



# Rateless Coding over Wireless Relay Networks Using Amplify/Decode and Forward Relays

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## ABSTRACT

In this paper two different rateless transmission schemes are developed. In the proposed scheme, relay node can decode and forward the message to the destination if they are able to decode it, or amplify and forward the message to the destination. Based on the analysis and simulation results provided in this paper, the proposed method has better transmission time than the scheme which only the relay nodes that have already decoded the message participate in the second phase. Due to the simplicity of the amplify and forward transmission, adding this feature to the network can result in better performance without any additional complexity.

## 1. INTRODUCTION

From the first realization of rateless codes, fountain codes such as LT and Raptor codes were designed for the purpose of efficient and reliable transmission over erasure channels (such as in the Internet) [1, 2]. In contrast to LDPC codes, which are capable of achieving capacity over the binary erasure channel (BEC) only when the transmitter and the receiver know the channel state information (CSI) a priori [2], fountain codes have been shown capable of universally achieving capacity over BEC without any CSI available at transmitter or receiver. In recent years, fountain codes have been studied in use for the binary symmetric channel (BSC), the additive white Gaussian noise channel (AWGNC) and the fading channel [3- 6].

One special branch of research on fountain coding over wireless channels is using fountain codes in a wireless network with cooperative relays. The first framework to use fountain codes in cooperative communications was proposed by Castura and Mao in [7]. Following [8], the rateless framework proposed in [7] was based on a quasi-static Rayleigh fading relay channel, where the channel gains remain constant over the period of an entire code word transmission that might last for several thousand channel uses, and

changes independently from one code word to the next.

When the numbers of relays in the network have increased, the rateless coding design and cooperative schemes will be even more challenging. In [10], the optimization problems of a cooperative scheme in decode and forward relays has been studied. The rateless code enables the network to automatically select the best source-relay channel, after which the second phase begins, i.e. forwarding the message to the destination, starts and so one or more relays can decode the message. If relays in the network can use different rateless codes, then an information combining decoder might be used at the receiver. Following the idea in [10], a queued cooperative communication based on rateless coding has been proposed in [11], in which each relay is supported with a buffer and all messages have been buffered until being transmitted. Considering an infinitely large buffer in [11] with no control on queued message in relays can lead to a very large delay in the network. The problem of controlling the queue size of the relays has been addressed in [12], and the system stability has also been analysed.

In [13], an efficient message relaying scheme has been proposed based on which soft decoding is made possible in the receiver side. Due to graph based design of rateless codes and the decoding algorithm such as message passing algorithm, in the case of using rateless codes soft decoding can be applied in relays. In [14], the problems related to optimal message combination in the receiver side using rateless code have been studied, and it is shown that the affiliation of information combining (IC) and energy combining (EC) methods have better delay and error performance than when being solely used. It is important to note that combining the methods in [14] requires the CSI to be at the transmitter.

Issue of amplify-forward rateless coding over relay networks has not been addressed in earlier work. In the current paper, two rateless transmission schemes, namely the decode-forward rateless transmission and decode-amplify-forward rate less transmission, are proposed and compared in terms of the average successful decoding time at the destination.

The rest of the paper is organized as follows. In section II, we describe the basic system model and the assumptions underlying our analysis. Section III describes the decode-forward rateless transmission scheme. Subsequently, decode-amplify-forward rateless scheme is developed in section IV. Simulation results are shown in section V, and finally, Section VI concludes the paper.

## 2. SYSTEM MODEL

Fig. 1 shows the basic system model of a relay network. A source wants to transmit a message consisting of  $m$  packets to the destination via  $N$  parallel decode-and-forward relays. We consider that each original packet has bandwidth-normalized entropy  $H$  (nats/Hz). In order to simplify the notation, we assume that the destination is not capable of obtaining information directly from the source, though inclusion of such a direct path in the performance analysis is straightforward.

All nodes operate in half-duplex mode, i.e., they can either transmit or receive, however, not simultaneously. In the following, we also assume the transmission is done through a direct-sequence spectrum spreading technique. Such an approach is useful for sensor networks as it allows different information streams to be transmitted in a flexible and decentralized way, as well as being distinguishable at the receiver. The transmission power of all nodes is defined by  $PT$ . The propagation channels between the different nodes are modelled as frequency-flat, block-fading channels. The channel

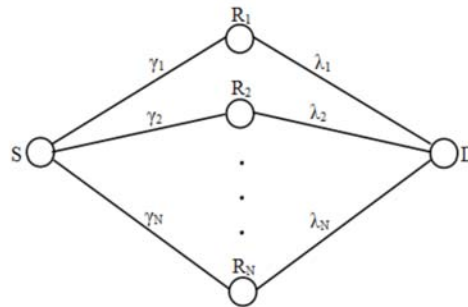


Figure 1: System model with one source,  $N$  parallel relay nodes and one destination

gains are independent and exponentially distributed, which corresponds to Rayleigh fading of their amplitudes. In the Rayleigh fading model, the probability density function (pdf) of the instantaneous channel SNR  $\gamma$  can be characterized as:

$$f_{\gamma}(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \text{ for } \gamma \geq 0 \quad (1)$$

We assume that different relay nodes can use the same rate less code or different ones. At the destination node, Rake receiver is used and then different signals that are received from different paths (transmitted from different relay nodes) are combined according to the maximum ratio combining criteria or Information combining criteria.

## 3. DECODE-FORWARD (DF) RATELESS TRANSMISSION

Initially, each packet, containing  $H$  information symbols, is encoded into an infinitely large number of bits using a rateless code and is then transmitted. Each relay node then tries to decode the message. According to the predetermined value of  $L$ , the source keeps on transmitting until  $L$  relay nodes can correctly decode the message and after receiving the  $L_{th}$  acknowledgment message from the relay nodes, the source stops its transmission. In the second phase, the  $L$  relay nodes that could have decoded the message in the first step, re-encode the message and then transmit it to the destination. First, we consider all relay nodes to transmit the source data encoded with the same rateless code, which can be the same as the one used by the source. We also assume that the Rake receiver is used to accumulate the energy from the signals transmitted by different nodes.

After decoding the message, the destination sends an acknowledgment, and relay nodes stop their transmission and the source transmits the next packet. The goal is to compute the average successful decoding time at the destination. Transmission from source to destination is done in two steps. We can calculate the average time the decoding takes in the

first step as follows.

According to Shannon's theorem of the channel capacity, the time,  $t_i$  which relay  $i$  requires to decode the message is calculated as below:

$$t_i = \frac{H}{\log(1 + \gamma_i)}, \quad \text{for } \gamma_i \geq 0 \quad (2)$$

Due to exponential distribution of  $\gamma_i$ , we can calculate the distribution of  $t_i$  as follows:

$$f_{t_i}(t) = \frac{H}{\bar{\gamma}t^2} \exp\left(\frac{1}{\bar{\gamma}} + \frac{H}{t} - \frac{e^{\frac{H}{t}}}{\bar{\gamma}}\right), \quad \text{for } t \geq 0 \quad (3)$$

We should find the time the  $L_{th}$  node requires to decode the message. Following the procedure in [10], the distribution of the time that the  $L_{th}$  relay can decode the message is:

$$f_{t(L)}(t) = \frac{N!}{(L-1)!(N-L)!} f_t(t) [F_t(t)]^{L-1} [1 - F_t(t)]^{N-L} \quad (4)$$

where  $F_t(t)$  is the cumulative distribution function (CDF) of the random variable  $t$  calculated as in (5).

$$F_t(t) = \frac{1}{H} \exp\left(\frac{1}{\bar{\gamma}} - \frac{e^{\frac{H}{t}}}{\bar{\gamma}}\right), \quad \text{for } t \geq 0 \quad (5)$$

Relay nodes use similar rateless codes, and a Rake receiver is used at the destination node for optimal combining. We assume relay nodes transmit with equal energy. Based on this assumption, the effective SNR of the channels is:

$$\lambda_{eff} = \sum_{i=1}^L \lambda_i \quad (6)$$

According to the fact that the SNR of each channel is an exponential random variable, the effective SNR is also a random variable with *Gamma* ( $L, \lambda$ ) distribution. The required time for the second phase can be calculated by (7).

$$s = \frac{H}{\log(1 + \lambda_{eff})} \quad (7)$$

And the probability distribution function of  $s$  is:

$$f_s(s) = \frac{H}{(L-1)! \bar{\lambda}^L s^2} (e^{\frac{H}{s}} - 1)^{L-1} \times \exp\left(\frac{1}{\bar{\lambda}} \left(1 - e^{\frac{H}{s}}\right) + \frac{H}{s}\right), \quad \text{for } s \geq 0 \quad (8)$$

For the case that the relay nodes use different rateless codes the required time for the second phase of transmission can be calculated as follows.

$$s = \frac{H}{\sum_{i=1}^L \log(1 + \lambda_i)} \quad (9)$$

It is important to note, in this case, information combining receiver is used at the destination.

#### 4. DECODE OR AMPLIFY-FORWARD (DAF) RATELESS TRANSMISSION

In this case all relay nodes participate in the transmission. The ones that have already decoded the packet encode it and then forward it to the destination; and also the other ones that were not able to decode the message, amplify and forward it to the destination. Due to the fact that relay nodes can use different rateless codes there are two possible rateless transmission schemes from relay nodes. One is that the relay nodes which have already decoded the packet use the same rateless code as the source is using. The other scheme is that these relay nodes use different rateless codes. Obviously, relay nodes that act as amplify-forward relays only forward the amplified version of the message to the destination and so, at the destination an amplified version of the packet that the source has encoded is received. In this paper these two proposed schemes are analysed and compared.

##### A. Same rateless code transmission

It is obvious that the first phase of the transmission is similar to the one in the decode-forward rateless transmission scheme that has been previously discussed in section II. As so, we only calculate the transmission time for the second phase. We assume that the relay  $j$  cannot decode the message in the first phase and so, it should amplify and forward the message to the destination. The receiving signal at relay  $j$  is shown in (10).

$$y_{SR}(i) = \alpha_{SR} x_s(i) + n_{SR}(i) \quad (10)$$

where  $x_s$  is the transmitted signal from the source,  $\alpha_{SR}$  and  $n_{SR}$  are the fading coefficient and the additive white Gaussian noise of the source to relay  $j$  channel, respectively. The relay node normalizes  $y_{SR}$  and then transmits it to the destination.

$$y_{R_j}(i) = \alpha_{SR_j} \frac{\alpha_{SR} \alpha_{RD}}{\sqrt{\bar{P}_y}} x_s(i) + \frac{\alpha_{RD}}{\sqrt{\bar{P}_y}} n_{SR_j}(i) + n_{RD}(i) \quad (11)$$

where  $\bar{P}_y$  is the average signal power at relay node and  $\alpha_{RD}$  and  $n_{RD}$  are the fading coefficient and the

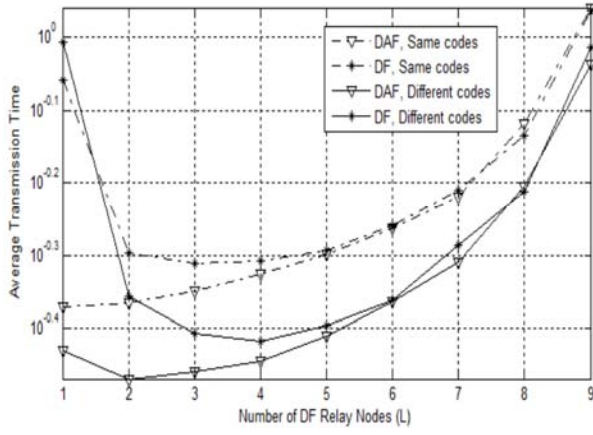


Figure 2: Average required time for successful decoding at the destination.  $H=1$ ,  $N=10$ , and average SNR of all channels are 10 dB

additive white Gaussian noise of the relay  $j$  to destination channel, respectively. Then, the overall SNR of the combined channel can be obtained from (12).

$$SNR_j = \frac{\gamma_j \lambda_j}{\gamma_j + \lambda_j + 1}, \quad (12)$$

The required time for the second phase of transmission can be calculated as follows.

$$s = \frac{H}{\log(1 + \sum_{i=1}^L \lambda_i + \sum_{i=L+1}^N SNR_i)}, \quad (13)$$

### B. Different rateless code transmission

In this transmission scheme the relay nodes that have been decoded the packet in the first phase use different rateless codes and so, at the destination, the information combining receiver can be used to decode the message. Obviously, AF relay nodes amplify and forward the packet that has been decode by a rateless code at the source; thus, an energy combining receiver should be used for packets from AF relay nodes. The required time for the second phase of transmission can be calculated as follows.

$$s = \frac{H}{\log(1 + \sum_{i=L+1}^N SNR_i) + \sum_{i=1}^L \log(1 + \lambda_i)}, \quad (14)$$

## 5. SIMULATION RESULTS

Fig. 2 shows the average transmission time for  $N=10$  when the average SNR of all channels is 10 dB. As we can see, DAF rateless transmission has better transmission time than DF rateless transmission in both scenarios, i.e., same rateless code scheme and different rateless code scheme. Furthermore, there is an optimum point for each graph which minimize the average transmission time. For the same rateless code scheme, the optimum  $L$  is 3 for DF and 1 for DAF, and for different rateless code scheme, the optimum  $L$  is 4 for DF and 2 for DAF.

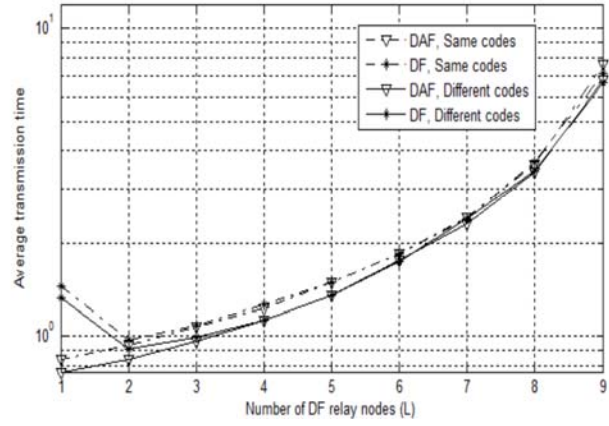


Figure 3: Average required time for successful decoding at the destination.  $H=1$ ,  $N=10$ , average SNR of S-R channels are 0 dB, and average SNR of R-D channels are 10 dB

Fig. 3 shows the average transmission time for the case that the average SNR's are 0 and 10 dB for S-R and R-D channels, respectively. The overall performance of DAF and DF schemes are the same, however, the average transmission time for DAF is optimum when  $L=1$ , but for DF it is optimum when  $L=2$ .

The average transmission time for the case that the SNR of R-D channel is 0 dB and SNR of S-R channel is 10 dB are shown in Fig. 4. It is obvious that DAF scheme has better performance than the DF one in the sense of average transmission time. For larger SNR's the performance of two different proposed schemes are similar but generally, the performance of the DAF rate less scheme is better than the DF one and in comparison with the same rate less code scenario, the different rate less code scenario has better average transmission time in both DF and DAF rate less schemes. It is important to note that, due to the simplicity of amplify and forward transmission, using the relay nodes that could not have decoded the message in the first phase to amplify and forward the message to the destination, results in lower transmission time in comparison with the decode and forward transmission scheme.

The average energy expenditure for  $N=10$  in case the SNR of all channels is 10 dB is shown in Fig. 6. Due to the fact that in DF schemes, relay nodes that have already decoded the message in the first phase only participate in the second phase, the average energy expenditure for DF schemes are lower than the one for DAF schemes. The average energy expenditure for the case that relay nodes use the same rateless code is relatively close for DF and DAF schemes, but there is a huge gap between them when relay nodes use different rateless codes. Moreover, the optimum value of  $L$  is 2 for DF schemes and for the DAF scheme, it is 6.

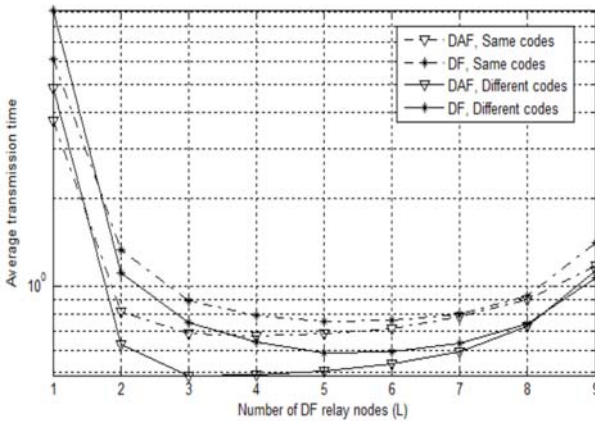


Figure 4: Average required time for successful decoding at the destination.  $H=1$ ,  $N=10$ , average SNR of S-R channels are 10 dB, and average SNR of RD channels are 0 dB

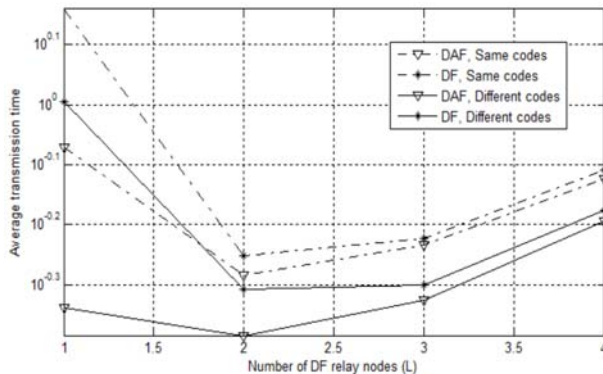


Figure 5: Average required time for successful decoding at the destination.  $H=1$ ,  $N=5$ , and average SNR of all channels are 10 dB

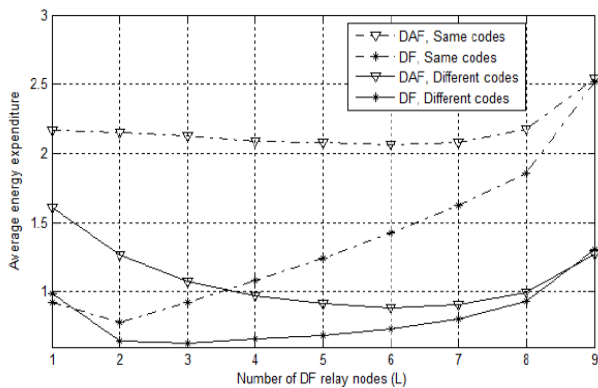


Figure 6: Average required time for successful decoding at the destination.  $H=1$ ,  $N=10$ , and average SNR of all channels are 10 dB

6. CONCLUSION

In this paper, two rateless transmission schemes have been discussed and their performance has been analysed. One scheme use Decode and Forward relay nodes and so, the ones that could have decoded the message in the first phase of the transmission

participate in the second phase. In the proposed method, i.e., DAF rateless scheme, all relay nodes can participate in the transmission and the relay nodes that have failed to decode the message in the first phase, amplify and forward the message to the destination. Due to the fact that relay nodes can use similar or different rateless codes, there are several transmission schemes that have been analysed and discussed in this paper. Simulation results show that the DAF rateless scheme have better transmission time than the DF rateless scheme. These two schemes are also compared from the perspective of average energy expenditure and it is shown that the DF scheme has better energy expenditure than the DAF scheme.

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## BIOGRAPHIES



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