \cdot K]dw, \qquad (41)$$

where $d(t) = [d_1^T, \ldots, d_N^T]^T$ and $\alpha(t) = diag\,[\alpha_1(t), \ldots, \alpha_m(t)]$. Defining the sliding trajectory as Eq. (18) and taking Ito derivation, one can write:

$$ds = d\varepsilon_2 + \mu d\varepsilon_1$$
$$= \left[\mu\varepsilon_2 + (L+B)\otimes I_m \cdot \left(H - \mathbf{1}\otimes h(p_o,q_o,t) - M^{-1}(q)\,(I_{mN} - \alpha)\tau(t) + d(t) - \mathbf{1}\otimes M_o^{-1}(q_o)\,\tau_o(t)\right)\right] dt + [(L+B)\otimes I_m \cdot G]dw. \qquad (42)$$

Assumption 4. [23]-[26]: The norm of bias fault is bounded as follows:

$$\|d(t)\| \leq \bar{d}, \qquad (43)$$

where $\bar{d}$ is a known positive scaler.

Theorem 2: Consider the Euler-Lagrange MAS multi-agent system defined with the followers and leader in given in (3), (4) under the assumptions 1-4. The fault-tolerant consensus input control in Eq. (39) guarantees that the consensus tracking errors are bounded in probability and after a finite-time remain in a desired area close to the origin in the mean-square senses.

$$\tau = \frac{1}{1-\alpha_{max}} M(q)(L+B)^{-1}\otimes I_m \cdot \gamma \qquad (44)$$

where:

$$\gamma = \left[b \otimes M_o^{-1}(q_o)\,\tau_o(t) - \mu\varepsilon_2 - sgn(s)\left(n_1\left(\rho_1\|\varepsilon_1\| + \rho_2\|\varepsilon_2\|\right) + \bar{d} + n_0 + 1\right)\right], \qquad (45)$$

and $b = [b_1, \ldots, b_N]^T$, $n_1 = \|(L+B)\|\|(L+B)^{-1}\|$.

Proof: By applying infinitesimal generator for following Lyapunov function:

$$V = \frac{1}{4}(s^T s)^2, \qquad (46)$$

we have:

$$LV = s^T s s^T((L+B)\otimes I_m \cdot (H - 1\otimes h(q_o,p_o,t) + (I-\alpha)\,M^{-1}(q)\,\tau(t) - 1\otimes M_o^{-1}(q_o)\,\tau_o(t) + d(t)) + \frac{1}{2}Tr[\left((L+B)\otimes I_m \cdot K\right)^T(2ss^T + s^T sI)\left((L+B)\otimes I_m \cdot K\right)]. \qquad (47)$$

Substituting Eqs. (24) and (25) into Eq. (47), one can write:

$$LV \leq \|s\|^3\left[n_1\left(\rho_1\|\varepsilon_1\| + \rho_2\|\varepsilon_2\|\right) + 1\right] + s^T s s^T\left[(L+B)\otimes I_m \cdot (I - \alpha)\, M^{-1}(q)\, \tau(t) - \mathbf{1}\otimes M_o^{-1}(q_o)\, \tau_o(t) + \mu\varepsilon_2\right] + \tfrac{1}{2} l_r^3\, \|L + B\|^6 \rho_3^6. \quad (48)$$

Replacing τ given in Eq. (44) into (48) yields:

$$LV \leq \|s\|^3\left[n_1\left(\rho_1\|\varepsilon_1\| + \rho_2\|\varepsilon_2\|\right) + 1\right] + s^T s s^T\left[(L+B)\otimes I_m \cdot \mu\varepsilon_2 - \mathbf{1}\otimes M_o^{-1}(q_o)\, \tau_o(t)\right] + \tfrac{1}{2} l_r^3\, \|L + B\|^6 \rho_3^6 - s^T s s^T\left[(L+B)\otimes I_m \cdot (I-\alpha)(L+B)^{-1}\otimes I_m \cdot \frac{1}{1-\alpha_{max}}\gamma sgn\,(s)\right]. \quad (49)$$

The last term of Eq. (49) can be rewritten as:

$$-\frac{1}{1-\alpha_{max}} s^T s s^T[(L+B)\otimes I_m \cdot (I-\alpha)\gamma sgn\,(s)]$$
$$\leq -\frac{1}{1-\alpha_{max}}\gamma\lambda_{min}\left((L+B)\otimes I_m \cdot (I-\alpha)(L+B)^{-1}\otimes I_m\right)\|s\|^3$$
$$\leq -\frac{1}{1-\alpha_{max}}\gamma[1-\lambda_{max}\left((L+B)\otimes I_m \cdot \alpha(L+B)^{-1}\otimes I_m\right)]\|s\|^3$$
$$\leq -\frac{1}{1-\alpha_{max}}\gamma(1-\alpha_{max})\|s\|^3 \leq -\gamma\|s\|^3. \quad (50)$$

Substituting Eq. (50) into Eq. (49), the Eq. (49) can be changed as:

$$LV \leq -n_0 V^{\frac{3}{4}} + \tfrac{1}{2} l_r^3\, \|L + B\|^6 \rho_3^6. \quad (51)$$

Similarly to the proof of Theorem 1, it is guaranteed that all trajectories of the closed-loop system will be converge to the small bound near the origin in finite-time with mean-square sense, and with the specified time and region that have given in Eq. (35) and (36).

Remark 2. The stochastic term in this paper is assumed to have a constant upper bound. While to improve the system performance and reduce conservatism, the upper bound of the stochastic term could be assumed to be related to the states of the system, which in turn provides higher amount of information for designing the controller [29].

Remark 3. In order to prevent a reduction in the useful lifespan of the actuators, which occurs because of the sign function in the control signal, the $tanh$ function could be used, which however, would result in the increase of the convergence region in Eq. (36).

C. Simulation Results

In this section, the simulation results are presented to validate the successfulness of the proposed theorems. To this end, a robotic manipulator with two revolute joints is considered.

Fig. 1 illustrates the schematics of this robotic arm.

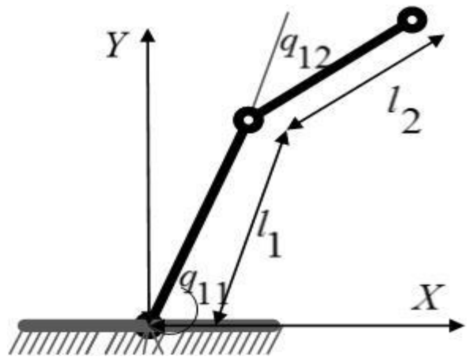

Fig. 1: Manipulator schematic with two revolute joints.

The kinematic of the robot and Jacobean vector are written as Eq. (2), where:

$$G_i(q) = \begin{bmatrix} G_{11} \\ G_{12} \end{bmatrix},\ M_i(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},$$
$$C_i(q,\dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},\ J_i(q) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \quad (52)$$

which is detailed in [30].

The dynamic of the agents can be expressed as Eq. (3) and (4).

The topology of the systems is shown in Fig. 2, which means $B = diag[\,1, 1\,, 0]$, and $L = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}$.

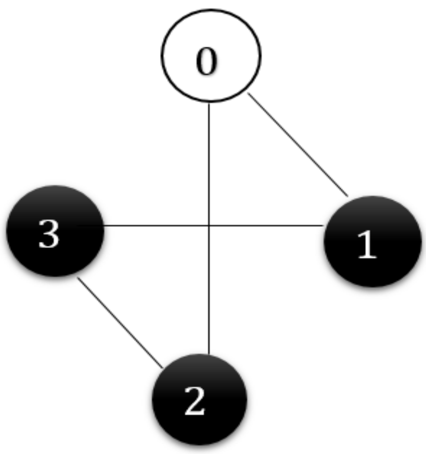

Fig. 2: The communication topology between manipulators.

The initial joint angles and angular velocities of the manipulators are given as:

$$q_1(0) = [20,\ -20]^T,\ q_2(0) = [40,\ -40]^T,$$
$$q_3(0) = [60,\ -60]^T,\ q_o(0) = [1,\ -1\ ]^T,$$
$$p_1(0) = [2,\ -2]^T\ ,\ p_2(0) = [4,\ -4]^T,$$
$$p_3(0) = [6,\ -6]^T\ ,\ p_o(0) = [2,\ -2\ ]^T, \qquad (53)$$

Let $\tau_o = 100\,sint$, $\rho_1 = \rho_2 = \rho_3 = 3$, $\mu = 0.7$, $m_1 = 0.1\,kg$, $m_2 = 0.75\,kg$, $l_1 = 0.3\,m$, $l_2 = 0.3\,m$, $I_1 = 0.06\,kgm^2$, $I_2 = 0.02\,kgm^2$, and $n_0 = 50$. The multiplicative faults of actuator are $a_1 = (0.5 - 0.5e^{-0.2t})eye(2)$, $a_2 = (0.4 - 0.4e^{-0.2t})eye(2)$, and $a_3 = (0.3 - 0.3e^{-0.1t})eye(2)$. Moreover, the bias fault of actuator is $d_i(t) = 10\,[u(t-2) - u(t-3)]ones(2)$, for $i = 1, \dots, 3$. Fig. 3 to Fig. 6 show the results derived by applying the fault tolerant consensus controller defined in Eq. (44), to the robot manipulators, when both the stochastic disturbance and actuator faults are present. The leader and followers' joint angles and angular velocities are shown in Fig. 3 and Fig. 4.

It is seen from these figures that the consensus has occurred between the agents in less than two seconds, which means that the follower manipulators follow the trajectory created by the leader manipulator. Moreover, the consensus of the robotic manipulators is robust to the actuator faults.

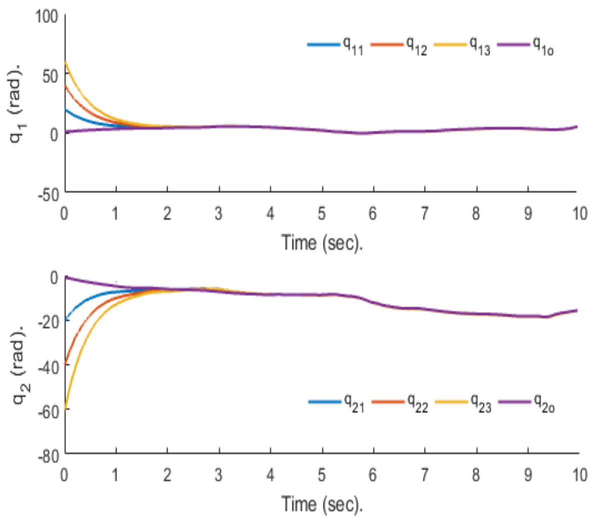

Fig. 3: The trajectories of the manipulators' positions.

The fast error convergence of the joint angles and angular velocities between the followers and the leader are demonstrated in Fig. 5 It is obvious that due to the greater difference in the position of the third manipulator from the leader compared to the other manipulators, its error would take longer to converge to zero.

Furthermore, in this figure, the reduction of disturbance effect on robotic manipulators is shown by magnification.

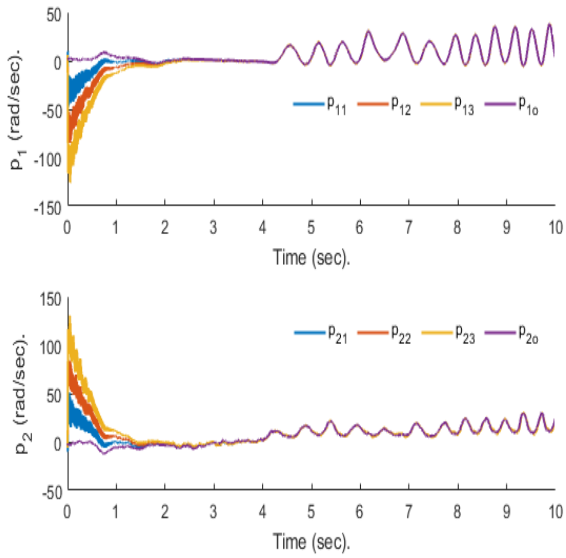

Fig. 4: The trajectories of the manipulators ' angular velocities.

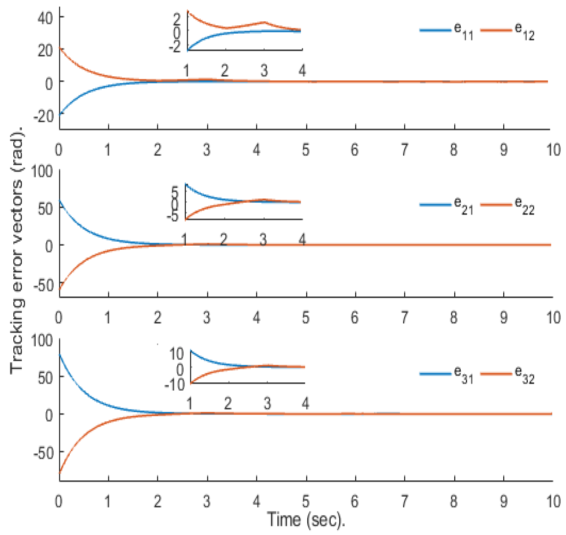

Fig. 5: The error vectors of joint angles and angular velocities for each follower.

In Fig. 6, all of the sliding trajectories corresponding to each robotic arm are illustrated and it is clear from this figure that they rapidly converge to zero, in less than 1 second.

Conclusion

In this study, the fault-tolerant finite-time consensus control of stochastic Euler-Lagrange MASs is investigated.

The proposed consensus controllers, both without actuator faults and with the actuator faults, are based on the sliding-mode approach. The proposed theorems guarantee that the consensus tracking errors are bounded in probability and after a finite-time remain in a desired area close to the origin in the mean-square senses.

The obtained theorems were applied to consensus control of the robotic manipulators to indicate the performance of the proposed controllers.

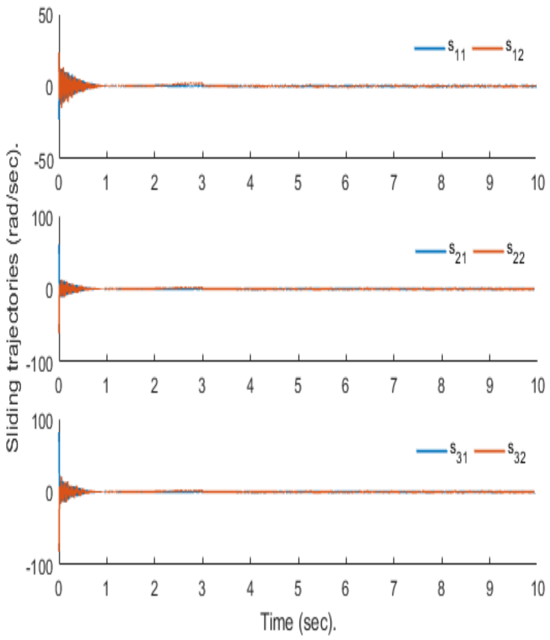

Fig. 6: The sliding trajectories of each follower.

Author Contributions

This paper is the result of M. Siavash PH.D thesis supervised and co-supervised by V. J. Majd and M. Tahmasebi, respectively.

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Conflict of Interest

There is no conflict of interests regarding the publication of this manuscript.

Abbreviations

$\otimes$	Kronecker product
$E[\cdot]$	Expectation value
$LV(\cdot)$	Infinitesimal generator
$\xi(t)$	Standard white noise
$w(t)$	standard Brownian motion
Eq.	Equation
$q, \dot{q}, \ddot{q}$	Position, angular velocity, and accretion vectors
$M(q)$	Mas matrix
$C(q, \dot{q})$	Centrifugal, Coriolis terms
$G(q)$	Gravity terms
$J^T(q)$	Nonsingular Jacobin matrix
Fig.	Figure

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