



Research paper

## Interference Alignment Using Difference of Convex-based Beamformer Design for MIMO Interference Channels

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### Abstract

**Background and Objectives:** To achieve significant throughput, interference alignment (IA) is an encouraging technique for wireless interference networks. In this study, we design an aligned beamformer based on the interference leakage minimization (ILM) method to reduce the interference power for a multiple-input multiple-output interference channel (MIMO-IC).

**Methods:** To deal with the non-convexity of ILM problem, we used a non-convex programming method (i.e., difference of convex [DC]). In this way, the interference leakage function is reformulated to a DC function including difference of two convex terms. Then, an additive function is defined that includes the DC objective function and a penalty function.

**Results:** We propose a novel DC-based IA algorithm that uses solutions of an upper bound of the additive function in each iteration; as the initial state for the next iteration. Through an iterative manner and for the large values of the penalty factor, the solutions of upper bound function converge to the solutions of the original DC objective function (i.e., interference leakage function).

**Conclusion:** In contrast to the frequent IA methods, the proposed DC-based IA algorithm updates transmit- and receive-beamformers in each iteration jointly (not alternately). Simulation results indicate that the proposed method outperforms some competitive IA algorithms by providing more throughputs and less interference leakage.

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### Introduction

The emergence of interference alignment in interference channels (ICs) attracts the researchers' interest [1]-[4] especially for the multiple-input multiple-output (MIMO) systems to achieve high spectral efficiency and improvement of the throughput [5]-[7]. By designing appropriate aligned transmit- and receive-beamformers, the number of interference-free signaling dimensions at high signal-to-noise (SNR) regimes, namely degree of freedom (DoF), is increased. To achieve the maximum possible of DoF, the beamforming strategies help IA techniques to increase the power and spectral

efficiencies. Variety of IA beamformer design methods are focused on the interference leakage minimization (ILM), i.e. reducing the sum of interference powers at the receivers. Also, some of these methods are focused on reducing the total rank of interference matrices [8]-[15]. For example, Gomadam et al. [8] proposed an IA algorithm, namely weighted leakage interference (WLI) that was concentrated on ILM methods. They presented a distributed numerical approach for minimizing the sum of interference powers at the receivers. They compared the proposed method with three schemes: a) orthogonal, b) simultaneous transmission, and c) selfish

interference avoidance. In this way, they showed the benefits of distributed IA algorithm. Implicitly concerned with ILM optimization problem, two adaptive algorithms based on least squares (LS) and minimum mean square error (MMSE) methods were proposed in [12] which both methods outperform WLI method under perfect channel state information (CSI). Also, it was shown that MMSE-based IA method outperforms the proposed distributed maximum signal-to-interference-plus-noise ratio (Max-SINR) method in [9], which is a special case of the weighted MMSE-based IA under the perfect CSI. Also in [13] two robust and non-robust IA approaches based on both MMSE and zero-forcing (ZF) criteria were proposed to deal with the interference in perfect and imperfect CSI cases. The proposed framework in [11] exploited relation between the algebraic independence and the feasibility of polynomial equation set to transform ILM problem to a polynomial form. To solve the IA polynomial optimization problem, an iterative IA algorithm was proposed that did not need to symbol extension over frequency- or time-space. However, that algorithm updates the transceiver beamformers alternately; so, it cannot ensure that all interference is eliminated. As compared to the methods which are concentrated on ILM and the reduction of sum of interference powers at the receivers, an algebraic alternative of IA has been introduced as a rank constrained rank minimization (RCRM) problem in [10] by minimizing the rank of the interference matrix subject to the full-rank affine constraints. Also, a suitable convex surrogate, namely nuclear norm, has been chosen to find the closed-form solutions (as the aligned beamformers). To provide the desired level of sparsity in [10] the introduced approaches in [14] namely reweighted nuclear norm minimization (RNNM) and reweighted Frobenius norm minimization (RFNM), iteratively minimize a series of weighted nuclear norms of the interference matrices instead of their nuclear norms. In this way, Mollaebrahim *et al.* [7] proposed a rank minimization method to obtain higher multiplexing gain (and to enhance IA), as well. For this purpose, they introduced a new class of convex relaxation to obtain lower rank solutions by expanding the feasibility set. Their proposed method obtained higher multiplexing gain and sum-rate as compared to Max-SINR [9] and ILM [1] approaches. Peter and Heath [1] proposed an IA algorithm in MIMO-IC with an arbitrary number of users, distribution antennas, and spatial streams. Their proposed algorithm was an alternating minimization over the precoding matrices at the TXs and the interference subspaces at the RXs. In this paper, we propose an IA algorithm to design aligned transmit- and receive-beamformers to reduce the interference power based on the defined mathematical framework for ILM

optimization problem in [11]. Noticeably, the algebraic feasibility conditions of IA were presented in the previous works [16], [17]; so, this work just concentrates on designing optimized aligned beamformers. Also, most of the mentioned iterative IA algorithms update beamformers alternately. In other words, through an iterative manner, first the transmit-beamformers are assumed fixed and the receive-beamformers are sought and then vice versa. They minimize the interference leakage at the receiver-side and then do the same at the transmitter-side. In this way, the assurance of inference elimination is so weak. Our proposed iterative algorithm compensates this flaw by determining the beamformer matrices simultaneously and jointly. To face the non-convex nature of IA problem, we present a convex surrogate of the objective function (i.e., interference leakage function) based on a non-convex programming method namely difference of convex (DC) programming [18]. The DC programming method demonstrates a non-convex function with a series of convex terms. Among several studies on the interference channels, only Tam *et al.* [19] designed precoding matrices for the maximum sum-information-rate subject to individual TX power constraints for MIMO interference channel. They transformed the maximum sum-rate problem (as a nonlinear non-convex problem) into an equivalent DC program. They compared the sum-rate performance of their method with Max-SINR and leakage minimization algorithms [2], [9] both published in 2011. In our work, the power of interference (as the cost function) is expressed in the form of the difference of two convex functions, while the sum-rate is represented as the difference of two convex functions in [19]. In addition, the constraints of the optimization problem are also represented in DC form beside the cost function in our work. We reformulate the interference leakage function to a difference of two convex DC components. In the first step of designing an algorithm to solve IA problem, an additive function is defined that includes the DC objective function and a penalty function. We define an upper bound for the nonlinear additive function and approximate it by the sub-gradient method linearly. Through an iterative manner, the solution of the upper bound function can be chosen as the initial state for the next iteration and then it converges to the solution of the original DC objective function for large values of the penalty parameter. This novel method is based on some mathematical reformulations used to turn our non-convex problem to a difference of convex problems which has not been introduced in other published researches in IA area (based on the knowledge of authors). The performance of the proposed method is acceptable as compared to some competitive methods, as well. As another aspect of the innovation in this

research, it is noted that most of the previous iterative IA algorithms [10]-[15] update the beamformers alternately. Therefore, they minimize the interference leakage at the receiver-side and then at the transmitter-side; so, the assurance of interference elimination is so weak. To compensate this flaw, our proposed iterative algorithm attempts to determine the beamformer matrices simultaneously and jointly. By this, it ensures that the interference leakage can be minimized almost completely. For the system performance analysis, we compare our proposed DC-based IA method to the non-robust IA design approach introduced in [13] and the ZF-based IA algorithm proposed in [20]. Since the papers that have been published in recent years have more focus on metrics such as bit error rate (BER), SINR, and interference leakage [13], [14], [20]-[22], we concentrate on these metrics as well. So, the evaluation of system performance in this study is presented in terms of BER, SINR, and interference leakage. The simulation experiments indicate that the proposed method outperforms some competitive IA algorithms by providing less BER and less interference leakage. This paper is organized as follows. We demonstrate the system model in Section 2. The DC-based reformulation of IA optimization problem is presented in Section 3. The proposed DC-based IA algorithm is developed in Section 4. The sensitivity formulation is presented in Section 5. The simulation results and performance comparison of the proposed algorithm and some previous works are presented in Section 6. The conclusion is provided in Section 7.

### System Model

The considered model is a K-user MIMO interference wireless network. It is assumed that each transmitter is equipped with  $M$  antennas and each receiver is equipped with  $N$  antennas (Fig. 1). For simplicity, the number of antennas is the same at all transmitters and receivers. However, the results can be easily carried to the case of different number of antennas at the transceivers. A number of  $d_k$  independent streams are sent to the  $k$ -th receiver from the  $k$ -th transmitter. It can be interpreted as the multiplexing gain desired by each transmitter-receiver pair or the number of free-interference signal space dimensions. The channel estimators at all transmitters and receivers monitor the channels consistently; so, CSI is available at both sides. Each user aims to communicate a symbol vector  $\mathbf{x}_k \in \mathbb{C}^{d_k \times 1}$  to its desired receiver.  $\mathbb{C}$  denotes the set of complex numbers. Before transmitting, pre-coding matrix  $\mathbf{V}_k \in \mathbb{C}^{M \times d_k}$  (with  $d_k$  linearly independent

columns) pre-codes each user symbol vector as  $\mathbf{e}_k \triangleq \mathbf{V}_k \mathbf{x}_k$  where  $\mathbf{e}_k \in \mathbb{C}^{M \times 1}$  and  $E\{\mathbf{e}_k \mathbf{e}_k^H\} = P\mathbf{I}$  where  $P$  is the transmit power for each symbol vector and  $\mathbf{I}$  is the identity matrix. Uniform power allocation is considered throughout this study for all users.

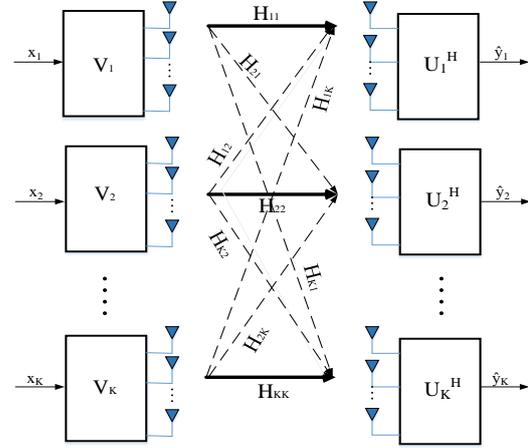


Fig. 1: A K-user MIMO interference channels system.

The received signal at the  $k$ -th receiver is given by:

$$\mathbf{y}_k = \mathbf{H}_{kk} \mathbf{V}_k \mathbf{x}_k + \sum_{j=1, j \neq k}^K \mathbf{H}_{kj} \mathbf{V}_j \mathbf{x}_j + \mathbf{w}_k, \quad (1)$$

where  $\mathbf{H}_{kj} \in \mathbb{C}^{N \times M}$  represents the channel between the  $j$ -th transmitter and the  $k$ -th receiver and  $\mathbf{w}_k$  shows the zero-mean additive white Gaussian noise  $\sim N(0, \sigma^2 \mathbf{I}_N)$ .

By linearly processing of the received signal using linear post-processing matrix  $\mathbf{U}_k \in \mathbb{C}^{(N \times d_k)}$  at the  $k$ -th receiver, we obtain:

$$\mathbf{U}_k^H \mathbf{y}_k = \mathbf{U}_k^H \mathbf{H}_{kk} \mathbf{V}_k \mathbf{x}_k + \mathbf{U}_k^H \sum_{j=1, j \neq k}^K \mathbf{H}_{kj} \mathbf{V}_j \mathbf{x}_j + \mathbf{U}_k^H \mathbf{w}_k. \quad (2)$$

We use  $(\cdot)^H$  to denote Hermitian (conjugate transpose) of a matrix. It is considered that the perfect IA requirements to linear interference alignment are established as:

$$\sum_{j=1, j \neq k}^K \mathbf{U}_k^H \mathbf{H}_{kj} \mathbf{V}_j = \mathbf{0}; \quad \forall k \neq j, \quad (3)$$

$$\text{rank}(\mathbf{U}_k^H \mathbf{H}_{kk} \mathbf{V}_k) = d_k; \quad \forall k. \quad (4)$$

As there is no direct relation between the leakage minimization and condition stated in (4), this condition only focuses on the maximum of DoF; so, just the condition given in (3) is satisfied in the interference leakage reduction.

### Difference of Convex Transformation of IA Optimization Problem

In this section, we focus on ILM problem and take the interference leakage function as the objective function.

We show a convex relaxation of the non-convex objective function to the difference of two convex components. In [11], the proposed optimization problem (based on ILM) alternatively updates the transmit-beamformers  $\mathbf{V}_j$  and the receive-beamformers  $\mathbf{U}_k$  to reduce the power of interference as follows:

$$\begin{aligned} & \underset{\mathbf{V}_j, \mathbf{U}_k}{\text{minimize}} \sum_{k=1}^K \sum_{j=1, j \neq k}^K \left\| \mathbf{U}_k^H \mathbf{H}_{kj} \mathbf{V}_j \right\|_F^2, \\ & \text{subject to:} \\ & a) \mathbf{U}_k = \begin{bmatrix} \mathbf{I}_{d_k \times d_k} \\ \bar{\mathbf{U}}_k \end{bmatrix}, \quad \mathbf{V}_k = \begin{bmatrix} \mathbf{I}_{d_j \times d_j} \\ \bar{\mathbf{V}}_j \end{bmatrix}; \quad \forall k, j \in K, \end{aligned} \quad (5)$$

where  $\bar{\mathbf{V}}_j \in \mathbb{C}^{(M-d_j \times d_j)}$  and  $\bar{\mathbf{U}}_k \in \mathbb{C}^{(N-d_k \times d_k)}$ . The Frobenius norm is presented by  $\|\cdot\|_F$ .

The objective function in (5) is a non-convex function of the optimization variables  $\mathbf{V}_j$  and  $\mathbf{U}_k$ . The problem can be solved firstly based on  $\{\bar{\mathbf{V}}_j, \bar{\mathbf{U}}_k\}$  and then the transceivers  $\mathbf{V}_j$  and  $\mathbf{U}_k$  can be constructed via constraint a). So, we attempt to reformulate the relations in (5) as the difference of two DC components.

Using Frobenius norm definition of the matrix, we expand the objective function stated in (5) as  $\left\| \mathbf{U}_k^H \mathbf{H}_{kj} \mathbf{V}_j \right\|_F^2 = \text{Tr}(\mathbf{U}_k^H \mathbf{H}_{kj} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{kj}^H \mathbf{U}_k)$ , where  $\text{Tr}(\cdot)$  represents the trace of a matrix. Now, let us define  $b) \mathbf{X}_j = \mathbf{V}_j \mathbf{V}_j^H$ ;  $\mathbf{X}_j \in \mathbb{C}^{M \times M}$  and  $c) \mathbf{Y}_k = \mathbf{U}_k \mathbf{U}_k^H$ ;  $\mathbf{Y}_k \in \mathbb{C}^{N \times N}$  which these constraints are added to the optimization problem conditions beside the constraints in (5). Using the trace property for the positive semi-definite matrices, the relation in (5) can be rewritten as  $\text{Tr}(\mathbf{U}_k^H \mathbf{H}_{kj} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{kj}^H \mathbf{U}_k) = \text{Tr}(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j)$ . The new reformulation of problem in (5) is:

$$\begin{aligned} & \underset{\mathbf{X}_j, \mathbf{Y}_k}{\text{minimize}} \sum_{k=1}^K \sum_{j=1, j \neq k}^K \text{Tr}(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j), \\ & \text{subject to:} \\ & a) \mathbf{U}_k = \begin{bmatrix} \mathbf{I}_{d_k \times d_k} \\ \bar{\mathbf{U}}_k \end{bmatrix}, \quad \mathbf{V}_k = \begin{bmatrix} \mathbf{I}_{d_j \times d_j} \\ \bar{\mathbf{V}}_j \end{bmatrix}; \quad \forall k, j \in K, \\ & b) \mathbf{X}_j = \mathbf{V}_j \mathbf{V}_j^H; \quad \forall j \in K, \\ & c) \mathbf{Y}_k = \mathbf{U}_k \mathbf{U}_k^H; \quad \forall k \in K. \end{aligned} \quad (6)$$

Then, let  $\mathbf{H}_{kj}^H \mathbf{Y}_k = \mathbf{A}$  and  $\mathbf{H}_{kj} \mathbf{X}_j = \mathbf{B}$ , where  $\mathbf{A}$  and  $\mathbf{B}$  are complex conjugate and in other words are Hermitian matrices. By applying another property of  $\text{Tr}(\cdot)$  as  $\text{Tr}(\mathbf{A}\mathbf{B}) = 0.25 \left( \left\| \mathbf{A} + \mathbf{B}^H \right\|_F^2 - \left\| \mathbf{A} - \mathbf{B}^H \right\|_F^2 \right)$ , we have:

$$\begin{aligned} \text{Tr}(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_k) &= 0.25 \left( \left\| \mathbf{H}_{kj}^H \mathbf{Y}_k + \mathbf{X}_j^H \mathbf{H}_{kj} \right\|_F^2 \right. \\ & \quad \left. - \left\| \mathbf{H}_{kj}^H \mathbf{Y}_k - \mathbf{X}_j^H \mathbf{H}_{kj} \right\|_F^2 \right). \end{aligned} \quad (7)$$

As shown, the objective function is converted to the difference of two Frobenius norm terms. Each term of the obtained Frobenius norms can be considered as a DC component. To reach an affine form, the second term of DC objective function can be approximated linearly by Taylor series. Let define

$g(\mathbf{X}_j, \mathbf{Y}_k) \triangleq \left\| \mathbf{H}_{kj}^H \mathbf{Y}_k - \mathbf{X}_j^H \mathbf{H}_{kj} \right\|_F^2$  where this function is satisfied in the following inequality for the convex differentiable functions:

$$\begin{aligned} g(\mathbf{X}_j, \mathbf{Y}_k) &\geq g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k) + \left\langle \nabla_{\mathbf{X}_j} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k), \mathbf{X}_j - \hat{\mathbf{X}}_j \right\rangle \\ &\quad + \left\langle \nabla_{\mathbf{X}_j^H} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k), \mathbf{X}_j - \hat{\mathbf{X}}_j \right\rangle \\ &\quad + \left\langle \nabla_{\mathbf{Y}_k} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k), \mathbf{Y}_k - \hat{\mathbf{Y}}_k \right\rangle \\ &\quad + \left\langle \nabla_{\mathbf{Y}_k^H} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k), \mathbf{Y}_k - \hat{\mathbf{Y}}_k \right\rangle, \end{aligned} \quad (8)$$

and

$$\begin{aligned} \nabla_{\mathbf{X}_j^H} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k) &= \left( \nabla_{\mathbf{X}_j} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k) \right)^H, \\ \nabla_{\mathbf{Y}_k^H} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k) &= \left( \nabla_{\mathbf{Y}_k} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k) \right)^H, \end{aligned} \quad (9)$$

where  $\hat{\mathbf{X}}_j$  and  $\hat{\mathbf{Y}}_k$  show the initial points of  $\mathbf{X}_j$  and  $\mathbf{Y}_k$ , respectively and  $\nabla_{\mathbf{A}}$  denotes the gradient versus matrix  $\mathbf{A}$ . All gradient expressions in (8) are the coefficients of the first order approximation terms of the Taylor series. The gradient terms are attained by deriving the complex-valued matrix function  $g(\mathbf{X}_j, \mathbf{Y}_k)$  with respect to the complex-valued matrix variables  $\mathbf{X}_j$  or  $\mathbf{Y}_k$  and its complex conjugate  $\mathbf{X}_j^H$  or  $\mathbf{Y}_k^H$  [23]. It is noticeable that matrices  $\mathbf{X}_j$  and  $\mathbf{X}_j^H$  are treated independently and this is applied to  $\mathbf{Y}_k$  and  $\mathbf{Y}_k^H$ , as well. Based on  $g(\mathbf{X}_j, \mathbf{Y}_k)$  definition, the linear approximation of the objective function in (7) can be written as follows:

$$\begin{aligned} \hat{G} &= 0.25 \left[ \sum_{k=1}^K \sum_{j=1, j \neq k}^K \left\| \mathbf{H}_{kj}^H \mathbf{Y}_k + \mathbf{X}_j^H \mathbf{H}_{kj} \right\|_F^2 \right. \\ & \quad \left. - \sum_{k=1}^K \sum_{j=1, j \neq k}^K g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k) \right] \\ & \quad + 0.5 \left[ \text{Re} \left( \sum_{j=1}^K \sum_{k=1, k \neq j}^K \left\langle \nabla_{\mathbf{X}_j} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k), \mathbf{X}_j - \hat{\mathbf{X}}_j \right\rangle \right) \right. \\ & \quad \left. - \text{Re} \left( \sum_{k=1}^K \sum_{j=1, j \neq k}^K \left\langle \nabla_{\mathbf{Y}_k} g(\hat{\mathbf{X}}_j, \hat{\mathbf{Y}}_k), \mathbf{Y}_k - \hat{\mathbf{Y}}_k \right\rangle \right) \right]. \end{aligned} \quad (10)$$

Then, we will focus on the constraints and attempt to reformulate them as some new appropriate forms. Using Schur compliment lemma [24], the constraints in b) and c) can be expressed as the semi-definite programming (SDP) inequality sub-constraints  $b_1)$ ,  $b_2)$  and  $c_1)$ ,  $c_2)$ , respectively as follows:

$$b_1) \begin{bmatrix} \mathbf{I}_{d_j \times d_j} & \mathbf{V}_j^H \\ \mathbf{V}_j & \mathbf{X}_j \end{bmatrix} \succeq \mathbf{0} \quad , \quad b_2) \quad Tr(\mathbf{X}_j) \leq d_j ; \forall j, \quad (11)$$

$$c_1) \begin{bmatrix} \mathbf{I}_{d_k \times d_k} & \mathbf{U}_k^H \\ \mathbf{U}_k & \mathbf{Y}_k \end{bmatrix} \succeq \mathbf{0} \quad , \quad c_2) \quad Tr(\mathbf{Y}_k) \leq d_k ; \forall k. \quad (12)$$

Therefore, the final reformulation of our proposed IA optimization problem consists of the objective function in (10) subject to the constraints mentioned in a),  $b_1)$ ,  $b_2)$ ,  $c_1)$ , and  $c_2)$  in (6), (11), and (12), respectively. To solve the proposed IA optimization problem, we optimize Tx and Rx beamformers using an iterative local search to achieve the aligned beamformers.

### DC-Based IA Algorithm

In this section, we focus on designing the optimized aligned beamformers by solving the proposed IA optimization problem using an iterative DC-based IA. First, we define a penalty function by scaling the proposed constraints in previous section by the factor  $\eta$ . By increasing the penalty factor  $\eta$  and substituting the current steps  $\mathbf{X}_k$  and  $\mathbf{Y}_k$  with the previous steps, respectively, the amount of penalty function reduces subject to the proposed constraints set iteratively. As this process is a decreasing iterative manner, the optimum solutions of an upper bound of the penalty function converge to the solutions of the original DC objective function. To prevent the complexity, all constraint transformations are applied to  $c_2)$  and then the results are referred to the constraints in  $b_2)$ . The rank-one constraint of a positive semi-definite matrix like  $\mathbf{A}$  can be written as  $Tr(\mathbf{A}) \leq \lambda_{\max}(\mathbf{A})$  [25] in which  $\lambda_{\max}$  represents the maximum eigenvalue of matrix  $\mathbf{A}$ ; so, the sub-constraint  $c_2)$  can be rewritten as

$$\alpha) \sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k) \geq Tr(\mathbf{Y}_k), \text{ where } \sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k) \text{ is the sum}$$

of  $k$  larger eigenvalues of the matrix  $\mathbf{Y}_k$ . Using the DC programming, it can be written as

$$\alpha) \quad Tr(\mathbf{Y}_k) - \sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k) \leq 0 ; \forall k, \text{ where the first term}$$

of it is an affine function, but the second term is not. However, the second one is satisfied in the following inequality:

$$\sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k) \leq \sum_{i=1}^{d_k} \lambda_i(\hat{\mathbf{Y}}_k) + Tr(\bar{\nabla}_{\mathbf{Y}} \sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k), \mathbf{Y}_k - \hat{\mathbf{Y}}_k). \quad (13)$$

As  $\sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k)$  is not a differentiable expression; so,

using the sub-gradient method,  $\bar{\nabla}_{\mathbf{Y}} \sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k)$  can be expressed as follows (refer to Appendix):

$$\bar{\nabla}_{\mathbf{Y}} \sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k)|_{\hat{\mathbf{Y}}_k} = \mathbf{S}_{\max}(\mathbf{Y}_k) \mathbf{S}_{\max}^H(\mathbf{Y}_k)|_{\hat{\mathbf{Y}}_k}, \quad (14)$$

where  $\bar{\nabla}_{\mathbf{Y}}$  denotes the sub-gradient versus matrix  $\mathbf{Y}$  and  $\mathbf{S}_{\max}$  is the matrix whose columns are the corresponding right eigenvectors of the maximum eigenvalues of  $\mathbf{Y}$ . Using  $\alpha)$  relation and  $Tr(\cdot)$  property besides inserting (14) to (13), the final transformation of sub-constraints in  $c_2)$  is shown as follows:

$$c_2^*) \quad G_2 = Tr(\mathbf{Y}_k) - Tr[\mathbf{S}_{\max}^H(\hat{\mathbf{Y}}_k) \mathbf{Y}_k \mathbf{S}_{\max}(\hat{\mathbf{Y}}_k)] - \sum_{i=1}^{d_k} \lambda_i(\hat{\mathbf{Y}}_k) + Tr[\mathbf{S}_{\max}^H(\hat{\mathbf{Y}}_k) \hat{\mathbf{Y}}_k \mathbf{S}_{\max}(\hat{\mathbf{Y}}_k)] \leq 0. \quad (15)$$

With the same approach, the sub-constraints in  $b_2)$  can be written equivalently as:

$$b_2^*) \quad G_1 = Tr(\mathbf{X}_k) - Tr(\mathbf{W}_{\max}^H(\hat{\mathbf{X}}_k) \mathbf{X}_k \mathbf{W}_{\max}(\hat{\mathbf{X}}_k)) - \sum_{i=1}^{d_j} \lambda_i(\hat{\mathbf{X}}_k) + Tr(\mathbf{W}_{\max}^H(\hat{\mathbf{X}}_k) \hat{\mathbf{X}}_k \mathbf{W}_{\max}(\hat{\mathbf{X}}_k)) \leq 0, \quad (16)$$

where  $\mathbf{W}_{\max}$  is the matrix of the eigenvectors corresponding to the maximum eigenvalues of  $\mathbf{X}_j$ .

We define an additive function as the following function:

$$q(\mathbf{X}, \mathbf{Y}, \eta_a) \triangleq \hat{G} + \eta_a \sum_{k=1}^K Tr(\mathbf{X}_k) - \eta_a \sum_{k=1}^K \sum_{i=1}^{d_j} \lambda_i(\mathbf{X}_k) + \eta_a \sum_{k=1}^K Tr(\mathbf{Y}_k) - \eta_a \sum_{k=1}^K \sum_{i=1}^{d_k} \lambda_i(\mathbf{Y}_k), \quad (17)$$

where  $a$  denotes the  $a$ -th iteration (i.e., iteration counter). The  $q$  function includes sum of the objective function in (10) and a penalty function. The penalty function is defined by scaling  $b_2^*)$  and  $c_2^*)$  by the factor  $\eta$ . For enough large values of  $\eta$ , the minimization process of  $q$  function over the defined constraints is equivalent to  $\hat{G}$  reduction over the same constraints. We also define function  $\hat{q}(\mathbf{X}, \mathbf{Y}, \hat{\mathbf{X}}, \hat{\mathbf{Y}}, \eta_a)$  which is an upper bound for the function  $q$  as follows:

$$\hat{q}(\mathbf{X}, \mathbf{Y}, \hat{\mathbf{X}}, \hat{\mathbf{Y}}, \eta_a) \triangleq \hat{G} + \eta_a \left( \sum_{k=1}^K G_1 + G_2 \right). \quad (18)$$

The nonlinear function  $q(\mathbf{X}, \mathbf{Y}, \eta)$  is approximated linearly by the function  $\hat{q}(\mathbf{X}, \mathbf{Y}, \hat{\mathbf{X}}, \hat{\mathbf{Y}}, \eta)$  from the initial

points  $\hat{\mathbf{X}}$ ,  $\hat{\mathbf{Y}}$  and using the sub-gradient method. By increasing the penalty factor  $\eta$ , we reduce function  $\hat{q}$  subject to the defined constraints set including a), b1),  $c_1)$ ,  $b_2^*)$ , and  $c_2^*)$  iteratively and substituting the current steps  $\mathbf{X}_k$  and  $\mathbf{Y}_k$  with the previous steps, respectively. Through a decreasing iterative manner, (18) is solved and by importing the penalty factor, the optimum solutions of the upper bound function converge to the solutions of the original DC objective function. As  $q(\mathbf{X}_k, \mathbf{Y}_k, \eta)$  is a non-decreasing function of  $k$ ; so, the convergence of the iterative approach can be proved for any fixed value of  $\eta$  [25].

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### Proposed DC-based IA Algorithm

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- 1: Initialize  $a \leftarrow 0$
  - 2: Choose an enough large  $\eta > 0$
  - 3: Arbitrarily initialize  $\tilde{\mathbf{V}}_j$  and  $\tilde{\mathbf{U}}_k; \forall j, k \in K$
  - 4: Construct  $\mathbf{V}_j, \mathbf{U}_k, \mathbf{X}_j$  and  $\mathbf{Y}_k; \forall j, k \in K$
  - 5: While  $a \leq a_{\max}$  do
  - 6: Solve (18) and put the solution to  $\mathbf{X}_{a+1}$  and  $\mathbf{Y}_{a+1}$  (solutions for  $(a+1)$ -th iteration)
  - 7: Set  $\eta_{a+1} \leftarrow \varepsilon \times \eta_a$
  - 8: Set  $a \leftarrow a + 1$
  - 9: end while
- 

### Sensitivity Formulation

In the following, we formulate the sensitivity of interference leakage with respect to the values of the trade-off parameters. It is assumed that the CSI mismatch is modeled as follows:

$$\mathbf{H}_{kj} = \tilde{\mathbf{H}}_{kj} + \Delta\mathbf{H}_{kj} \quad \text{and} \quad \tilde{\mathbf{H}}_{kj} = \left(1 - \frac{\gamma}{\|\mathbf{H}\|_F}\right)\mathbf{H}_{kj}. \quad (19)$$

where  $\Delta\mathbf{H}$  is the channel measurement error and  $\mathbf{H}_{kj}$  is the actual channel. It is assumed that the elements of error matrix  $\Delta\mathbf{H}$  have uniform distribution in the  $[-\gamma, \gamma]$  interval. The value of  $\gamma$  is equivalent to the high limit of quantization error and therefore is equal to the step size between the two levels of quantization.

The following problem reformulation was mentioned previously in Section 3:

$$\begin{aligned} & \underset{\mathbf{X}_j, \mathbf{Y}_k}{\text{minimize}} \quad \sum_{k=1}^K \sum_{j=1, j \neq k}^K \text{Tr}(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j), \\ & \text{subject to:} \quad \text{a) } \mathbf{U}_k = \begin{bmatrix} \mathbf{I}_{d_k \times d_k} \\ \tilde{\mathbf{U}}_k \end{bmatrix}, \quad \mathbf{V}_k = \begin{bmatrix} \mathbf{I}_{d_j \times d_j} \\ \tilde{\mathbf{V}}_j \end{bmatrix}; \quad \forall k, j \in K, \\ & \quad \quad \quad \text{b) } \mathbf{X}_j = \mathbf{V}_j \mathbf{V}_j^H; \quad \forall j \in K, \\ & \quad \quad \quad \text{c) } \mathbf{Y}_k = \mathbf{U}_k \mathbf{U}_k^H; \quad \forall k \in K. \end{aligned} \quad (20)$$

where the interference leakage function is represented as  $Q = \text{Tr}(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j)$ . Using the replacement property of  $E(\cdot)$  and Trace operators, as  $E(\text{Tr}(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j)) = \text{Tr}(E(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j))$ , we have:

$$\begin{aligned} & \text{Tr}(E(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j)) = \\ & \text{Tr}(E(\tilde{\mathbf{H}}_{kj}^H \mathbf{Y}_k E(\tilde{\mathbf{H}}_{kj}) \mathbf{X}_j)) + \text{Tr}(E(\tilde{\mathbf{H}}_{kj}^H \mathbf{Y}_k E(\Delta\mathbf{H}_{kj}) \mathbf{X}_j)) + \\ & \text{Tr}(E(\Delta\mathbf{H}_{kj}^H \mathbf{Y}_k E(\tilde{\mathbf{H}}_{kj}) \mathbf{X}_j)) + \text{Tr}(E(\Delta\mathbf{H}_{kj}^H \mathbf{Y}_k E(\Delta\mathbf{H}_{kj}) \mathbf{X}_j)). \end{aligned} \quad (21)$$

it is assumed that  $\tilde{\mathbf{H}}$  and  $\Delta\mathbf{H}$  are independent, so:

$$E(\tilde{\mathbf{H}}_{kj}^H \mathbf{Y}_k E(\Delta\mathbf{H}_{kj}) \mathbf{X}_j) = E(\Delta\mathbf{H}_{kj}^H \mathbf{Y}_k E(\tilde{\mathbf{H}}_{kj}) \mathbf{X}_j) = 0. \quad (22)$$

Finally, we have:

$$\begin{aligned} E(\text{Tr}(\mathbf{H}_{kj}^H \mathbf{Y}_k \mathbf{H}_{kj} \mathbf{X}_j)) &= \text{Tr}(\tilde{\mathbf{H}}_{kj}^H \mathbf{Y}_k \tilde{\mathbf{H}}_{kj} \mathbf{X}_j) + \\ & \text{Tr}(\Delta\mathbf{H}_{kj}^H \mathbf{Y}_k \Delta\mathbf{H}_{kj} \mathbf{X}_j). \end{aligned} \quad (23)$$

Using the Lagrange duality method [24] as follows:

$$\begin{aligned} L(\alpha_j, \beta_k, \mathbf{X}_j, \mathbf{Y}_k) &= \\ & \sum_{k=1}^K \sum_{j=1, j \neq k}^K \text{Tr}(\tilde{\mathbf{H}}_{kj}^H \mathbf{Y}_k \tilde{\mathbf{H}}_{kj} \mathbf{X}_j) + \text{Tr}(\Delta\mathbf{H}_{kj}^H \mathbf{Y}_k \Delta\mathbf{H}_{kj} \mathbf{X}_j) \\ & + \alpha_j (\mathbf{X}_j - \mathbf{V}_j \mathbf{V}_j^H) + \beta_k (\mathbf{Y}_k - \mathbf{U}_k \mathbf{U}_k^H), \end{aligned} \quad (24)$$

and differentiating in (24) with respect to the trade-off parameters, the sensitivity formulas can be obtained as follows:

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{X}_j} &= \sum_{k=1}^K \sum_{j=1, j \neq k}^K \text{Tr}(\tilde{\mathbf{H}}_{kj}^H \mathbf{Y}_k \tilde{\mathbf{H}}_{kj}) + \text{Tr}(\Delta\mathbf{H}_{kj}^H \mathbf{Y}_k \Delta\mathbf{H}_{kj}) + \alpha_j \mathbf{I}_j = 0, \\ \frac{\partial L}{\partial \mathbf{Y}_k} &= \sum_{k=1}^K \sum_{j=1, j \neq k}^K \text{Tr}(\tilde{\mathbf{H}}_{kj}^H \tilde{\mathbf{H}}_{kj} \mathbf{X}_j) + \text{Tr}(\Delta\mathbf{H}_{kj}^H \Delta\mathbf{H}_{kj} \mathbf{X}_j) + \beta_k \mathbf{I}_k = 0. \end{aligned} \quad (25)$$

### System Performance Analysis

In this section, we evaluate the system performance of the proposed DC-based IA design method as compared to the non-robust IA design approach introduced in [13] and the ZF-based IA algorithm [20] for the case of perfect CSI and single cell (single relay). In [20], the IA filters were determined based on the minimization of total interference leakage at the receivers.

We consider uniform power allocation for all input data streams of all users. Each channel element has i.i.d Gaussian distribution with zero mean and unity variance. Also, all the simulations were conducted using 4-QAM constellation. In order to evaluate the system performance, the system configuration of a 3-user 6x2 MIMO ( $M = 6$  and  $N = 2$ ) transmission is taken into account. For the case of approaches in [13] and [20], the relay was equipped with six antennas ( $N_{re} = 6$ ). Table 1 shows the simulation parameters at a glance. Our

established convex optimization problem was solved by using CVX toolbox [26]-[28] which is a package for solving convex problems. In each iteration of our approach, one SDPT3 (a CVX-solver) [26] optimized transmit- and receive-beamformers jointly and simultaneously. It is noticeable that both reference algorithms in [13] and [20] do not require any solver per iteration.

The appropriate choices  $\eta=10^3$  and  $\varepsilon=2$  were set for the proposed algorithm by trial and error. In order to reduce the running time, the maximum number of iterations ( $a_{\max}$ ) was set to 10. Figure 2 depicts the convergence of a) the objective function (i.e., interference leakage) of the proposed DC-based IA algorithm, b) the non-robust IA design approach in [13], and c) the ZF-based IA algorithm in [20] versus number of iterations. It is seen that all three methods are non-increasing functions over iteration, so that gradually reduction of the objective function results in the convergence of them to local minimum. However, it depicts that our proposed approach gives slightly lower amount of interference powers as compared to other two methods at the receiver's side.

Table 1: Simulation parameters

Parameter	Description
$\sigma^2$	Variance of noise
$K$	Number of users
$M$	Number of transmitter's antennas
$N$	Number of receiver's antennas
$\eta$	Penalty factor
$a$	Counter of iteration
$\varepsilon$	Step size of penalty function
$N_{re}$	Number of relay's antennas

The proposed method offers lower leakage but the running time depends on the required iterations and the complexity order. The complexity of the DC-based IA algorithm increases dramatically as the number of users and antennas increases. Since our proposed algorithm can achieve a low interference leakage level after sufficient iterations; so in high SNR regimes where the sum-rate is limited by the interference rather than the noise, the DC-based IA algorithm serves as a good method to decrease the interference leakage.

Inspired by what is presented in [13], the end-to-end SINR of our proposed system model for the  $k$ -th user is determined by:

$$SINR = \frac{\|\mathbf{H}_{kk}\|_F^2}{\sum_{j=1, j \neq k}^K \|\mathbf{H}_{kj}\|_F^2 + \sigma_k^2 \|\mathbf{U}_k\|_F^2} \quad (26)$$

where  $\sigma_k^2$  is defined at the  $k$ -th user.

The performance of system in terms of the effective SINR as a function of  $1/\sigma^2$  is evaluated as shown in Fig. 3, where  $\sigma^2$  is the noise power at each user. It is seen that considering noise in IA beamformer design improves the end-to-end effective SINR considerably for all users and an SINR gain about 5 dB can be achieved.

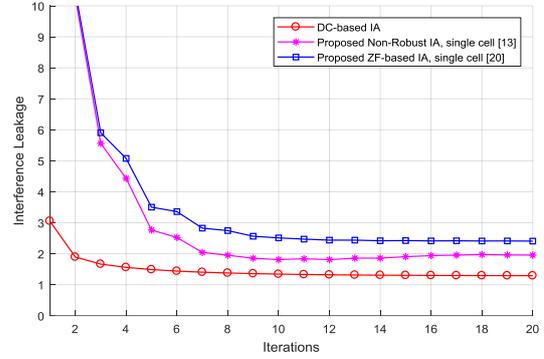


Fig. 2: Interference leakage versus number of iterations.

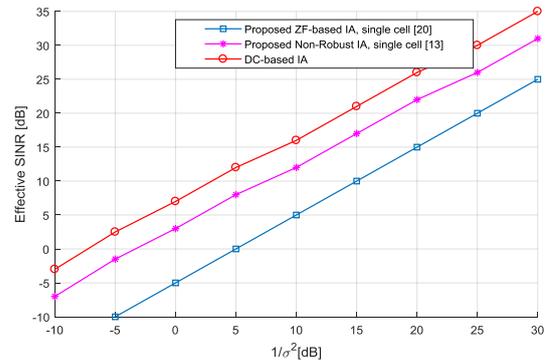


Fig. 3: Effective end-to-end SINR.

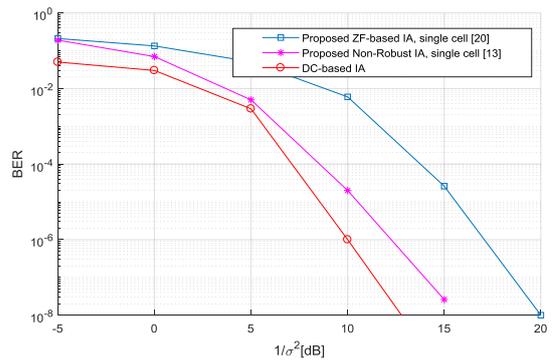


Fig. 4: BER performance comparison of proposed method.

Also in Fig. 4, by considering the total noise at the receiver, the system performance in terms of BER indicates that the proposed DC-based IA approach provides better performance in terms of BER as compared to the mentioned algorithms introduced in [13] and [20]. In low and moderate SNR regimes, an SNR

gain of about 4 dB can be achieved and for high SNRs, the BER of the DC-based IA is lower than other two competitive methods.

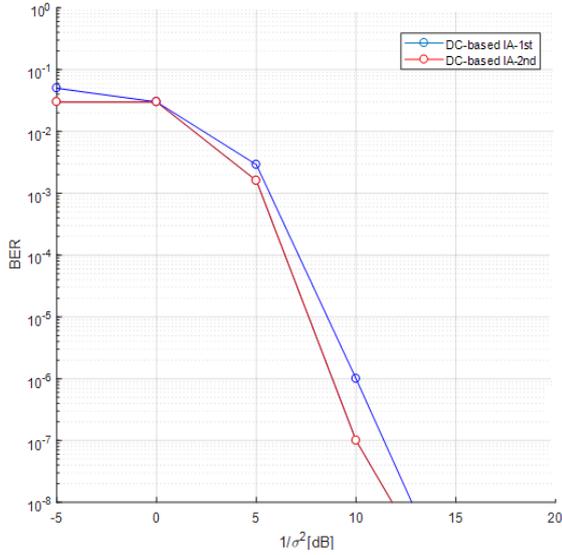


Fig. 5: Performance comparison of a 3-user 6×2 MIMO (DC-based-1st) and a 3-user 8×4 MIMO (DC-based-2nd) in terms of BER.

Since our proposed algorithm is much more complex as compared to competitive methods in this study, by increasing the number of transmitters and receivers, the running time will be increased considerably, as well. In Fig. 5, by extending the system configuration from a 3-user 6×2 MIMO (called DC-based-1st) to a 3-user 8×4 MIMO (called DC-based-2nd) configuration, the system performance is shown in terms of BER and as seen in this figure, the performance is improved.

For the DC-based-2nd configuration, the running time is about 5 times higher than the DC-based-1st configuration. Noticeably, our computing machine (Intel (R) Corei5-3210M, CPU 2.50 GHz and RAM 8 GB) supported the calculations for mentioned numbers of transmitters and receivers in a reasonable time.

### Conclusion

In this work, using a method in the non-convex programming, namely difference of convex (DC), we proposed a DC-based IA algorithm to design optimized aligned beamformers for a MIMO interference channel. The non-convex objective function according to the sum of interference powers at the receivers was reformulated as the difference of two convex terms. An additive function was developed which includes the resultant DC objective function and a defined penalty function. We proposed a DC-based IA method in which for large values of the penalty factor, the solutions of upper bound additive function converged to the solutions of the original DC interference leakage function. In each iteration, the proposed DC-based IA

algorithm optimized transmit- and receive-beamformers jointly and simultaneously (in contrast to the previous algorithms which update beamformers, alternately). For different scenarios, the simulation results showed that the DC-based IA algorithm outperforms the proposed non-robust IA design approach in [13] and the ZF-based IA method in [20] in terms of interference leakage, SINR and BER.

### Appendix

According to the Rayleigh-Ritz relation, the sum of  $m$  largest eigenvalues of the matrix  $\mathbf{D}$  can be expressed as follows [22]:

$$\sum_{i=1}^m \lambda_i(\mathbf{D}) = \sup\{Tr(\mathbf{Q}^H \mathbf{D} \mathbf{Q}) \mid \mathbf{Q} \in R^{K \times n}, \mathbf{Q}^H \mathbf{Q} = \mathbf{I}\}, \quad (\text{A-1})$$

where  $\mathbf{Q}$  is the matrix that includes the eigenvectors corresponding to the maximum eigenvalues of  $\mathbf{D}$ .

Let define  $F = Tr(\mathbf{Q}^H \mathbf{D} \mathbf{Q})$ . We have:

$$F = Tr(\mathbf{Q}^H \mathbf{D} \mathbf{Q}) = Tr(\mathbf{Q}^H (\mathbf{D} + \Delta \mathbf{D}) \mathbf{Q}). \quad (\text{A-2})$$

the derivation of  $F$  with respect to the variable  $\mathbf{D}$  is expressed as follows:

$$\begin{aligned} dF(\mathbf{D}) &= F(\mathbf{D} + \Delta \mathbf{D}) - F(\mathbf{D}) \\ &= Tr[\mathbf{Q}^H (\mathbf{D} + \Delta \mathbf{D}) (\mathbf{D} + \Delta \mathbf{D}) \mathbf{Q} (\mathbf{D} + \Delta \mathbf{D}) - \mathbf{Q}^H (\mathbf{D}) \mathbf{D} \mathbf{Q} (\mathbf{D})]. \end{aligned} \quad (\text{A-3})$$

It is noted that:

$$\mathbf{Q}(\mathbf{D} + \Delta \mathbf{D}) = \mathbf{Q} + \Delta \mathbf{Q}, \quad (\text{A-4})$$

and

$$\mathbf{Q}^H (\mathbf{D} + \Delta \mathbf{D}) = (\mathbf{Q} + \Delta \mathbf{Q})^H. \quad (\text{A-5})$$

So:

$$\begin{aligned} dF(\mathbf{D}) &= Tr[(\mathbf{Q} + \Delta \mathbf{Q})^H \mathbf{D} (\mathbf{Q} + \Delta \mathbf{Q}) + \\ &\quad (\mathbf{Q} + \Delta \mathbf{Q})^H (\Delta \mathbf{D}) (\mathbf{Q} + \Delta \mathbf{Q}) \\ &\quad - Tr(\mathbf{Q}^H \mathbf{D} \mathbf{Q})]. \end{aligned} \quad (\text{A-6})$$

To save simplicity, we withdraw  $\Delta \mathbf{D} \Delta \mathbf{Q}$  term. Then, using the trace property, we have:

$$\begin{aligned} dF(\mathbf{D}) &= Tr[(\mathbf{Q} + \Delta \mathbf{Q})^H \mathbf{D} \mathbf{Q} + (\mathbf{Q} + \Delta \mathbf{Q})^H \Delta \mathbf{D} \mathbf{Q} \\ &\quad + (\mathbf{Q} + \Delta \mathbf{Q})^H \Delta \mathbf{D} \mathbf{Q}] - Tr(\mathbf{Q}^H \mathbf{D} \mathbf{Q}), \\ dF(\mathbf{D}) &= Tr[(\Delta \mathbf{Q}^H \mathbf{Q} + \mathbf{Q}^H \Delta \mathbf{Q}) \mathbf{D}] - Tr(\mathbf{Q}^H \mathbf{D} \mathbf{Q}) \\ &\quad + Tr(\mathbf{Q} \mathbf{Q}^H \Delta \mathbf{D}), \\ dF(\mathbf{D}) &= Tr(\mathbf{Q} \mathbf{Q}^H \Delta \mathbf{D}). \end{aligned} \quad (\text{A-7})$$

So:

$$\tilde{\nabla}_{\mathbf{Q}} \sum_{i=1}^m \lambda_i(\mathbf{D}) = Tr(\mathbf{Q} \mathbf{Q}^H). \quad (\text{A-8})$$

### Author Contributions

B. Mahboobi, M. Sheikhan, and N. Danesh proposed the idea. N. Danesh and B. Mahboobi derived the formulas. N. Danesh, M. Sheikhan, and B. Mahboobi interpreted the results and wrote the manuscript.

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## Conflict of Interest

The author declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy has been completely observed by the authors.

## References

- [1] S. W. Peters, R. W. Heath, Jr., "Interference alignment via alternating minimization," presented at the IEEE Int. Conf. Acoustics, Speech and Signal Processing (ICASSP), Taipei, Taiwan, 19-24, 2009.
- [2] K. Gomadam, V. R. Cadambe, S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Information Theory*, 57(6): 3309-3322, Jun. 2011.
- [3] R. F. Guiazon, K. K. Wong, M. Fitc, "Coverage probability of cellular networks using interference alignment under imperfect CSI," *Digital Communications and Networks*, 2(4): 162-166, 2016.
- [4] N. Zhao, B. Chen, "Joint optimization of power splitting and allocation for SWIPT in interference alignment networks," *Physical Communication*, 29: 67-77, 2018.
- [5] H. Y. Lu, "Cyclic interference alignment for MIMO interference channels: A hybrid approach of MTLI and PSO," *Applied Soft Computing*, 50: 158-165, Jan. 2017.
- [6] P. G. Sudheesh, M. Magarini, P. Muthuchidambaramathan, "Interference alignment with iterative channel estimation for the reciprocal Mx2 MIMO X Network," *Physical Communication*, 27: 188-196, 2018.
- [7] S. Mollaebrahim, P. M. Ghari, M. S. Fazel, M. A. Imran, "Designing precoding and receive matrices for interference alignment in MIMO interference channels," presented at the IEEE Global Telecommunications Conference (GLOBECOM), Singapore, 2017.
- [8] K. Gomadam, V. R. Cadambe, S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," presented at the IEEE Global Telecommunications Conference (GLOBECOM), New Orleans, USA, 2008.
- [9] S. W. Peters, R. W. Heath, "Cooperative algorithms for MIMO interference channels," *IEEE Trans. Vehicular Technology*, 60(1): 206-218, 2011.
- [10] D. S. Papailiopoulos, A. G. Dimakis, "Interference alignment as a rank constrained rank minimization," *IEEE Trans. Signal Processing*, 60(8): 4278-4288, 2012.
- [11] L. Ruan, M. Z. Win, V. K. N. Lau, "Designing interference alignment algorithms by algebraic geometry analysis," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*: 1796-1801, 2013.
- [12] S. M. Razavi, T. Ratnarajah, "Adaptive LS-and MMSE-based beamformer design for multiuser MIMO interference channels," *IEEE Transactions on Vehicular Technology*, 65(1): 132-144, 2016.
- [13] C. Le, S. Moghaddamnia, J. Peissig, "A hybrid optimization approach for interference alignment in multi-user MIMO relay networks under different CSI," *IEEE Trans. Wireless Communications*, 16: 7834-7847, 2017.
- [14] G. Sridharan, W. Yu, "Linear beamformer design for interference alignment via rank minimization," *IEEE Trans. Signal Processing*, 63(22): 5910-5923, 2015.
- [15] S. M. Razavi, "Beamformer design for MIMO interference broadcast channels with semi-definite programming," *IEEE Trans. Signal Processing*, 66(17): 4504-4515, 2018.
- [16] O. González, C. Beltrán, I. Santamaría, "A feasibility test for linear interference alignment in MIMO channels with constant coefficients," *IEEE Trans. Information Theory*, 60(3): 1840-1856, 2014.
- [17] D. C. G. Bresler, D. Tse, "Feasibility of interference alignment for the MIMO interference channel," *IEEE Trans. Information Theory*, 60(9), p. 5573-5586, 2014.
- [18] H. A. L. Thi, T. P. Dinh, "The DC (difference of convex functions) programming and DCA revisited with DC models of real world nonconvex optimization problems," *Annals Operations Research*, 133(1-4): 23-46, 2005.
- [19] H. H. M. Tam, E. Che, H. D. Tuan, "Optimized linear precoder in MIMO interference channel using D.C. programming," presented at the IEEE 7<sup>th</sup> Int. Conf. Signal Processing and Communication Systems (ICSPCS), Carrara, Australia, 2013.
- [20] X. Chen, S. H. Song, K. B. Letaief, "Interference alignment in dual-hop MIMO interference channel," *IEEE Trans. Wireless Communications*, 13(3): 1274-1283, 2014.
- [21] T. Ketsoglu, E. Ayanoglu, "Zero-forcing per-group precoding (ZF-PGP) for robust optimized downlink massive MIMO performance," *IEEE Trans. Communications*, 67(10): 6816-6828, 2019.
- [22] T. Ketsoglu, E. Ayanoglu, "Downlink precoding for massive MIMO systems exploiting virtual channel model sparsity," *IEEE Trans. Communications*, 66(5): 1925-1939, 2018.
- [23] A. Hjørungnes, D. Gesbert, "Complex-valued matrix differentiation: Techniques and key results," *IEEE Trans. Signal Processing*, 55(6): 2740-2746, 2007.
- [24] S. Boyd, L. Vandenberghe, *Convex optimization*, New York: Cambridge, 2004.
- [25] B. Mahboobi, E. Soleimani-Nasab, M. Ardebilipour, "Outage probability based robust distributed beam-forming in multi-user cooperative networks with imperfect CSI," *Wireless Personal Communications*, 77(3): 1629-1658, 2014.
- [26] CVX research, "Matlab software for disciplined convex programming, version 2.0 beta," 2012.
- [27] M. C. Grant, S. P. Boyd, "Graph implementations for Non-smooth convex programs," in *Recent Advances in Learning and Control*, Springer: 95-110, 2008.
- [28] B. S. Grant, "CVX: Matlab software for disciplined convex programming (web page and software)," 2009.

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