



Research paper

# Cooperative Distributed Constrained Adaptive Generalized Predictive Control for Uncertain Nonlinear Large-Scale Systems: Application to Quadruple-Tank System

A. Mirzaei<sup>1</sup>, A. Ramezani<sup>1,\*</sup>

<sup>1</sup>Center of advanced Control systems, School of Electrical and Computer Engineering, Tarbiat Modares University, Tehran, Iran.

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\*Corresponding Author's Email Address:  
ramezani@modares.ac.ir

## Extended Abstract

**Background and Objectives:** In this paper, a constrained cooperative distributed model predictive control (DMPC) is proposed. The proposed DMPC is based on linear adaptive generalized predictive control (AGPC) to control uncertain nonlinear large-scale systems.

**Methods:** The proposed approach, has two main contributions. First, a novel cooperative optimization strategy is proposed to improve the centralized global cost function of each local controller. Second, using the proposed linear distributed AGPC (DAGPC), the mismatch between linearized and nonlinear models is compensated via online identification of the linearized model in each iteration of optimization.

**Results:** The proposed novel cooperative optimization strategy decreases the computational burden of optimization process compared to conventional cooperative DMPC strategies. Moreover, the proposed linear DAGPC decreases the satisfaction time of the terminal condition compared to conventional DMPC methods. The paper establishes sufficient conditions for the closed-loop stability. The performance and effectiveness of proposed method is demonstrated through simulation of a quadruple-tank system for both certain and uncertain situations. The imposed uncertainty changes the system from minimum phase to nonminimum-phase situation. Closed-loop stability and proper convergences are concluded from simulation results of both situations.

**Conclusion:** Most important advantages of proposed linear cooperative DAGPC are its less design complexity and consequently less convergence time compared to fully nonlinear DMPC methods, due to its online identification of the linearized model.

## Introduction

Most practical plants are nonlinear dynamic, so a precise dynamic of the simplest systems such as a DC motor would still be nonlinear. On the other hand, many industrial systems are large-scale systems which consist of several nonlinear subsystems. Controlling these nonlinear large-scale systems is one of the most important challenges. Model predictive control (MPC) is

one of the desirable approaches to deal with this challenge for linear and nonlinear large-scale systems which leads us to distributed MPC (DMPC) algorithms. Different linear DMPC approaches namely cooperative, non-cooperative and agent negotiation which is a kind of cooperative approach, are theoretically investigated and can be extended to nonlinear large-scale systems [1] and [2]. In non-cooperative algorithms, each local

controller optimizes its own local cost function, whereas in cooperative algorithms, it optimizes global cost function which is a combination of all local cost functions [3], [4], and [5]. Dual mode DMPC is developed based on the proximity of the state variables to the origin. When the state variables are far from origin, the nonlinear DMPC algorithm is considered. On the other hand, when the state variables are in the neighborhood of the origin, the linear DMPC algorithm is applied. Compared to the methods in which nonlinear algorithms are used when the states are in a neighborhood of the origin, the computational burden is reduced using proposed dual mode DMPC or intelligent methods like genetic algorithm [6] and [7]. Gradient projection optimizer is designed as a subsidiary nonlinear nonconvex algorithm that improves the objective function. Therefore, there would be no need for a coordinator layer [3] and [8]. On the contrary, some methods require coordination between their subsystems because the subsystems are optimized independently [9], [10], and [11]. In these methods each subsystem exchanges its information to its interconnected neighbors via coordinator. In some other approaches, the interconnection between subsystems is considered as a constraint in each local optimization instead of using the coordinator [12] and [13]. The network based DMPC algorithm with multi rate sampling is a combination approach which is designed for a nonlinear large-scale uncertain system composed of coupled subsystems. Local controllers are interconnected through a network-based coordination structure that uses iterative strategy to control the entire large-scale system [14]. The key factor in approaches with coordination layer or approaches in which each local controller optimizes its cost function separately and exchanges information to its neighbors, is communication delay. In most of these approaches, communication delay is considered as a constraint [15] and [16]. A two-layer robust DMPC is another method which is designed for nonlinear constrained systems coupled through cost functions [17]. Each subsystem receives information of control trajectories of its neighbors and solves its local optimization that involves interconnected coupling terms and computes its local optimal control signal. Sufficient conditions for convergence of states of all subsystems are provided in the first layer. In the second layer, robust DMPC approach is developed which uses a shorter prediction horizon and also tolerates larger disturbances [18] and [19]. Moreover, robust distributed control methods are useful to control a group of nonlinear subsystems being exposed to constraints of the control inputs and external disturbances [20] and [21]. Another DMPC approach is sequential nonlinear DMPC for constrained

large-scale systems. In this approach, each local controller solves its optimization problem at each sampling time, and exchanges its information via the communication channel to achieve the global objective of the overall system. The performance of the proposed nonlinear DMPC is similar to the centralized nonlinear MPC. However, they are more effective compared to the centralized approaches [22] and [23]. By applying the contraction theory in DMPC algorithms, larger sampling intervals and stronger coupling between subsystems are addressed. The conservative conditions are also reduced using contraction theory [24] and [25].

Hierarchical based DMPC is a useful algorithm for uncertain large-scale systems. A two level DMPC approach is designed based on hierarchical framework, which takes unreliable fault information into account to the system. The objective is to compensate the identified actuator faults of the subsystems which include the detected time delays and uncertainties. In first level, faults are recovered to maintain the design characteristics for all subsystems. In second level, recovery process is applied by increasing the whole system performance. The designed distributed method satisfies the recovery design characteristics, and also imposes lower fault compensation and consequently, lower cost compared to the centralized and decentralized methods [26] and [27].

Nonlinear large-scale systems could be controlled using both linear and nonlinear DMPC algorithms. Linear algorithms are less complicated than nonlinear ones. However, there are restrictions to apply the linear algorithms for nonlinear systems. When the nonlinearity degree is high or it has several equilibrium points, the closed loop nonlinear system may get unstable by applying linear algorithms. Moreover, in all reviewed researches, the computational burden of optimization and convergence time are two main challenges. In this manuscript, a novel cooperative DMPC strategy is proposed which reduces the computational burden of optimization process and convergence time due to its main algorithm's reconfiguration. Additionally, to exploit the advantages of linear algorithms for controlling the uncertain nonlinear large-scale systems, a new Distributed Adaptive Generalized Predictive Control is proposed.

The remainder of current paper is organized as follows. In second section, problem statement is given for nonlinear interconnected large-scale systems. In third section, the new cooperative optimization strategy is proposed. The reconstructed distributed model predictive controller is proposed in fourth section. Fifth section presents the results and discussion for a quadruple-tank system. Finally, concluding remarks are expressed in last section.

## Problem Statement: Mathematical Model of Nonlinear Large-Scale Systems

### A. Notation

The operator  $|\cdot|$  denotes the absolute value of a variable. Variables  $\alpha_i$  and  $\alpha_{ii}$  belong to subsystem  $i$ , and  $\alpha_{ij}$  denotes the interconnection between subsystem  $i$  and its neighboring subsystem  $j$ . Function  $F^l$  represents the identified version of  $F$ . Function  $\hat{F}(k+s)$  is the prediction of  $F$  in  $s$  seconds ahead of the current time  $k$ . Function  $\Delta F(k)$  denotes the increment of  $F(k)$ ;  $\Delta F(k) = F(k) - F(k-1)$ .

### B. Mathematical model

Consider a large-scale system with nonlinear dynamic whose centralized model is decomposed into  $M$  coupled subsystems. Each subsystem can be presented by the following nonlinear continuous-time model:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t), x_j(t)), \quad i = 1, 2, \dots, M \quad (1)$$

where,  $f_i$  is a nonlinear function,  $x_i \in R^{n_{x_i}}$  and  $u_i \in R^{n_{u_i}}$  present the vectors of state and input of subsystem  $i$  respectively,  $x_j \in R^{n_{x_j}}$  presents the vector of state of subsystem  $j$  which is the interconnected neighboring subsystem of subsystem  $i$ , and  $t$  is the continuous-time index. The overall dynamic,  $x(t)$ , is described as follows:

$$\dot{x}(t) = f(x(t), u(t)) \quad (2)$$

where,  $f = [f_1^T \ \dots \ f_M^T]^T$ . It is assumed that  $f_i$  is a Lipschitz function. The nonlinear continuous-time model of subsystem  $i$ , can be discretized using the following Euler derivative approximation:

$$\dot{x}_i(t) = \frac{x_i(k+1) - x_i(k)}{T_s} \quad (3)$$

where,  $k$  is the discrete-time index and  $T_s$  is the sampling time. Hence, according to (1) and (3), the nonlinear discrete-time dynamic of subsystem  $i$ ,  $x_i(k+1)$ , is represented as:

$$x_i(k+1) = h_i(x_i(k), u_i(k), x_j(k)) \quad (4)$$

where,  $h_i$  is a nonlinear function. The overall discrete-time dynamic,  $x(k+1)$ , is represented as:

$$x(k+1) = h(x(k), u(k)) \quad (5)$$

In this paper, a quadruple-tank system, consisting of four interconnected subsystems which have first order linearized models, is analyzed using proposed approach. Thus, each state is considered as the output of its corresponding subsystem. As a result, the model for each subsystem is implemented in transfer function form.

## Approach: Proposed Cooperative Optimization Strategy

DMPC methods are divided into two main groups,

cooperative and non-cooperative, based on communication structure between local controllers. Unlike the non-cooperative group, in cooperative DMPC the same centralized global cost function, which is defined based on a combination of all cost functions, is optimized in each local controller to consider the effect of control input of each local controller on the entire plant. Each local controller optimizes its own control input by minimizing the global cost function at each iteration, supposing that other control inputs are equal to their last optimal values [1], [3], and [4]. In the proposed cooperative DMPC, the global cost function will be modified. Each local controller optimizes the global cost function which is defined based on a convex combination of its own cost function and the cost function of its neighbors and not cost functions of all other subsystems. The proposed approach assumes that if two subsystems are not neighbors, it is not necessary to consider their cost functions in each other's corresponding global cost functions. The effect of control input of each local controller is still taken into account on the entire plant which is mathematically provable. The proposed idea helps us to significantly reduce the computational burden of optimization processes of all subsystems. The proposed cooperative DMPC uses the following strategy:

1. All local controllers receive the state measurement of overall large-scale system in (5) ( $x(k)$ ) from the sensors at time  $k$ .
2. At iteration  $p$ :
  - 2.1. Each local controller  $i$  calculates its vector of future inputs,  $U_{+i}$ , along the control horizon based on  $x(k)$  and the vectors of control inputs of its neighboring local controllers (not all other controllers) which are constant and equal to their latest optimal input vectors.
  - 2.2. The neighboring local controllers exchange their vectors of control inputs, and each local controller  $i$  calculates the current optimal input vector,  $U_{+i}^{Opt^p}$ .
  - 2.3. According to receding horizon criteria the current optimal control input  $u_i^{Opt^p}$  is the first element of the current optimal input vector,  $U_{+i}^{Opt^p}$ .
3. If a termination condition which is considered in corresponding global cost function, is satisfied in current iteration, each controller  $i$  dispatches its optimal control input to its actuators; otherwise it will go to step 2 and let  $p+1 \rightarrow p$ .
4. All subsystems' dynamics are updated using obtained optimal control inputs.
5. Whenever a new measurement of  $x(k)$  is received from sensors, go to step 1 and let  $k+1 \rightarrow k$ .

Each local controller  $i$  optimizes following optimization process in each iteration:

$$\min_{\Delta u_i(k), \dots, \Delta u_i(k+N_p-1)} J(k) \quad (6-a)$$

$$U_{+i} = [\Delta u_i(k) \dots \Delta u_i(k+N_p-1)]^T$$

Subject to (5).

$$u_i(k+s) \in U_i, s = 0, \dots, N_p - 1 \quad (6-b)$$

$$U_{+j}^p = U_{+j}^{p-1}, \forall j \in M' \text{ and } j \neq i \quad (6-c)$$

$$\|\hat{y}_i(\tau) - W_i(\tau)\|_{Q_i}^2 \in \frac{N_p}{\tau - (k-1)} r_i, \quad \tau \in [k, k+N_p] \quad (6-d)$$

with:

$$J(k) = \alpha_i J_i + \sum_j \alpha_j J_j \quad (6-e)$$

$$\alpha_i, \alpha_j > 0, \alpha_i + \sum_j \alpha_j = 1 \quad (6-f)$$

and:

$$J_i(k) = \sum_{j=1}^{N_p} \|\hat{y}_i(k+j) - W_i(k+j)\|_{Q_i}^2 + \|\Delta u_i(k+j-1)\|_{R_i}^2 \quad (7)$$

where,  $N_p$  is the prediction horizon, and  $M' \subset M$  is the set of neighboring subsystems of subsystem  $i$ . The termination condition is represented in (6-d) where  $r_i$  is a small enough positive constant. The termination condition states that if the closed loop system is stable, the prediction error tends to zero over the time.  $\hat{y}_i(k+s), s = 1, \dots, N_p$  is the predicted output of subsystem  $i$  which calculates via particular manner in each linear MPC algorithm.  $W_i$  is the reference trajectory.  $Q_i$  and  $R_i$  are diagonal positive definite weighting matrices of prediction errors and increment control inputs respectively with appropriate dimensions. To achieve global optimality,  $J(k)$  is represented as a convex combination of subsystem  $i$  and its neighbors' cost functions with appropriate  $\alpha_i$  and  $\alpha_j$  coefficients.

### Approach: Proposed Cooperative Constrained DMPC Algorithm

There are two key reasons for using linear algorithms to control the nonlinear large-scale systems. First of all, the identification of linear subsystems, based on empirical data is a lot easier than nonlinear one. Secondly, most industrial nonlinear systems like liquid level control, internal temperature control of a furnace, power system components, Arrhenius temperature based chemical reaction and pressure control have only one equilibrium point. Thus, they can be identified by a precise first order linear model. On the other hand, MPC algorithms are model based control strategies, so a complicated model significantly increases the computational burden of applied MPC strategy. Considering this, in many industrial processes the first order model is used to take advantage of the MPC useful

characteristics.

In this paper, a reconstructed cooperative constrained DAGPC algorithm is proposed to control uncertain nonlinear large-scale systems. The proposed DAGPC algorithm identifies the transfer function of first order linearized system which actually presents the nonlinear behavior of the plant.

#### A. Proposed cooperative constrained DAGPC algorithm

The linear GPC algorithm is developed based on the transfer function model which can be applied to non-minimum phase and unstable linear systems [28]. The mismatch between nonlinear and linearized models will be compensated using online identification process in the proposed DAGPC approach. The numerator and denominator polynomials of first order linearized version will be identified using online RLS algorithm. The identified polynomials, free responses of nominal nonlinear subsystems, and GPC algorithm are applied to the proposed cooperative optimization strategy to calculate the optimal control trajectories. The calculated optimal control trajectories will be applied to nominal nonlinear system and this process iterates in the next time sample. The free response of each nominal nonlinear subsystem is derived from its past information of input and output.

Assume that the identified linearized model of subsystem  $i$  is:

$$y_i(k) = \frac{B_{ii}^l(z^{-1})}{A_i^l(z^{-1})} u_i(k) + \frac{\sum_{j \neq i} B_{ij}^l(z^{-1}) u_j(k)}{A_i^l(z^{-1})} \quad (8)$$

where:

$$B_{ii}^l(z^{-1}) = b_{0ii} \rightarrow n_{b_{ii}}^l = 0$$

$$A_i^l(z^{-1}) = 1 + a_{i1} z^{-1} \rightarrow n_{a_i}^l = 1$$

$$B_{ij}^l(z^{-1}) = b_{0ij} \rightarrow n_{b_{ij}}^l = 0$$

where,  $n_{a_i}^l, n_{b_{ii}}^l$  and  $n_{b_{ij}}^l$  are the orders of the identified polynomials,  $A_i^l, B_{ii}^l$  and  $B_{ij}^l$ , respectively. The DAGPC matrices will be calculated using the new identified polynomials of linearized model and finally the optimal control input will be obtained using these new matrices.

The proposed DAGPC algorithm solves the optimization problem for subsystem  $i$  at each iteration  $p$  using the novel proposed cooperative optimization strategy in which, the new identified numerator and denominator of linearized model of each subsystem are calculated in the beginning of each time sample. In this process, the cost function, which is presented in (6-a), is being subjected to (8) which presents linearized model with new identified numerator and denominator.

The cost function of subsystem  $i$  is represented as matrix form as follows:

$$J_i = (Y_i - W_i)^T Q_i (Y_i - W_i) + U_{+i}^T R_i U_{+i} \quad (9)$$

in which the predicted output matrix is:

$$Y_i = \Phi_i^l Y_{-i} + \Omega_i^l U_{+i} + \Psi_{ij}^l V_{+j} \quad (10)$$

where:

$$Y_i = \begin{bmatrix} \hat{y}_i(k+1) \\ \hat{y}_i(k+2) \\ \vdots \\ \hat{y}_i(k+N_p) \end{bmatrix}_{N_p \times 1}$$

$$\Psi_{ij}^l = [\Omega_{i1}^l \ \dots \ \Omega_{ij}^l \ \dots \ \Omega_{iM'}^l], \quad j \neq i, M' \subset M$$

$$V_{+j} = [U_{+1} \ \dots \ U_{+j} \ \dots \ U_{+M'}]^T, \quad j \neq i, M' \subset M$$

The local controller  $i$  analytically evaluates its closed form solution of future optimal control trajectory by computing the following derivative equation:

$$\frac{\partial J_i}{\partial U_{+i}} = 0 \rightarrow U_{+i} = (\Omega_i^{lT} Q_i \Omega_i^l + R_i)^{-1} \Omega_i^{lT} Q_i (W_i - \Phi_i^l Y_{-i} - \Psi_{ij}^l V_{+j}) \quad (11)$$

where:

$$Y_{-i} = \begin{bmatrix} y_i(k) \\ y_i(k-1) \end{bmatrix}_{(n_{a_i}^l+1) \times 1}$$

$$U_{+i} = \begin{bmatrix} \Delta u_i(k) \\ \Delta u_i(k+1) \\ \vdots \\ \Delta u_i(k+N_p-1) \end{bmatrix}_{N_p \times 1}$$

$$U_{+j} = \begin{bmatrix} \Delta u_j(k) \\ \Delta u_j(k+1) \\ \vdots \\ \Delta u_j(k+N_p-1) \end{bmatrix}_{N_p \times 1} = U_{+j}^{p-1}$$

Similar to the conventional GPC, the predicted output,  $\hat{y}_i(k+s)$  ( $s=1,2,\dots,N_p$ ) will be calculated using developed CARIMA and Diophantine relations as following procedure, assuming the delay is zero [28]:

$$\Delta A_i^l(z^{-1})y_i(k) = b_{0ii}\Delta u_i(k-1) + \sum_{j \neq i} b_{0ij}\Delta u_j(k-1) + C_i^l(z^{-1})e_i(k) \quad (12)$$

$$1 = E_{i_s}^l(z^{-1})\Delta A_i^l(z^{-1}) + z^{-j}F_{i_s}^l(z^{-1}) \quad (13)$$

where:

$$F_{i_s}^l(z^{-1}) = f_i^{(s,0)} + f_i^{(s,1)}z^{-1} \quad (14)$$

$$E_{i_s}^l(z^{-1}) = e_i^{(s,0)} + e_i^{(s,1)}z^{-1} + \dots + e_i^{(s,s-1)}z^{-(s-1)} \quad (15)$$

where, the coefficients of  $F_{i_s}^l(z^{-1})$  and  $E_{i_s}^l(z^{-1})$  are calculated using the following recursive equations:

$$E_{i_{s+1}}^l(z^{-1}) = E_{i_s}^l(z^{-1}) + f_i^{(s,0)}z^{-s} \quad (16)$$

$$f_i^{(s+1,s')} = f_i^{(s,s'+1)} - f_i^{(s,0)}\tilde{a}_{i(s'+1)}, \quad s' = 0, 1 \quad (17)$$

with following initial values:

$$F_{i_1}^l = z(1 - \Delta A_i^l(z^{-1})) \quad (18)$$

$$E_{i_1}^l = 1 \rightarrow e_i^{(s,0)} = 1 \quad (19)$$

where,  $\tilde{a}_{i(s'+1)}$  ( $s' = 0, 1$ ) are coefficients of the  $\Delta A_i^l(z^{-1})$ . The future outputs are calculated using (12) and (13) as follows:

$$y_i(k+s) = F_{i_s}^l(z^{-1})y_i(k) + E_{i_s}^l(z^{-1}) \left[ b_{0ii}\Delta u_i(k+s-1) + \sum_{j \neq i} b_{0ij}\Delta u_j(k+s-1) + E_{i_s}^l(z^{-1})e_i(k+s) \right] \quad (20)$$

The proper estimation of  $y_i(k+s)$  is its average so the predicted outputs are obtained using the average of  $y_i(k+s)$  in (20):

$$\hat{y}_i(k+s) = G_{i_s}^l(z^{-1})\Delta u_i(k+s-1) + \sum_{j \neq i} G_{ij_s}^l(z^{-1})\Delta u_j(k+s-1) + F_{i_s}^l(z^{-1})y_i(k), \quad s = 1, \dots, N_p \quad (21)$$

$$G_{i_s}^l(z^{-1}) = b_{0ii}E_{i_s}^l(z^{-1}) = b_{0ii}(1 + e_i^{(s,1)}z^{-1} + \dots + e_i^{(s,s-1)}z^{-(s-1)}) \quad (22)$$

$$G_{ij_s}^l(z^{-1}) = b_{0ij}E_{i_s}^l(z^{-1}) = b_{0ij}(1 + e_i^{(s,1)}z^{-1} + \dots + e_i^{(s,s-1)}z^{-(s-1)}) \quad (23)$$

$e_i(k+s)$  is the white noise, so its average is zero.

The new DAGPC matrices of predicted output's vector of the form of (10), are presented with appropriate dimensions as:

$$\Phi_i^l = \begin{bmatrix} f_i^{(1,0)} & f_i^{(1,1)} \\ f_i^{(2,0)} & f_i^{(2,1)} \\ \vdots & \vdots \\ f_i^{(N_p,0)} & f_i^{(N_p,1)} \end{bmatrix}_{N_p \times (n_{a_i}^l+1)}$$

$$\Omega_i^l = \begin{bmatrix} g_i^{(1,0)} & 0 & 0 & \dots & 0 \\ g_i^{(2,1)} & g_i^{(2,0)} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_i^{(N_p, N_p-1)} & g_i^{(N_p, N_p-2)} & \dots & \dots & g_i^{(N_p, 0)} \end{bmatrix}_{N_p \times N_p}$$

$$g_i^{(1,0)} = b_{0ii} \\ g_i^{(s,s-1)} = - \sum_{v=1}^{s-1} a_{iv}g_{i(s-v, s-1-v)} + b_{0ii}$$

$$\Omega_{ij}^l = \begin{bmatrix} g_{ij}^{(1,0)} & 0 & 0 & \dots & 0 \\ g_{ij}^{(2,1)} & g_{ij}^{(2,0)} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{ij}^{(N_p, N_p-1)} & g_{ij}^{(N_p, N_p-2)} & \dots & \dots & g_{ij}^{(N_p, 0)} \end{bmatrix}_{N_p \times N_p}$$

$$g_{ij}^{(1,0)} = b_{0ij} \\ g_{ij}^{(s,s-1)} = - \sum_{v=1}^{s-1} a_{iv}g_{ij(s-v, s-1-v)} + b_{0ij}$$

where,  $g_i^{(s,l)}$  and  $g_{ij}^{(s,l)}$  ( $s = 1, 2, \dots, N_p, l = 0, 1, \dots, N_p - 1$ ) are the coefficients of  $G_{i_s}^l(z^{-1})$  and  $G_{ij_s}^l(z^{-1})$  polynomials respectively.

### B. Stability analysis

Since the GPC is developed based on the transfer function, it is possible for it to obtain the closed-loop transfer function and derive some properties such as closed-loop stability. In (11), it is assumed that:

$$K_i = \left( \Omega_i^{lT} Q_i \Omega_i^l + R_i \right)^{-1} \Omega_i^{lT} Q_i = \begin{bmatrix} k_{i1} \\ \vdots \\ k_{iN_p} \end{bmatrix}_{N_p \times N_p} \quad (24)$$

$$= \begin{bmatrix} k_{i11} & \dots & k_{i1N_p} \\ \vdots & \ddots & \vdots \\ k_{iN_p1} & \dots & k_{iN_pN_p} \end{bmatrix}_{N_p \times N_p}$$

sufficient conditions for the closed-loop stability of the system are provided next.

**Theorem 1:** Let the overall constrained nonlinear system in (2) with the DMPC controller in (6), if:

$$|-a_{i1}| < (1 + b_{0ii}\theta_{i2}) \quad (25)$$

$$|a_{i1} + b_{0ii}\theta_{i3} - 1| < 2(1 + b_{0ii}\theta_{i2}) \quad (26)$$

hold for subsystem  $i$  with  $1 + b_{0ii}\theta_{i2} > 0$ , and if:

$$|-a_{i1}| < (1 + b_{0ij}\delta_{ij2}) \quad (27)$$

$$|a_{i1} + b_{0ij}\delta_{ij3} - 1| < 2(1 + b_{0ij}\delta_{ij2}) \quad (28)$$

hold for its neighboring subsystem  $j$  with  $1 + b_{0ij}\delta_{ij2} > 0$ , then the dynamic of subsystem  $i$  satisfies the terminal condition (6-d) and the closed loop system will be stable, where:

$$\theta_{i2} = (k_{i11}f_i^{(1,0)} + \dots + k_{i1N_p}f_i^{(N_p,0)})$$

$$\theta_{i3} = (k_{i11}f_i^{(1,1)} + \dots + k_{i1N_p}f_i^{(N_p,1)})$$

$$\delta_{ij2} = (fr_{ij11}^l f_i^{(1,0)} + \dots + fr_{ij1N_p}^l f_i^{(N_p,0)})$$

$$\delta_{ij3} = (fr_{ij11}^l f_i^{(1,1)} + \dots + fr_{ij1N_p}^l f_i^{(N_p,1)})$$

in which,  $fr_{ij11}^l, \dots, fr_{ij1N_p}^l$  are the elements of the first row of  $(\Omega_{ij}^l)^{-1}$  matrix.

**Proof.** From (11) and (24), it follows that:

$$U_{+i} = K_i (W_i - \Phi_i^l Y_{-i} - \Psi_{ij}^l V_{+j}) \quad (29)$$

From (8), it is clear that we are dealing with a multiple input model for each subsystem  $i$ , thus for calculating the closed loop system the control input of subsystem  $i$  and the control vector of its neighbouring subsystems should be taken into consideration separately. First, the control of subsystem  $i$  is considered, hence the substitution  $u_j(k) = 0$  and  $V_{+j} = 0$  in (8) and (29) respectively leads to:

$$y_i(k) = \frac{b_{0ii}}{1 + a_{i1}z^{-1}} u_i(k) \quad (30)$$

$$U_{+i} = K_i (W_i - \Phi_i^l Y_{-i}) \quad (31)$$

Current increment of optimal control input of subsystem  $i$ ,  $\Delta u_i(k)$ , is the first row of  $U_{+i}$ :

$$\Delta u_i(k) = k_{i1} (W_i - \Phi_i^l Y_{-i}) \quad (32)$$

after being substituted from (30), (32) obtains:

$$\frac{1 + a_{i1}z^{-1}}{b_{0ii}} \Delta y_i(k) = k_{i1} (w_i 1_{N_p \times 1} - \Phi_i^l Y_{-i}) \quad (33)$$

where,  $1_{N_p \times 1}$  is the column matrix in which all its elements are one, and  $w_i 1_{N_p \times 1} - \Phi_i^l Y_{-i}$  is expanded as:

$$w_i 1_{N_p \times 1} - \Phi_i^l Y_{-i} = w_i 1_{N_p \times 1} - (f_i^0 + f_i^1 z^{-1}) y_i(k) \quad (34)$$

where:

$$f_i^0 = \begin{bmatrix} f_i^{(1,0)} \\ \vdots \\ f_i^{(N_p,0)} \end{bmatrix}, f_i^1 = \begin{bmatrix} f_i^{(1,1)} \\ \vdots \\ f_i^{(N_p,1)} \end{bmatrix}$$

after being substituted from (24) and (34), (33) obtains:

$$\frac{1 + a_{i1}z^{-1}}{b_{0ii}} \Delta y_i(k) = w_i \theta_{i1} - (\theta_{i2} + \theta_{i3} z^{-1}) y_i(k) \quad (35)$$

where:

$$\theta_{i1} = (k_{i11} + \dots + k_{i1N_p})$$

$$\theta_{i2} = (k_{i11}f_i^{(1,0)} + \dots + k_{i1N_p}f_i^{(N_p,0)})$$

$$\theta_{i3} = (k_{i11}f_i^{(1,1)} + \dots + k_{i1N_p}f_i^{(N_p,1)})$$

and consequently, the closed loop transfer function is analytically yielded as:

$$\frac{y_i(k)}{w_i} = \frac{\theta_{i1} b_{0ii}}{(1 + b_{0ii}\theta_{i2}) + (a_{i1} + b_{0ii}\theta_{i3} - 1)z^{-1} - a_{i1}z^{-2}} \quad (36)$$

Second, the control vector of neighboring subsystems of subsystem  $i$  is considered, thus the substitution  $U_{+i} = 0$  in (29) leads to:

$$0 = K_i (w_i 1_{N_p \times 1} - \Phi_i^l Y_{-i} - \Psi_{ij}^l V_{+j}) \quad (37)$$

It is sufficient to calculate only the effect of one of the neighboring inputs and extend the result to the rest of them, i. e., the control of subsystem  $j$  is considered and set all other elements of  $V_{+j}$  to zero, thus the substitution  $u_i(k) = 0$  in (8) leads to:

$$y_i(k) = \frac{b_{0ij}}{1 + a_{i1}z^{-1}} u_j(k) \quad (38)$$

and also (37) lead to:

$$0 = K_i (w_i 1_{N_p \times 1} - \Phi_i^l Y_{-i} - \Omega_{ij}^l U_{+j}) \quad (39)$$

it is clear that:

$$w_i 1_{N_p \times 1} - \Phi_i^l Y_{-i} - \Omega_{ij}^l U_{+j} = 0 \quad (40)$$

the control trajectory of subsystem  $j$ ,  $U_{+j}$ , yields:

$$U_{+j} = (\Omega_{ij}^l)^{-1} (w_i 1_{N_p \times 1} - \phi_i^l Y_{-i}) \quad (41)$$

where:

$$(\Omega_{ij}^l)^{-1} = \begin{bmatrix} fr_{ij11}^l & \cdots & fr_{ij1N_p}^l \\ \vdots & \ddots & \vdots \\ fr_{ijN_p1}^l & \cdots & fr_{ijN_pN_p}^l \end{bmatrix}_{N_p \times N_p}$$

Current increment of optimal control input of subsystem  $j$ ,  $\Delta u_j(k)$ , is the first row of  $U_{+j}$ :

$$\Delta u_j(k) = fr \left( (\Omega_{ij}^l)^{-1} \right) (w_i 1_{N_p \times 1} - \phi_i^l Y_{-i}) \quad (42)$$

where,  $fr \left( (\Omega_{ij}^l)^{-1} \right)$  indicates the first row of the  $(\Omega_{ij}^l)^{-1}$  and after the substitution, (42) leads to:

$$\Delta u_j(k) = [fr_{ij11}^l \quad \cdots \quad fr_{ij1N_p}^l] (w_i 1_{N_p \times 1} - (f_i^0 + f_i^1 z^{-1}) y_i(k)) \quad (43)$$

From (38) and (43), it follows that:

$$\frac{1 + a_{i_1} z^{-1}}{b_{0ij}} \Delta y_i(k) = w_i \delta_{ij1} - (\delta_{ij2} + \delta_{ij3} z^{-1}) y_i(k) \quad (44)$$

where:

$$\delta_{ij}^1 = fr_{ij11}^l + \cdots + fr_{ij1N_p}^l$$

$$\delta_{ij}^2 = (fr_{ij11}^l f_i^{(1,0)} + \cdots + fr_{ij1N_p}^l f_i^{(N_p,0)})$$

$$\delta_{ij}^3 = (fr_{ij11}^l f_i^{(1,1)} + \cdots + fr_{ij1N_p}^l f_i^{(N_p,1)})$$

the closed loop transfer function is analytically yielded as:

$$\frac{y_i(k)}{w_i} = \frac{\delta_{ij1} b_{0ij}}{(1 + b_{0ij} \delta_{ij2}) + (a_{i_1} + b_{0ij} \delta_{ij3} - 1) z^{-1} - a_{i_1} z} \quad (45)$$

**Remark 1.** The closed-loop transfer function in (45) holds for all other elements of  $V_{+j}$ .

Therefore, based on (36), (45), and remark 1, the overall characteristic equation is obtained as follows:

$$Q(z^{-1}) = Q_{ii}(z^{-1}) \times \prod_{\substack{j=1 \\ j \neq i}}^{M'} Q_{ij}(z^{-1}) \quad (46)$$

where,  $Q_{ii}(z^{-1})$  and  $Q_{ij}(z^{-1})$  are denominators of (36) and (45) respectively. The closed loop stability conditions are investigated by applying the Jury criteria for following equations:

$$Q_{ii}(z^{-1}) = q_{ii_2} + q_{ii_1} z^{-1} - q_{ii_0} z^{-2} \quad (47)$$

$$Q_{ij}(z^{-1}) = q_{ij_2} + q_{ij_1} z^{-1} - q_{ij_0} z^{-2} \quad \forall j = 1, \dots, M' (j \neq i) \quad (48)$$

where:

$$q_{ii_2} = (1 + b_{0ii} \theta_{i_2})$$

$$q_{ii_1} = (a_{i_1} + b_{0ii} \theta_{i_3} - 1)$$

$$q_{ii_0} = q_{ij_0} = (-a_{i_1})$$

$$q_{ij_2} = (1 + b_{0ij} \delta_{ij_2})$$

$$q_{ij_1} = (a_{i_1} + b_{0ij} \delta_{ij_3} - 1)$$

First,  $Q_{ii}(z^{-1})$  is investigated. According to Jury criteria, the necessary and sufficient conditions for the polynomial  $Q_{ii}(z^{-1})$  to have no roots outside or on the unit circle are:

$$Q_{ii}(1) > 0 \quad (49)$$

$$Q_{ii}(-1) > 0 \quad (50)$$

$$|q_{ii_0}| < q_{ii_2} \quad (51)$$

$$|e_{ii_0}| > |e_{ii_1}| \quad (52)$$

$$q_{ii_2} > 0 \quad (53)$$

where:

$$e_{ii_0} = \begin{vmatrix} q_{ii_0} & q_{ii_2} \\ q_{ii_2} & q_{ii_0} \end{vmatrix}, e_{ii_1} = \begin{vmatrix} q_{ii_0} & q_{ii_1} \\ q_{ii_2} & q_{ii_1} \end{vmatrix}$$

Inequality (51) holds if inequality (53) holds. Inequalities (49), (50) and (52) are represented as:

$$q_{ii_0} + q_{ii_1} + q_{ii_2} > 0 \quad (54)$$

$$q_{ii_0} - q_{ii_1} + q_{ii_2} > 0 \quad (55)$$

$$|(q_{ii_0})^2 - (q_{ii_2})^2| > |q_{ii_1}(q_{ii_0} - q_{ii_2})| \quad (56)$$

From inequality (56) we have:

$$|q_{ii_1}| < |q_{ii_0} + q_{ii_2}| \quad (57)$$

From (54) and (54) it follows that  $q_{ii_0} + q_{ii_2} > 0$ , so the inequality (57) leads to:

$$|q_{ii_1}| < q_{ii_0} + q_{ii_2} \quad (58)$$

after being substituted from (51), inequality (58) yields:

$$|q_{ii_1}| < 2q_{ii_2} \quad (59)$$

So, the necessary and sufficient conditions for the polynomial  $Q_{ii}(z^{-1})$  to have no roots outside or in the unit circle, with  $q_{ii_2} > 0$ , are:

$$|q_{ii_0}| < q_{ii_2} \quad (60)$$

$$|q_{ii_1}| < 2q_{ii_2} \quad (61)$$

The necessary and sufficient conditions for the  $Q_{ij}(z^{-1}), j = 1, \dots, M' (j \neq i)$  in order not to have roots outside or on the unit circle, with  $q_{ij_2} > 0$ , are inspired from the above mentioned procedure of  $Q_{ii}(z^{-1})$  and are represented as:

$$|q_{ij_0}| < q_{ij_2} \quad (62)$$

$$|q_{ij_1}| < 2q_{ij_2} \quad (63)$$

for  $j = 1, \dots, M' (j \neq i)$ . This complete the proof.

### Results and Discussion

A quadruple-tank system is taken into account as a nonlinear interconnected large-scale system which consists of four coupled subsystems to demonstrate the performance and effectiveness of the proposed cooperative DAGPC algorithm. The proposed algorithm is developed based on transfer function model. Thus, each state variable that denotes height of water in corresponding tank is considered as the output of the corresponding subsystem ( $k_c = 1$ ) [6]. Hence, the following continuous-time implementation is considered as:

$$\begin{aligned} \dot{y}_1(t) &= -\frac{a_1}{A_1}\sqrt{2gy_1(t)} + \frac{a_3}{A_1}\sqrt{2gy_3(t)} + \frac{\gamma_1 k_1}{A_1}u_1(t) \\ \dot{y}_2(t) &= -\frac{a_2}{A_2}\sqrt{2gy_2(t)} + \frac{a_4}{A_2}\sqrt{2gy_4(t)} + \frac{\gamma_2 k_2}{A_2}u_2(t) \\ \dot{y}_3(t) &= -\frac{a_3}{A_3}\sqrt{2gy_3(t)} + \frac{(1-\gamma_2)k_2}{A_3}u_2(t) \\ \dot{y}_4(t) &= -\frac{a_4}{A_4}\sqrt{2gy_4(t)} + \frac{(1-\gamma_1)k_1}{A_4}u_1(t) \end{aligned}$$

The discrete-time model can be obtained using Euler derivative approximation, presented in (3), and sampling time  $T_s$ :

$$\begin{aligned} y_1(k+1) &= y_1(k) \\ &\quad + (T_s/A_1) \left( -a_1\sqrt{2gy_1(k)} \right. \\ &\quad \left. + a_3\sqrt{2gy_3(k)} + \gamma_1 k_1 u_1(k) \right) \\ y_2(k+1) &= y_2(k) \\ &\quad + (T_s/A_2) \left( -a_2\sqrt{2gy_2(k)} \right. \\ &\quad \left. + a_4\sqrt{2gy_4(k)} + \gamma_2 k_2 u_2(k) \right) \\ y_3(k+1) &= y_3(k) \\ &\quad + (T_s/A_3) \left( -a_3\sqrt{2gy_3(k)} \right. \\ &\quad \left. + (1-\gamma_2)k_2 u_2(k) \right) \\ y_4(k+1) &= y_4(k) \\ &\quad + (T_s/A_4) \left( -a_4\sqrt{2gy_4(k)} \right. \\ &\quad \left. + (1-\gamma_1)k_1 u_1(k) \right) \end{aligned}$$

where,  $y_i$  ( $i = 1, \dots, 4$ ) is the output of the subsystem  $i$  which presents its water level, and  $u_i$  ( $i = 1, 2$ ) is the control input which presents input voltage to the pump  $i$ . Definitions of  $a_i, A_i$ , ( $i = 1, \dots, 4$ ),  $\gamma_i, k_i$  ( $i = 1, 2$ ) and  $g$  are given in [6]. Typical experimental values of parameters are given in [29] as:

$$\begin{aligned} A_1 &= A_3 = 28 \text{ cm}^2 \\ A_2 &= A_4 = 32 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} a_1 &= a_3 = 0.071 \text{ cm}^2 \\ a_2 &= a_4 = 0.057 \text{ cm}^2 \\ g &= 981 \text{ cm/s}^2 \\ (k_1, k_2) &= (3.33, 3.35) \text{ cm}^3/Vs \\ (\gamma_1, \gamma_2) &= (0.7, 0.6) \end{aligned}$$

The quadruple-tank system is investigated with following set-points:

$$\begin{aligned} y_1^* &= 12.5 \text{ cm} \\ y_2^* &= 12.7 \text{ cm} \\ y_3^* &= 1.45 \text{ cm} \\ y_4^* &= 1.6 \text{ cm} \end{aligned}$$

According to experimental data, above mentioned  $k_i, \gamma_i$  and set-point values are related to minimum-phase model of the quadruple-tank system [29].

Following constraint is imposed to control inputs:

$$0 \leq u_i \leq 6, i = 1, 2$$

The objective of the control process is to maintain the water levels at the set-points. The prediction horizon is 5, the weighting matrices are  $Q_i = 1$  and  $R_i = 4$ , the sampling time is  $T_s = 0.1$ , and the initial water levels are considered to be zero. The linearized models of subsystems are identified with first order models. The simulation results of predicted outputs of subsystems are illustrated in Fig. 1, Fig. 2, Fig. 3, and Fig. 4.

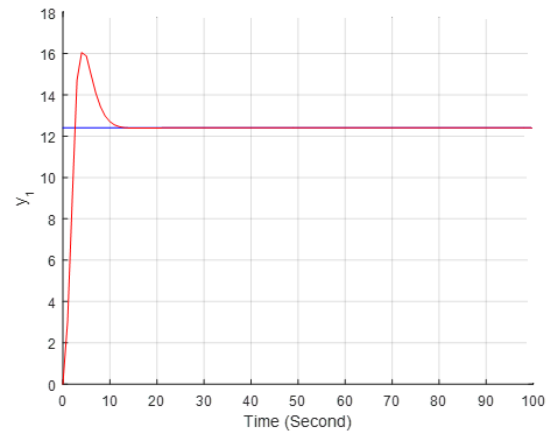


Fig. 1: The predicted output of subsystem 1.

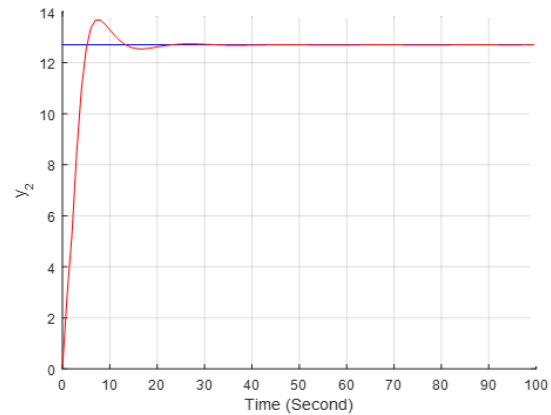


Fig. 2: The predicted output of subsystem 2.



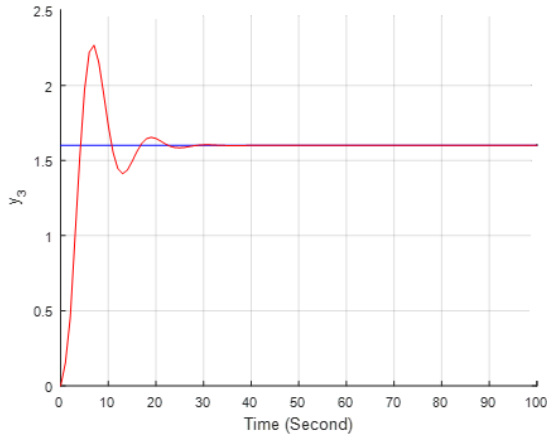


Fig. 3: The predicted output of subsystem 3.

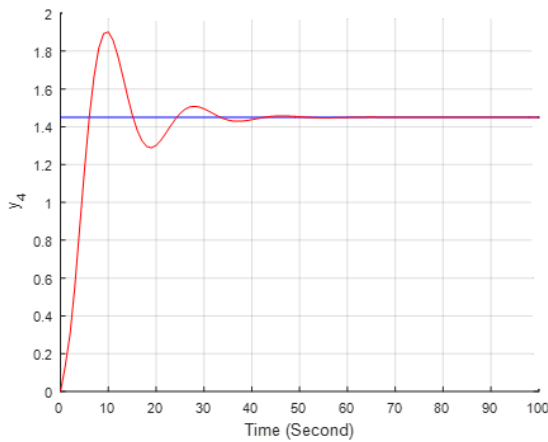


Fig. 4: The predicted output of subsystem 4.

It can be concluded that the proposed cooperative DAGPC algorithm leads to stable and feasible responses which solve proposed cooperative optimization strategy at each time instant. Based on the proposed cooperative optimization strategy, each local controller solves a global cost function that is a convex combination of its own and its neighboring subsystems' cost functions. For example, in quadruple-tank system, subsystem 1 is neighbor to the subsystem 3, but not to the subsystems 2 and 4. Thus, the corresponding global cost function which is minimized in local controller of subsystem 1 is defined as:

$$J = \alpha_1 J_1 + \alpha_3 J_3, \quad \alpha_1 + \alpha_3 = 1$$

while this global cost function, in conventional cooperative DMPCs, is defined based on all four subsystems [1], [3], and [4]:

$$J = J_1 + J_2 + J_3 + J_4$$

Thus, it is expected to decrease the computational burden of optimization and convergence time using proposed cooperative optimization strategy. Simulation results of predicted outputs of subsystems 1 and 2, are illustrated in Fig. 5 and Fig. 6 respectively using conventional cooperative DMPC, proposed in [1], [3], and [4]. It is clear from Fig. 1 and Fig. 2 that the

proposed method, in this paper, decreases the convergence times compared to the conventional cooperative DMPC which are illustrated in Fig. 5 and Fig. 6.

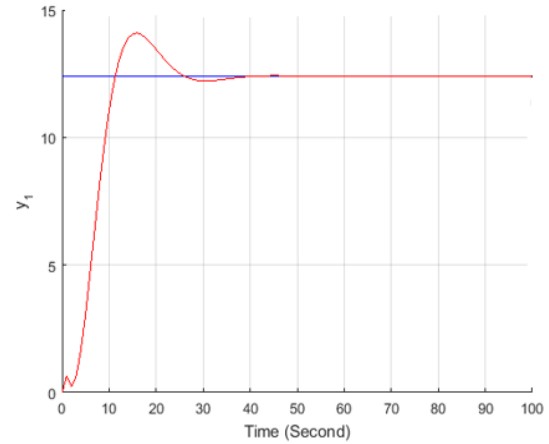


Fig. 5: The predicted output of subsystem 1, using the conventional DMPC.

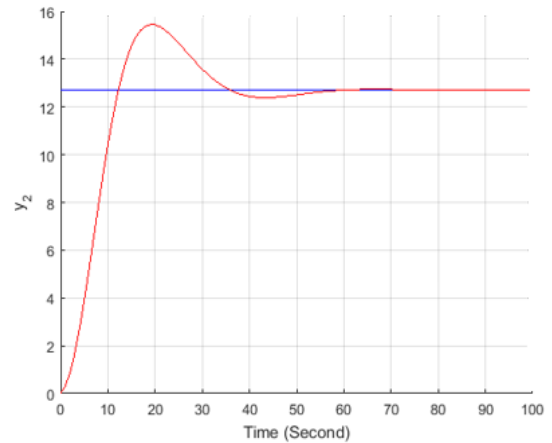


Fig. 6: The predicted output of subsystem 2, using the conventional DMPC.

In [6], the dual mode DMPC approach is proposed to investigate the quadruple-tank system. According to simulation results which are depicted in Fig. 1, Fig. 2, Fig. 3, and Fig. 4, convergence times of predicted outputs are decreased using proposed approach, in this paper, in comparison with the results which are presented in [6]. Because the proposed Cooperative DAGPC method in this paper is based on transfer function model of the system, the state variable of each subsystem is considered as the output of the corresponding subsystem.

Now, the proposed approach is examined when an uncertainty is applied. Assuming that in  $t = 60$  (sec) coefficients of corresponding pumps' flows change to following new values:  
 $(k_1^{new}, k_2^{new}) = (3.14, 3.29) \text{ cm}^3/Vs$

in which, according to the experimental data, above mentioned new coefficients are related to nonminimum-phase model of the quadruple-tank system [29]. The simulation results of predicted outputs of uncertain subsystems are drawn in Fig. 7, Fig. 8, Fig. 9, and Fig. 10. The results emphasize performance and effectiveness of proposed algorithm in dealing with the uncertainty which represents the nonminimum-phase specifications of the quadruple-tank system. Although responses fluctuate when the uncertainties are imposed, they quickly converge back to their set-points.

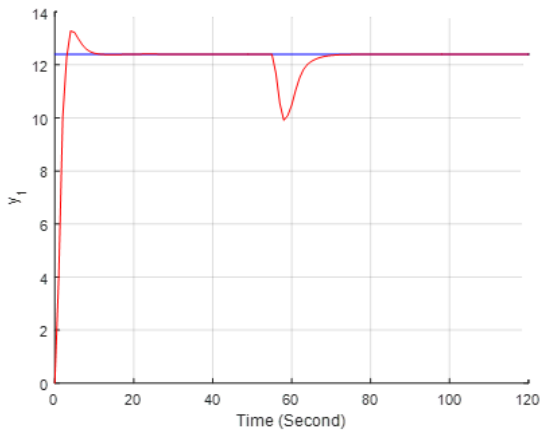


Fig. 7: The predicted output of uncertain subsystem 1.

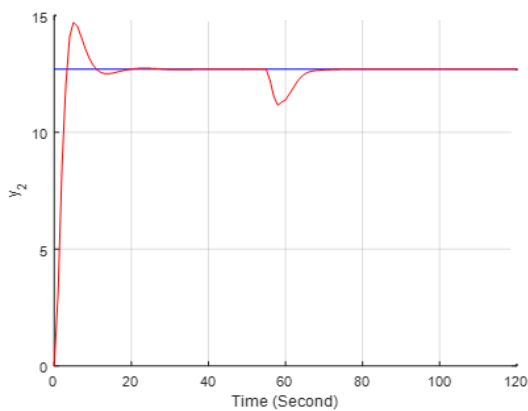


Fig. 8: The predicted output of uncertain subsystem 2.

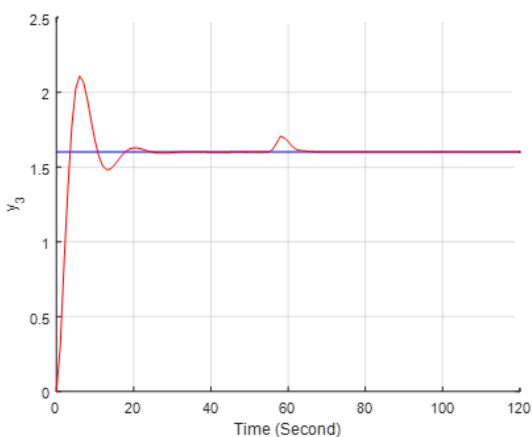


Fig. 9: The predicted output of uncertain subsystem 3.

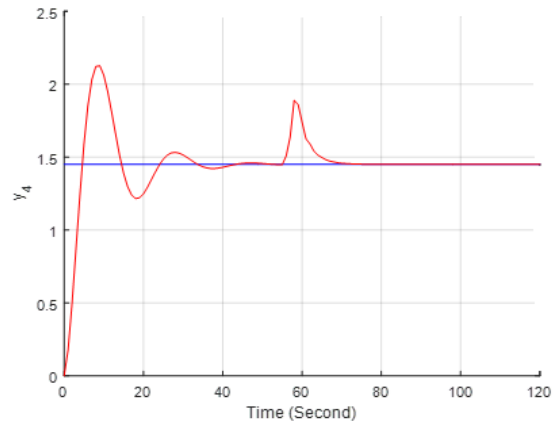


Fig. 10: The predicted output of uncertain subsystem 4.

### Conclusion

In this paper a novel cooperative DMPC strategy is proposed which improves the global cost function of each local controller. In proposed strategy, each local controller optimizes the global cost function that is a convex combination of its own and its neighboring subsystems' cost functions. So the computational burden of optimization and convergence time is reduced compared to conventional cooperative DMPC methods. Because, in these conventional cooperative DMPCs, each local controller optimizes the global cost function which is defined as a combination of its own and all other subsystems' cost functions. A linear distributed model predictive controller; DAGPC is presented which employs the proposed cooperative optimization strategy to control the uncertain coupled nonlinear large-scale systems. The simulation results of a quadruple-tank system consisting of four interconnected nonlinear subsystems with and without imposed uncertainty indicate the performance of the proposed approach. According to simulation results, proposed cooperative DAGPC algorithm leads to stable and feasible responses for both certain and uncertain models in which the imposed uncertainty represents the nonminimum-phase specifications of the quadruple-tank system.

### Author Contributions

A. Mirzaei, and A. Ramezani designed the approach. A. Mirzaei carried out the MATLAB coding. A. Mirzaei, and A. Ramezani interpreted the results and wrote the manuscript.

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### Conflict of Interest

A. Mirzaei and A. Ramezani declare that there is no conflict of interests regarding the publication of this

paper. A. Mirzaei and A. Ramezani have participated in design, analysis and interpretation of the approach and preparing the paper. This manuscript has not been submitted to, nor is under review at, another journal or other publishing venue. The authors have no affiliation with any organization with a direct or indirect financial interest in the subject matter discussed in the manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by the authors.

### Abbreviations

$J$	Cost function
$N_p$	Prediction horizon
$t$	Continuous-time index
$k$	Discrete-time index
$T_s$	Sampling time
$A^l$	Identified version of $A$
$\hat{y}$	Predicted of $y$
$Y_i$	Value of each individual observation
$\hat{Y}_i$	Predicted value

### References

- [1] P. D. Christofides, R. Scattolini, D. M. De la Pena, and J. Liu, "Distributed model predictive control: A tutorial review and future research directions," *Journal of Computers and Chemical Engineering*, vol. 51, pp. 21-41, 2013.
- [2] I. Necoara, V. Nedelcu, and I. Dumitrache, "Parallel and distributed optimization methods for estimation and control in networks," *Journal of Process Control*, vol. 21, no. 5, pp. 756-766, 2011.
- [3] B. T. Stewart, S. J. Wright, and J. B. Rawlings, "Cooperative distributed model predictive control for nonlinear systems," *Journal of Process Control*, vol. 21, no. 5, pp. 698-704, 2011.
- [4] B. T. Stewart, A. N. Venkat, J. B. Rawlings, S. J. Wright, and G. Pannocchia, "Cooperative distributed model predictive control," *Systems & Control Letters*, vol. 59, no. 8, pp. 460-469, 2010.
- [5] T. Bai, S. Li, and Y. Zheng, "Distributed Model Predictive Control for Networked Plant-wide Systems With Neighborhood Cooperation," *IEEE/CAA Journal of Automatica Sinica*, vol. 6, no. 1, pp. 108-117, 2019.
- [6] A. Grancharova, T. A. Johansen, and S. Olaru, "Dual-Mode distributed model predictive control of a quadruple-tank system," *Journal of Chemical Technology and Metallurgy*, vol. 53, no. 4, pp. 674-682, 2018.
- [7] H. N. Soloklo, and N. Bigdeli, "A PFC-based Hybrid Approach for Control of Industrial Heating Furnace," *Journal of Electrical and Computer Engineering Innovations*, vol. 7, no. 1, pp. 81-92, 2019.
- [8] Y. Gao, L. Dai, Y. Xia, and Y. Liu, "Distributed model predictive control for consensus of nonlinear second-order multi-agent systems," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 5, pp. 830-842, 2017.
- [9] N. Sadati, M. Rahmani, and M. Saif, "Two-Level Robust Optimal Control of Large-Scale Nonlinear Systems," *IEEE Systems Journal*, vol. 9, no. 1, pp. 242-251, 2015.
- [10] M. Zhao, and B. Ding, "Distributed model predictive control for constrained nonlinear systems with decoupled local dynamics," *ISA Transactions*, vol. 55, pp. 1-12, 2015.
- [11] T. Keviczky, F. Borrelli, K. Fregene, D. Godbole, and G. J. Balas, "Decentralized Receding Horizon Control and Coordination of Autonomous Vehicle Formations," *IEEE Transactions on Control Systems Technology*, vol. 16, no. 1, pp. 19-33, 2008.
- [12] X. Liu, Y. Shi, and D. Constantinescu, "Distributed model predictive control of constrained weakly coupled nonlinear systems," *Systems & Control Letters*, vol. 74, pp. 41-49, 2014.
- [13] W. B. Dunbar, "Distributed Receding Horizon Control of Dynamically Coupled Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 52, no. 7, pp. 1249-1263, 2007.
- [14] M. Heidarinejad, J. Liu, D. M. De la Pena, J. F. Davis, and P. D. Christofides, "Multirate Lyapunov-based distributed model predictive control of nonlinear uncertain systems," *Journal of Process Control*, vol. 21, no. 9, pp. 1231-1242, 2011.
- [15] H. Li, and Y. Shi, "Distributed model predictive control of constrained nonlinear systems with communication delays," *Systems & Control Letters*, vol. 62, no. 10, pp. 819-826, 2013.
- [16] J. Liu, D. M. De la Pena, and P. D. Christofides, "Distributed model predictive control of nonlinear systems subject to asynchronous and delayed measurements," *Automatica*, vol. 46, no. 1, pp. 52-61, 2010.
- [17] X. Liu, Y. Shi, and D. Constantinescu, "Robust Distributed Model Predictive Control of Constrained Continuous-Time Nonlinear Systems Using Two-Layer Invariant Sets," *Journal of Dynamic Systems, Measurement, and Control*, vol. 138, pp.1-7, 2016.
- [18] A. Ma, K. Liu, Q. Zhang, and Y. Xia, "Distributed MPC for linear discrete-time systems with disturbances and coupled states," *Systems & Control Letters*, vol. 135, pp. 1-8, 2020.
- [19] Z. Guo, S. Li, and Y. Zheng, "Feedback Linearization Based Distributed Model Predictive Control for Secondary Control of Islanded Microgrid," *Asian Journal of Control*, vol. 22, no. 1, pp. 460-473, 2020.
- [20] H. Li, and Y. Shi, "Robust Distributed Model Predictive Control of Constrained Continuous-Time Nonlinear Systems: A Robustness Constraint Approach," *IEEE Transactions on Automatic Control*, vol. 59, no. 6, pp. 1673-1678, 2014.
- [21] M. Siavash, V. J. Majd, and M. Tahmasebi, "Finite-Time Consensus Control of Euler-Lagrange Multi-agent Systems in the Presence of Stochastic Disturbances and Actuator Faults," *Journal of Electrical and Computer Engineering Innovations*, vol. 7, no. 2, pp. 163-170, 2019.
- [22] Y. Zheng, S. E. Li, K. Li, F. Borrelli, and J. K. Hedrick, "Distributed Model Predictive Control for Heterogeneous Vehicle Platoons under Unidirectional Topologies," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 3, pp. 899-910, 2017.
- [23] S. G. Abokhatwa, and R. Katebi, "Sequential Distributed Model Predictive Control for State-Dependent Nonlinear Systems," 2013 IEEE International Conference on Systems, Man, and Cybernetics, pp. 565-570.
- [24] Y. Long, S. Liu, L. Xie, and K. H. Johansson, "Distributed nonlinear model predictive control based on contraction theory," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 2, pp. 492-503, 2018.
- [25] X. Liu, Y. Shi, and D. Constantinescu, "Robust distributed model predictive control of constrained dynamically decoupled nonlinear systems: A contraction theory perspective," *Systems & Control Letters*, vol. 105, pp. 84-91, 2017.
- [26] N. R. Esfahani and K. Khorasani, "A Distributed Model Predictive Control (MPC) Fault Reconfiguration Strategy for Formation Flying Satellites," *International Journal of Control*, vol. 89, no. 5, pp. 960-983, 2016.

- [27] X. Kong, X. Liu, L. Ma, and K. Y. Lee, "Hierarchical Distributed Model Predictive Control of Standalone Wind/Solar/Battery Power System," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 8, pp. 1570-1581, 2019.
- [28] E. F. Camacho, and C. Bordons, *Model Predictive Control*, vol. II. London: Springer, 2007, p.47.
- [29] K. H. Johansson, "The Quadruple-Tank Process: A Multivariable Laboratory Process with an Adjustable Zero," *IEEE Transactions on Control Systems Technology*, vol. 8, no. 3, 2000.

## Biographies



**Ahmad Mirzaei** was born in Tehran, Iran, in 1983. He received the B.E. degree in Electrical Engineering majoring in Control from University of Tabriz, Tabriz, Iran, in 2007, and the M.sc in Control systems from Tarbiat Modares University, Tehran, Iran, in 2013. Currently he is a Ph.D. candidate in Tarbiat Modares University, Tehran, Iran. He is also a research assistant at Advanced Control Systems Laboratory in Tarbiat Modares University. His research interests include

Distributed Model Predictive Control, Large-Scale Systems, Cooperative Optimization, and Robust Control.



**Amin Ramezani** was born in Khuzestan, Iran, in 1979. He received his B.E. degree in electrical engineering from Shahid Beheshti University of Tehran, Tehran, Iran, in 2001, and his M.sc in Control systems from Sharif University of Technology, Tehran, Iran, in 2004. He received his Ph.D. degrees in electrical engineering from the University of Tehran, Tehran, Iran, in 2011. In 2012, he joined the Department of Electrical Engineering, Tarbiat Modares University,

Tehran, Iran, as a Lecturer. He is the chair of Advanced Control Laboratory and cooperates as the PC members of some conferences and special editor and reviewer of international journals in the field of control systems. His current research interests include Fault Tolerant Control Systems, Model-Based Predictive Control Systems, Stochastic Control Systems, and Hybrid Systems. He is a member of IEEE since 2005 and a member of Iranian Community Instrument and Control Engineers since 2012.

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