Research paper

On the Design of Coherent Zero-Forcing Receiver for the Flat Fading MIMO Multiple-Access Channels

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**Extended Abstract**

**Background and Objectives:** Design of low-complexity receiver for space-time block coded (STBC) transmission over multiple-input multiple-output (MIMO) multiple-access channels has been subject of interest over the years. In this regard, zero-forcing receiver, as a low complexity receiver needing as many receive antennas as the numbers of users, has received increasing attention.

**Methods:** This paper investigates multiuser detection for STBC transmission over a flat-fading MIMO multiple-access channel consisting of $J$ co-channel users each with $N$ antennas and a zero-forcing coherent receiver equipping with $M$ receiving antennas. For the cases in which $M = J$, it was previously claimed that it is impossible to extend this receiver to general scenarios of orthogonal STBC transmission with $J > 2$ and $N > 2$.

**Results:** We provide a theorem allowing this extension to any scenarios satisfying the theorem condition. Describing in more details, we first prove that zero-forcing receiver of $M = J$ antennas can successfully extend to any STBC transmission over MIMO multiple-access systems which provides an Alamouti-like structure for the inner product of equivalent channels between different receive antennas and users. Then, in order to gain more insight, the theorem role on extending zero-forcing receiver for transmission of orthogonal STBC over MIMO multiple-access systems with $J > 2$ and $N > 2$, and also to other STBC schemes like generalized coordinate interleaved orthogonal design and Quasi-orthogonal STBC is investigated in more details. Finally, the average symbol error rate of considered scenarios are numerically evaluated and compared for different STBC schemes with various numbers of $J$ and $N$.

**Conclusion:** Generally speaking, it is concluded that extension of zero-forcing receiver to any scenarios of OSTBC transmissions over MIMO multiple-access channels exactly depends on satisfaction of the provided theorem and this receiver can be successfully employed in all scenarios providing an Alamouti-like structure for the inner product of equivalent channels between different receiving antennas and users.
techniques have been proposed in literature and as an effective design, space-time block coding (STBC) has reached great scholarly attention [5, 6]. In parallel with this development, multiser detection for STBC-based MIMO systems has grown significantly [7]-[9]. In general, depending on the availability of channel state information (CSI), three main classes of multiser detection schemes are i) the scheme needing CSI at both transmitter and receiver terminals, ii) the case in which neither the transmitters nor the receivers needing CSI, and iii) coherent scheme where the CSI is only needed by the receiver terminals. The first two schemes fall beyond the scope of this paper, and research works of [10] and [11], [12] can be considered as their examples in a respective order. The research filed of this paper belongs to the latter category and is dedicated to design a low-complexity coherent detection method for STBC transmission over flat-fading MIMO multiple-access channels. In this regard, zero-forcing receiver, as a low complexity receiver needing as many receive antennas as the numbers of users, is employed and its application on STBC transmission over flat-fading MIMO multiple-access channels is investigated.

To the best of our knowledge, coherent detection of flat fading MIMO multiple-access systems has first addressed in [13] in which a single receiver of \( M = N J \) antennas are employed for detection of \( J \) co-channel users each with \( N \) transmit antennas. To reduce receiving antennas to \( M = J \), an interesting scheme based on STBC transmission at the transmitters and zero-forcing interference suppression at the receiver has attracted great attention in the literature. Receiver of this scheme, known as zero-forcing receiver, utilizes two-step detection method for interference suppression and separate detection of users. In [14], this receiver has been applied to a MIMO multiple-access system in which users having two transmit antennas and utilizing Alamouti design of [5]. In [15], by exploiting half-rate orthogonal STBC (OSTBC) structures given in [6], this receiver has also been extended to a two-user MIMO multiple-access system having \( N > 2 \) transmitting antennas. Finally, in [16], it was clarified that it is impossible to extend this receiver to a general scenario of complex OSTBC with \( J > 2 \) and \( N > 2 \), and Quasi-orthogonal STBC (QO-STBC) scheme has been employed as an alternative for increasing \( J \) and \( N \) to the values greater than two.

Furthermore, it is important to note that the STBC concept, which originally designed for the flat-fading environments, has found its way to frequency-selective multipath channels. As a result, many multiser detection approaches have been suggested for transmission of STBC designs over such channels, where the methods given in [17] and [18] are two examples investigating STBC transmission over frequency-selective MIMO multiple-access channels. However, the research filed of these works is fundamentally different from that one of current manuscript. Two main aspects of this difference are as: First, since signals of different antennas in frequency-selective channels are mixed in both space and time, a block-transmission anti-multipath technique should be utilized for physical layer of such channels. In this regard, the work of [17] is dedicated based on orthogonal frequency division multiplexing and single-carrier frequency-domain equalization approach, as the physical layer technique of LTE standards, has employed in [18]. Second, the original symbol-level STBC schemes must be redesigned according to utilized anti-multipath technique and extended to their block-level versions for multipath channels. This is while original STBC schemes are directly utilized in flat-fading channels using single-carrier modulations.

In this paper, we present a theorem and prove that development and extension of zero-forcing receiver to any STBC transmissions over arbitrary number of users exactly depends on the satisfaction of this theorem. More specifically, it is concluded that when theorem condition is satisfied for a specific two-user system of \( N \) transmitting antennas, then this satisfaction will also be guaranteed for increasing users more than two. Consequently, it is possible to successfully extend zero-forcing receiver to some general scenarios of STBC transmission with \( J > 2 \) and \( N > 2 \). Considering the provided theorem as the main contribution of the paper, its secondary results and modifications over previously published works can also be detailed as below:

- For a two-user MIMO multiple-access system utilizing Alamouti code, it is demonstrated that the provided theorem is satisfied and resultant zero-forcing receiver is similar to that one of [14].
- For a two-users MIMO multiple-access system transmitting half-rate OSTBC structures of [6], we will show that the provided theorem is satisfied only for OSTBC structures with \( N = 3 \) and \( N = 4 \) transmitting antennas, and failed for \( N > 4 \). This result is in contrary to that one of [15] which considered zero-forcing receiver for any numbers of transmitting antennas.
- In contrary to results of [16], it is demonstrated that there exist some examples for OSTBC transmission over MIMO multiple-access systems of \( J > 2 \) and \( N > 2 \) in which the theorem condition is satisfied and zero-forcing receiver works efficiently.
- Furthermore, it has also confirmed that the provided theorem will be satisfied for some other STBC structures such as QO-STBC [19] and generalized
coordinate interleaved orthogonal design (GCIOD)-STBC [20] schemes.

The rest of this paper is organized as follows: in Section II, the preliminaries include a brief review of OSTBC scheme and the system model are presented. Details of our main results are provided in Section III. Sections IV and V include the numerical results and conclusion, respectively.

**Notation:** Through this paper super-scripts \((\cdot)^{\dagger}\), \((\cdot)^{T}\), \((\cdot)^{*}\) and \((\cdot)^{H}\) indicate transpose, conjugate, Hermitian conjugate and inverse operations, respectively; non-boldface, boldface lower-case and boldface upper-case letters are used to denote scalar quantities, vectors and matrices, respectively; \(I_{n}\) and \(0_{n\times n}\) are \(Q\times Q\) Identity and \(Q\times Q\), all-zero matrices, respectively.

**Preliminaries**

This section provides a brief review to OSTBC concept and presents the signal model for the considered system.

**A. A Brief Review on STBC Concept**

In general, a linear STBC with rate of \(R = K / P\), \(G(s_{1},...,s_{K})\), is a \(P\times N\) matrix whose entries are complex linear combinations of \(K\) complex indeterminate symbols \(s_{i}\) and their complex conjugates, and are transmitted through \(N\) antennas during \(P\) symbol periods. Alamouti has proposed the first STBC design, as following two OSTBC structures [5]:

\[
G(s_{1},s_{2}) = \begin{bmatrix} s_{1} & s_{2} \\ -s_{2} & s_{1} \end{bmatrix}
\]

\[
G(s_{1},s_{2}) = \begin{bmatrix} s_{1} & s_{2} \\ s_{2} & -s_{1} \end{bmatrix}
\]

Note that through this paper an Alamouti-like matrix is also defined as a generalization of above structures, which obtained by substituting scalar quantities \(s_{i}\) and \(s_{2}\) with two different matrices, and \((\cdot)^{*}\) with \((\cdot)^{T}\).

The work of Alamouti on OSTBC design has paved the way for many subsequent study on different STBC schemes, where [6], [21]-[23] are generalized Alamouti work to higher order, [19] have introduced QO-STBC scheme for the first time, and [20] is the basic work of GCIOD-STBC structures. It is important to be noted that Alamouti structures are the only rate-one designs of complex OSTBCs and this rate is less than unity for any other designs of complex OSTBC with \(N > 2\). Furthermore, design of half-rate structures for complex OSTBC scheme has also addressed in [6] and these structures are available for any \(N\).

**B. System Model**

Block diagram of considered flat-fading MIMO multiple-access system has been illustrated in Fig. 1. As seen, this system includes \(J\) co-channel users each utilizing a \(P\times N\) STBC matrix for data transmission through its \(N\) antennas and a base station (receiver) equipping with \(J\) antennas.

The received signal of \(m^{th}\) receive antenna at \(i^{th}\) time index, i.e. \(r_{m}^{i}\), can be represented as:

\[
r_{m}^{i} = \sum_{j=1}^{J} \sum_{n=1}^{N} H_{n,j}^{m} g_{n}^{j,m} + n_{m}^{i}
\]

\(m = 1,\ldots,J; \quad i = 1,\ldots,P\)

in which \(H_{n,j}^{m}\) represents the channel gain between the \(m^{th}\) receive antenna and the \(n^{th}\) antennas of \(j^{th}\) user at \(i^{th}\) time index. As shown in Fig. 1, \(g_{n}^{j,m}\) denotes the transmitted symbol from the \(n^{th}\) antenna of \(j^{th}\) user at \(i^{th}\) time index. Finally, \(n_{m}^{i}\) represents noise sample of \(m^{th}\) receive antenna at \(i^{th}\) time index and models as a zero-mean, complex Gaussian random variable with variance of \(N_{o}\).

Considering Quasi-static fading model, i.e. \(H_{n,j}^{m} = H_{n,j}^{m}\) for \(i = 1,\ldots,P\), and rearranging the received symbols of (3) as \(r_{m} = \left[r_{m}^{1},\ldots,r_{m}^{I},(r_{m}^{1})^{\dagger},\ldots,(r_{m}^{I})^{\dagger}\right]^{T}\), the received block of \(m^{th}\) antenna can be formulated as follows:

\[
r_{m} = \sum_{j=1}^{J} H_{n,j}^{m} s_{j} + n_{m}^{i}, \quad m = 1,\ldots,M = J
\]

where \(r_{m}\) and \(n_{m}\) are the \(2P\)-length received and noise blocks of \(m^{th}\) antenna. \(H_{n,j}^{m}\) is a \(2P\times K\) matrix consisting of \(H_{n,j}^{m}\) and \(H_{n,j}^{m}\), and we defined it as the equivalent channel between the \(m^{th}\) receive antenna and the \(j^{th}\) user. As shown in Fig. 1, \(S_{j}\) represents the \(K\)-length data stream of \(j^{th}\) user. Detection method of this signal model is detailed in the next section.

**Main Results**

In order to avoid long computations and provide linear complexity, zero-forcing receiver as a low-complexity two-step detection method, has been employed for detection of users information from \(r_{m}\).

Note that these steps are interference suppression and separate detection of users [14]-[16]. In the sequel, in order to provide better understanding, this receiver is first employed for transmission of OSTBC structures over
two-user MIMO multiple-access systems and then will be extended to more users and other STBCs as well.

A. Receiver Design for OSTBC transmission over Two-User MIMO multiple-access systems

Using (4), the received blocks for OSTBC transmission over a two-user MIMO multiple-access system can be arranged as below:

\[
\begin{bmatrix}
\mathbf{r}_1 \\
\mathbf{r}_2
\end{bmatrix} =
\begin{bmatrix}
\mathbf{H}_{11} & \mathbf{H}_{12} \\
\mathbf{H}_{21} & \mathbf{H}_{22}
\end{bmatrix}
\begin{bmatrix}
\mathbf{s}_1 \\
\mathbf{s}_2
\end{bmatrix} +
\begin{bmatrix}
\mathbf{n}_1 \\
\mathbf{n}_2
\end{bmatrix}
\]  

(5)

As outlined earlier, \(\mathbf{r}_m\) and \(\mathbf{n}_m\) represent received and noise vectors of the \(m^{th}\) antenna, respectively. \(\mathbf{s}_j\) is the information stream of the \(j^{th}\) user. \(\mathbf{H}_{mj}\), \(m, j = 1, 2\) are the equivalent channels between the two receive antennas and the two users. Note that due to transmission of OSTBC design, all \(\mathbf{H}_{mj}\) have orthogonal structure, and all \(\mathbf{H}_m^\dagger\mathbf{H}_n\) are diagonal matrices with equal elements on main diagonal.

As the first step of detection, co-channel interference is suppressed by multiplying following interference cancellation matrix with (5):

\[
\mathbf{W} =
\begin{bmatrix}
\mathbf{W}_{11} & \mathbf{W}_{12} \\
\mathbf{W}_{21} & \mathbf{W}_{22}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{I}_{2p} & -\mathbf{H}_{12} (\mathbf{H}_{22}^\dagger\mathbf{H}_{22})^{-1} \mathbf{H}_{22}^\dagger \\
-\mathbf{H}_{21} (\mathbf{H}_{11}^\dagger\mathbf{H}_{11})^{-1} \mathbf{H}_{11}^\dagger & \mathbf{I}_{2p}
\end{bmatrix}
\]  

(6)

where the resultant inter-user interference-free system is:

\[
\tilde{\mathbf{r}} = \mathbf{W}\mathbf{r} =
\begin{bmatrix}
\tilde{\mathbf{r}}_1 \\
\tilde{\mathbf{r}}_2
\end{bmatrix} =
\begin{bmatrix}
\mathbf{\Sigma}_1 & \mathbf{0}_{2p\times K} \\
\mathbf{\Sigma}_2
\end{bmatrix}
\begin{bmatrix}
\mathbf{s}_1 \\
\mathbf{s}_2
\end{bmatrix} +
\begin{bmatrix}
\tilde{\mathbf{n}}_1 \\
\tilde{\mathbf{n}}_2
\end{bmatrix}
\]  

(7)

in which \(\mathbf{\Sigma}_1 = \mathbf{H}_{11} + \mathbf{W}_{12}\mathbf{H}_{21}\) and \(\mathbf{\Sigma}_2 = \mathbf{H}_{22} + \mathbf{W}_{21}\mathbf{H}_{12}\) are two matrices of size \(2p \times K\) channel matrices. \(\tilde{\mathbf{n}}_1 = \mathbf{W}_{11}\mathbf{n}_1 + \mathbf{W}_{12}\mathbf{n}_2\) and \(\tilde{\mathbf{n}}_2 = \mathbf{W}_{21}\mathbf{n}_1 + \mathbf{W}_{22}\mathbf{n}_2\) are the noise vectors of resultant interference-free sub-systems \(\tilde{\mathbf{r}}_1\) and \(\tilde{\mathbf{r}}_2\), respectively.

Structure of \(\mathbf{\Sigma}_j\) plays dominating role in second phase of detection. Indeed, to enable decoding of \(\mathbf{s}_j\) from \(\tilde{\mathbf{r}}_j = \mathbf{\Sigma}_j\mathbf{s}_j + \tilde{\mathbf{n}}_j\) without any intra-user interferences, it is necessary that each \(\mathbf{\Sigma}_j\), \(j = 1, 2\) has orthogonal structure (or \(\mathbf{\Sigma}_j^\dagger\mathbf{\Sigma}_j\) be a diagonal matrix). The conditions that guarantee orthogonal structure for each \(\mathbf{\Sigma}_j\) are presented below in Theorem 1. Assuming these conditions are met, the second stage of detection can be simply done using \(\tilde{\mathbf{r}}_j = \mathbf{\Sigma}_j^\dagger\tilde{\mathbf{r}}_j\) and MMSE equalization of \(\tilde{\mathbf{r}}_j\).

Theorem 1: If inner product of the equivalent channels between different receive antennas and users, i.e. \(\mathbf{H}_{mj}^\dagger\mathbf{H}_{mj}\), for any \(m_1, m_2, j_1\) and \(j_2\), has Alamouti-like structure, then each \(\mathbf{\Sigma}_j\) is an orthogonal matrix and \(\mathbf{\Sigma}_j^\dagger\mathbf{\Sigma}_j\) is diagonal.

Remark 1: Notes we outlined earlier just after (5) that \(\mathbf{H}_{mj}^\dagger\mathbf{H}_{mj}\) is diagonal matrix for any \(m_1 = m_2\) and \(j_1 = j_2\). Since diagonal matrices can be considered as a
special form of Alamouti-like matrices, Theorem 1 is generally presented for any \( m_s, m_t, j_1 \) and \( j_2 \).

**Proof:** To prove Alamouti-like structure of \( H_{m_s}^t H_{m_t}^s \), leading to diagonality of \( \Sigma_s^t \Sigma_r \), this matrix is first decomposed into three distinct terms as below:

\[
\Sigma_s^t \Sigma_r = H_{11}^t H_{11} + (W_{12}^t H_{21}) W_{12}^t H_{21} + \left[ H_{11}^t (W_{12}^t H_{21}) \right] H_{11}^t H_{21}
\]

(8)

As pointed out earlier just after equation (5), the first term, i.e. \( H_{11}^t H_{11} \), is a diagonal matrix with equal elements on main diagonal.

Diagonality of two remaining terms can also be resulted from interesting properties of Alamouti-like matrices. Indeed, if \( A \) indicates an Alamouti-like matrix, then \( A^t A \) and \( A^t + A \) will be two diagonal matrices. In addition, the product of two different square Alamouti-like matrices leading to an Alamouti-like matrix as well [15], [24], [25].

Now, to prove diagonality of second term given in (8), the value of \( W_{12} \) from (6) is first substituted into

\[
(W_{12} H_{21})
\]

where the result is as follows:

\[
W_{12} H_{21} = -H_{12} (H_{12}^t H_{22})^{-1} H_{22}^t H_{21}
\]

(9)

where \( \alpha \) is due to the fact that \( H_{12} H_{22}^t \) is a diagonal matrix with equal elements on main diagonal. Using (9), diagonality of second term of (8) can be concluded as below:

\[
(W_{12} H_{21}) W_{12} H_{21} = \alpha_1^2 \left( H_{12} H_{22}^t H_{12} H_{22}^t H_{12} \right) H_{21}
\]

\[
= \alpha_1^2 \left( H_{12} H_{22}^t \right)^2 H_{21}
\]

(8)

where all \( \alpha \) are constant numbers, and \( \alpha \) is due to the fact that \( A^t A \) is diagonal for any Alamouti-like matrix of \( A \).

Finally, to prove diagonality of last term of (8), \( W_{12} H_{21} \) from (9) is substituted into \( H_{11}^t (W_{12} H_{21}) \) as below:

\[
H_{11}^t (W_{12} H_{21}) = H_{11}^t (-\alpha_1 H_{12} H_{22} H_{21})
\]

\[
= -\alpha_1 \left( H_{11}^t H_{12} \right) \left( H_{22}^t H_{21} \right)
\]

The above structure reveals that \( H_{11}^t (W_{12} H_{21}) \) has also Alamouti-like structure. This is because all \( H_{m_s}^t H_{m_t}^s \) are Alamouti-like matrices of size \( K \times K \), and product of two different square Alamouti-like matrices is an Alamouti-like matrix as well. Thereby, diagonality of last term of (8) can be easily concluded from the fact that \( A^t + A \) is diagonal for any Alamouti-like matrices like \( A = H_{11}^t (W_{12} H_{21}) \). This proof is also valid for \( \Sigma_r \).

In the sequel, the dominating role of Theorem 1 on application of zero-forcing receiver over two-user MIMO multiple-access systems is more clarified using two different corollaries for transmission of OSTBC designs.

**Corollary 1:** For a two-user MIMO multiple-access system utilizing Alamouti structure of (1) for the first user and (2) for the second one, the input/output model is as:

\[
\begin{bmatrix}
    r_1
    \\
    \vdots
    \\
    r_s
\end{bmatrix}
= \begin{bmatrix}
    s_1
    \\
    \vdots
    \\
    s_r
\end{bmatrix} + \begin{bmatrix}
    n_1
    \\
    \vdots
    \\
    n_s
\end{bmatrix}
\]

(10)

As outlined earlier, \( r_m \) and \( n_m \) represent received and noise blocks of \( m \)th antenna. \( H_{mj} \) is the equivalent channel between the \( m \)th receive antenna and \( j \)th user. \( s_j = [s_{j1}, s_{j2}] \) indicates the data stream of \( j \)th user.

As can be seen, all equivalent channels of (10) have Alamouti structures and these structures will also maintain for their inner products as \( H_{m_t}^t H_{m_t}^s \). In a similar manner, these structures are also valid for all \( H_{m_t}^t H_{m_t}^s \) of the case in which the users employing the same structure of Alamouti design. Therefore, theorem 1 will be satisfied for transmission of same or different structures of Alamouti designs over two-user MIMO multiple-access systems and zero-forcing receiver can successfully detect these structures as also numerically illustrated in Fig. 2.

**Corollary 2:** The structure of equivalent channels for a two-user MIMO multiple-access system in which users are equipped with four transmitting antennas and utilize half-rate OSTBC structures of (11) is detailed in (12):
\[ G(s_{1}, \ldots, s_{4}) = \begin{bmatrix} s_{1} & s_{2} & s_{3} & s_{4} \\
-s_{1} & s_{2} & -s_{3} & s_{4} \\
-s_{1} & s_{2} & s_{3} & -s_{4} \\
-s_{1} & -s_{2} & s_{3} & s_{4} \end{bmatrix} \]

(11)

\[ H_{m_{1}} = \begin{bmatrix} H_{m_{1}} & H_{m_{2}} & H_{m_{3}} & H_{m_{4}} \\
H_{m_{1}} & -H_{m_{2}} & H_{m_{3}} & -H_{m_{4}} \\
H_{m_{1}} & H_{m_{2}} & -H_{m_{3}} & H_{m_{4}} \\
H_{m_{1}} & -H_{m_{2}} & -H_{m_{3}} & -H_{m_{4}} \end{bmatrix} \]

(12)

According to (12), inner product of equivalent channels has following Alamouti-like structure:

\[ H'_{m_{1}}H_{m_{2}} = \begin{bmatrix} a_{1} & a_{2} & \cdots & a_{1} & a_{2} \\
-a_{1} & a_{2} & \cdots & -a_{1} & a_{2} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-a_{1} & -a_{2} & \cdots & a_{1} & a_{2} \end{bmatrix} \begin{bmatrix} s_{1} \\
\bar{s}_{1} \\
\vdots \\
\bar{s}_{1} \end{bmatrix} \]

(13)

in which \( a_{i} \) are real quantities and \( s_{i} \) represents a \( 2 \times 2 \) matrix.

Furthermore, by omitting the forth column of (11), a half-rate OSTBC structure for three transmit antennas is obtained. Now, if the two considered users exploits this new half-rate design, the resulting equivalent channels are obtained from (12) by substituting \( H_{m_{4}} \) with zero, and inner product of these channels is as (13). Consequently, it is concluded that Theorem 1 is satisfied for these two cases and zero-forcing receiver can successfully work for two-user MIMO multiple-access systems utilizing half-rate OSTBC designs of three and four antennas.

It is worth to be noted that the half-rate design of (11) is not unique and similar to Alamouti code, we can consider some other structures for this design. However, when different structure of this design are utilized by assumed two-user system, the theorem is not satisfied and Alamouti-like structure of (13) will destroy for inner product of equivalent channel.

More importantly, the theorem is not satisfied for the case in which the users equipping with \( N > 4 \) transmitting antennas and utilizing same or different structures of half-rate OSTBC of [6]. This is because this scenario does not provide an Alamouti-like structure for \( H'_{m_{1}}H_{m_{2}} \), and as evidence from Fig. 4, zero-forcing receiver will fail in this scenario due to existence of intra-user interference. To come with conclusion, when a two-user MIMO multiple-access system utilizing half-rate OSTBC designs of [6], Theorem 1 is only satisfied for transmission of same structure of half-rate design having \( N = 3 \) and \( N = 4 \) antennas. This results is in contrast to that one of [15] in which this limitation on the numbers of transmit antennas has not been taken into account and zero-forcing receiver has generaly considered for any \( N \).

B. Theorem Extension to More Users and Other OSTBC Schemes

In two previous corollaries, the dominating role of Theorem 1 on application of zero-forcing receiver for OSTBC transmission over two-user MIMO multiple-access systems has been addressed. Here, we generalize these corollaries and investigate extension of the receiver for transmission of any STBC designs over an arbitrary number of co-channel users.

Generally speaking, similar to previous corollaries, this extension entirely depends on satisfaction of Theorem 1 regarding to structure of inequivalent channels, and zero-forcing receiver of \( M = J \) antennas can successfully detect any STBC transmissions of MIMO multiple-access systems providing an Alamouti-like structure for the inner product of equivalent channels between different receive antennas and users. Next two corollaries are examples of this extension for transmission of OSTBC structures over general scenarios of \( J > 2 \) and \( N > 2 \), and also transmission of other STBC schemes.

Corollary 3: As outlined earlier in introduction, it is claimed by [16] that is impossible to extend zero-forcing receiver to a general scenario of complex OSTBC with \( J > 2 \) and \( N > 2 \). This corollary briefly investigates application of zero-forcing receiver to general scenarios of OSTBC designs and provides an example contradicting this claim.

In this regard, it is worth to be noted that when theorem condition is satisfied for a specific two-user system of \( N \) transmitting antennas, then this satisfaction will also be guaranteed for increasing users more than two. Therefore, it is concluded that it is possible to increase number of users more than two for any scenarios of previous corollaries satisfying Theorem 1. As a result, zero-forcing receiver can successfully decode any number of users which are equipped with i) two transmitting antennas and transmit same or different structures of Alamouti design ii) three or four transmitting antennas and transmit the same structure...
of half-rate OSTBC of [6]. As can be seen, the case ii) indicates transmission of OSTBC structures over MIMO multiple-access systems of $J > 2$ and $N > 2$, and is in contrary to claim of [16] in this regard. Note that this issue has also addressed numerically in Fig. 3.

**Corollary 4:** In previous corollaries, the dominating role of Theorem 1 for transmission of OSTBC designs over MIMO multiple-access channels is addressed. This corollary investigates this issue for two other STBC schemes including QO-STBC and GCIOD-STBC schemes.

Frist, we imply to the detection method employed by [16] for transmission of QO-STBC scheme over the MIMO multiple-access system. In fact, the successful function of this method is directly in connection with the utilized QO-STBC structure. This structure is given in equation (5) of [19]. Since this structure has constructed based on Alamouti structures, it provides Alamouti-like structure for inner product of equivalent channels and satisfying theorem 1 as evidence from Fig. 3.

GCIOD-STBC scheme is another category of STBC techniques, where generated based on OSTBC designs. Thereby, some structures of GCIOD-STBC may satisfy Theorem 1, where the full-rate designs given in (40) and (41) of [20] are two examples which employed for data transmission over MIMO multiple-access systems in Fig. 2 and Fig. 3 of the next section.

**Simulation Results and Discussion**

This section provides simulation results for different scenarios of STBC transmission over flat fading MIMO multiple-access channels. In these simulations, average Symbol Error Rate (SER) of zero-forcing receiver has been illustrated versus average Signal to Noise Ratio (SNR) per each receive antenna, and the results are presented for a single-user system having one and two receive antennas, a two-user system with two-receive antennas and a three-user system of three-receive antennas. Similar to [16], [26]-[27], to fairly compare SER of different STBC schemes, the modulation order of have chosen so that leading to the same transmission rate per hertz, i.e. [bit/sec/Hz], for all considered cases. Furthermore, due to Quasi-statistic fading model, the channel gains are considered invariant during each STBC transmission and modeled as zero-mean, complex Gaussian random variables of unit variance.

In Fig. 2, number of transmitting antennas has been set to $N = 2$ and SER of zero-forcing receiver has been compared for transmission of two different STBC designs. These are Alamouti design of (1) and rate-one GCIOD-STBC structure given in equation (40) of [20]. In Fig. 3, number of transmitting antennas is equal to $N = 4$ and the SER of receiver has been compared for transmission of three different STBC designs. These are half-rate OSTBC design of (11), rate-one QO-STBC structure given in equation (5) of [19] and rate-one GCIOD-STBC given in equation (41) of [20]. To guarantee transmission rate of $1$ [bit/sec/Hz], 2-PSK constellation has been chosen for both rate-one designs of Fig. 2; however, in Fig. 3, 4-PSK is employed for the half-rate OSTBC design and 2-PSK for both rate-one structures of QO-STBC and GCIOD-STBC. As can be seen from Fig. 2 and Fig. 3, the results confirm the issues discussed in corollaries 1, 2, 3 and 4 on satisfaction of Theorem 1 for STBC transmission over MIMO multiple-access systems of $N = 2$ and $N = 4$ transmitting antennas. As demonstrated, due to satisfaction of Theorem 1, zero-forcing receiver functions well for $J \geq 2$ users and detects information with small amount of SER. Indeed, due to success in cancellation of inter-user interference, the SER performances of both two-user and three-user systems are equal to their single-user counterpart with one receive antenna. Furthermore, as expected, due to having more transmit diversity gain, OSTBC scheme performs better than that of QO-STBC and GCIOD-STBC ones. In a similar way, as can be seen, increasing the number of receive antennas provides more receive diversity gain and leads to a significant improvement in SER performance of single-user scenario of Fig. 2 as well.

![Fig. 2: SER comparison of receiver for transmissions of OSTBC and GCIOD-STBC schemes over different scenarios having $N = 2$ transmitting antennas.](image1)

![Fig. 3: SER comparison of receiver for transmissions of OSTBC, QO-STBC and GCIOD-STBC schemes over different scenarios having $N = 4$ transmitting antennas.](image2)
To verify the discussion of corollary 2 regarding limitation on number of transmit antennas, SER performances of zero-forcing receiver for transmission of two half-rate OSTBC designs of $N = 5$ and $N = 8$ are illustrated in Fig. 4. As seen, while the receiver works well for both cases in single-user scenarios, it has been failed for multiuser scenarios of two users. As outlined earlier in corollary 2, this result is due to the fact that the theorem condition will only be satisfied for transmission of half-rate designs of $N = 3$ and $N = 4$ transmit antennas, and it is impossible to extend zero-forcing receiver for multiuser scenarios of $N > 4$.

In order to investigate the receiver performance for a more complex constellation, the modulation type of Fig. 1 has been changed from 2-PSK into 16-QAM and the SER performance of Alamouti scheme has been illustrated in Fig. 5. As can be seen, similar to Fig. 1, zero-forcing receiver functions well in this situation. However, as an expected result, utilization of 16-QAM, as a more complex modulation, has increased the error rate of Fig. 5 compared with that one of Fig. 1.

Finally, it is worth to be noted that all above simulations have been performed base on assumption of perfect estimation of channel. Here, impact of channel estimation error on the performance of suggested receiver is numerically investigated. Similar to [28], this error is modeled as $\hat{H} = H + \tau \phi$, where $\hat{H}$ and $H$ represent the estimated and actual channel gains, respectively. Parameter $\phi$ is a zero-mean, complex Gaussian random variable of unit variance, and $\tau$ is a constant number which measures the accuracy of channel estimation. In Fig. 6, we choose two values of $\tau = 0.15$ and $\tau = 0.2$, i.e. considering channel estimation accuracy equal to 85% and 80%, and compare the results with that one of perfect estimation, i.e. $\tau = 0$. In Fig. 6, the SER performance for the Alamouti design of Fig. 1 has been compared for these three values of channel estimation accuracy. Note that this comparison has been performed for three different scenarios: a single-user system having two receive antennas, a two-user system with two receive antennas and a three-user system of three receive antennas. As can be seen, the effect of channel estimation error on single-user system is negligible. However, similar to that one of [28], this error will be noticeable for multiuser scenarios and the more accuracy of channel estimation is decreased, the more symbol errors is caused. It is also important to be noted that performance degradation due to this error is different for other scenarios depending on assumed system, utilized STBC design and modulation type.

Conclusion

In this paper, application of zero-forcing receiver for coherent detection of different STBC over flat-fading MIMO multiple-access systems is investigated. For $J$ co-channel users each with $N$ antennas, it was proved
that the zero-forcing receiver of $M = J$ antennas can successfully detects any STBC transmissions in which the inner product of equivalent channels between different receive antennas and users is an Alamouti-like matrix. This proof is more clarified via four different corollaries highlighting its main results and modifications with respect to previous works. Finally, SER performance of all considered scenarios are also numerically illustrated and compared from many different aspects, where the results highlight the dominating role of the provided theorem on application of zero-forcing receiver in all considered scenarios.

**Author Contributions**
M. Sheikh-Hosseini designed and analyzed all parts of the manuscript.

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**Conflict of Interest**
The author declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by author.

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<tr>
<td>STBC</td>
<td>Space-Time Block Code/Coding</td>
</tr>
<tr>
<td>OSTBC</td>
<td>Orthogonal STBC</td>
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<tr>
<td>QO-STBC</td>
<td>Quasi-Orthogonal STBC</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<td>PSK</td>
<td>Phase Shift Keying</td>
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<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<td>LTE</td>
<td>Long term Evolution</td>
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<td>GCID-STBC</td>
<td>Generalized Coordinate Interleaved STBC</td>
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**References**


Biographies

Mohsen Sheikh-Hosseini was born in Iran in 1983. He received the B.S. degree in electrical engineering from Shahid Bahonar University of Kerman, Kerman, Iran in 2007, and then he received the M.S. and Ph.D. degrees both in telecommunications field of electrical engineering from Ferdowsi University of Mashhad, Mashhad, Iran in 2009 and 2014, respectively. He is currently with the Department of Computer and Information Technology, Institute of Science and High Technology and Environmental Sciences, Graduate University of Advanced Technology, Kerman, Iran. His research interests include wireless communications, power line communications, smart grids, and machine learning applications in telecommunications.