Responsive Load Model Integration with SCUC to Design Time-of-Use Program

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Introduction

The concept of “demand response” (DR) that has been brought forward by the U.S. department of energy (DOE) is defined as follows: "Changes in electric usage by end-use customers from their normal consumption patterns in response to the changes in the price of electricity over time, or to the incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized" [1].

According to the DOE classification, DR programs can be categorized into time-based and incentive-based programs. Time-of-use (TOU), critical peak pricing (CPP)
and real-time pricing (RTP) programs are three of famous time-based programs. Moreover, different incentive-based programs have been introduced in the forms of long-term, short-term and even real-time schemes, each of which is designed to achieve specific goals for specific group of customers. A prominent research on DR technologies, including standards and target customers and their attributes (e.g., response frequency, response time, the load curtailment cost, and the magnitude of load curtailment) in the United States has been done [2].

Both time-based and incentive-based programs are used to motivate customers to shift or reduce their energy consumptions. The former motivates customers to respond to varying electricity price while the latter cause reaction to incentive values. Proper design of these programs enables ISOs/RTOs to improve power system operation economically and technically by decreasing operation cost, peak shaving, providing participants’ financial profit, alleviating market power, improving load factor along with managing reliability [3][4].

In electricity markets, proper designing of DR programs is one of the most important issues for market operators and retailers in such a way that it brings considerable improvement of some important technical and economic factors.

To investigate on advantages and disadvantages of the DR programs, many researches have been devoted to model responsive loads based on the linear representations [5][6][7][8][9][10]. These widely used models assume demand will vary linearly proportional to the electricity price based on the demand elasticity factors. The pioneer research has discussed various aspects of the models completely [9][10]. One of the main advantages of the DR programs is the reliability enhancement of the power system that has been studied in [8][11][12]. To simulate various DR programs under different conditions, the representation of responsive loads through a linear model has been promoted by including tariffs, incentive and penalty rates into the model. The main purpose is to study technical and economic benefits can be achieved by DR programs implementation [5][7]. The authors of [5][13] have prioritized DR programs’ merits from different viewpoints. The effect of presence of responsive loads on reducing local marginal prices has been studied [6]. Previous studies generally have examined of the DR programs impressions, assuming that elasticity, incentive and penalty rates as DR programs attributes are pre-specified. This is while less attention has been paid toward pricing DR programs as an important issue. In this paper, among different DR programs, TOU program has been selected for pricing.

TOU program is the most widespread time-based program. Based on the power utilities objective to manage daily, weekly or yearly peak demand, the countries define and employ different types of TOU program to encourage consumers for energy consumption mostly at non-peak hours. In daily TOU, a day is often divided to three parts called peak, off-peak, and low periods. The TOU programs have effects on mitigating market power (e.g.,[14]) and satisfying reliability criteria (e.g., our previous research, Error! Reference source not found., which studied the daily TOU using the stochastic SCUC framework from reliability view point) as well as economic benefit incensement [5]. A critical question and favorite requirement is how to determine electricity price for each interval. The present paper is an endeavor to address how to price a daily TOU in the framework of using security constrained unit commitment (SCUC) problem. The SCUC plays a fundamental role in operating electricity markets in which scheduling of the generating units is determined aiming at minimizing the operational costs by satisfying the prevalent constraints such as load balance limit, system spinning reserve, ramp rate limitations, minimum up and down time limitations as well as the transmission lines flow over a 24-hour period [16]. Significant efforts have been made to solve the SCUC problem [17][18][19][20]. A stochastic SCUC based on the point estimation (PEM) approach and Bender’s decomposition procedure to mitigate the computational burden, have been presented in[20]. In addition, a stochastic programming framework for coupled energy and reserve auctions with participation of wind power producers has been proposed in [19].

Through two non-cooperative games, the interaction among utility companies and customers have been modeled in a smart grid context [21]. In the utility game as a first game, by maximizing utility companies’ profit, the electricity price is determined and is transmitted to the customers. In the second game, customers regulate optimal load profile to achieve their maximum payoff. Finally, the Nash equilibrium concept of the games has been studied.

Ref. [22], for efficient participation of energy hub in the open power market with volatile prices, the electric loads have been considered to be responsive. The optimal operation of energy hubs based on a 2m+1 point estimation probabilistic scheduling model has been presented. The effect of both responsive electric demand and thermal loads in decreasing costs has been investigated.

TOU from demand side has been intelligently joined with the economic dispatch (ED) problem from supply side and its optimal tariffs during different periods namely valley, off-peak, and peak periods have been determined so that the fuel costs of generation units
have been minimized [23]. Linear and nonlinear model of incentive-based DR programs have been considered and joined with the ED problem [24]. The fuel costs have been minimized based on the same procedure of [23], and the optimal incentives of DR programs have been determined. The authors of Refs. [23] and [24] have acclaimed that the optimal prices of considered DR programs have been determined via their proposed procedure. Because of the following reasons, the calculated prices are not optimal. (i) To determine optimal prices, some predefined values have been employed. They must be considered as independent variables of an optimization problem and determined based on solving an optimization problem. (ii) The demand and supply sides have not been solved simultaneously. Indeed, at the first step, the load response to predetermined price values has been determined and then the ED problem has been solved for new load curves. In this paper, the response of loads to TOU are linearly modeled and this model is embedded to SCUC problem. The proposed TOUSCUC optimization model is linearized and solved in one step and the optimal TOU tariffs are determined.

Since we have assumed that loads will participate in TOU programs as a part of SCUC the load, behavioral model should be included. The operation costs of generating units and customers’ benefits are merged through developing a linear optimization model, which has been proposed, in harmony with a SCUC model. A linear representation has been used for responsive loads to be included as new constraints of the SCUC formulation and on this basis, TOU programs have been simulated. Moreover, requirements on daily energy consumption and limitations of DR potentials are also accounted as supplementary constraints into the model. The model serves for reserve requirement of a fixed and predetermined value. The proposed formulation is mathematically stated as follows: in Section 2, the responsive loads’ model based on the price elasticity concept is reviewed. The proposed approach for TOU pricing will be explained in Section 3. In Section 4, the presented method is verified over the IEEE 24-bus test system, and finally, the paper is concluded in Section 5.

### Linear Demand Response Model

The meaning of linear model for DR representation is to express linearly the change of a load in change of the electricity price; and it is always conducted by utilizing elasticity factors.

The elasticity of a demand is achieved from information provided by relative slope of a demand curve, which is mathematically stated by the following relationship where $\Delta d$ and $d^0$ respectively indicate an increment in load demand and initial value of the demand. The same meaning is true for $p$ as price variable.

$$e = \frac{\Delta d / d^0}{\Delta p / p^0} \quad (1)$$

In a realistic power system, loads are continuously varying over the time. The change of the price in specific periods regularly may cause to alter the loads at that time or even other intervals. Hence, cross-time elasticity factors or mutual elasticities could be determined using the cross-time coefficients. The self-elasticity coefficient $e_{tt}$, which is normally negative number, represents elasticity of a load when the price is changing at time $t$, while mutual elasticity coefficient $e_{tt'}$, which has normally positive value, represents load elasticity at time $t$ when the price has changed at time $t'$. These two concepts for demand elasticity are formulated mathematically by the following relations:

$$e_{tt} = \frac{\Delta d_t / d_t^0}{\Delta p_t / p_t^0} \quad (2)$$

$$e_{tt'} = \frac{\Delta d_t / d_t^0}{\Delta p_{t'} / p_{t'}^0} \quad (3)$$

Considering a time varying load, the elasticity factors can be prepared in a NT×NT matrix $E$.

$$E = \begin{bmatrix}
    e_{1,1} & \cdots & e_{1,NT} \\
    \vdots & \ddots & \vdots \\
    e_{NT,1} & \cdots & e_{NT,NT}
\end{bmatrix} \quad (4)$$

With linearity assumption, DR model imitates the following relation where the changes of demand are connected to the changes of electricity price at any
period via elasticity matrix.

\[
\begin{bmatrix}
\Delta d_i / d_i^0 \\
\vdots \\
\Delta d_{NT} / d_{NT}^0
\end{bmatrix} = E 
\begin{bmatrix}
\Delta \rho_i / \rho_i^0 \\
\vdots \\
\Delta \rho_{NT} / \rho_{NT}^0
\end{bmatrix}
\] (5)

**Problem Definition**

Here, first, an approximate model for customers’ benefit representation is expressed to integrate it into a linear model and implement it in the SCUC problem. Then, the considered problem is formulated based on a SCUC framework constrained with a new set of linear equations for TOU modeling.

A. Approximation of Customers’ benefit

Customers’ benefit is determined as the costs that customers will save due to participate in TOU program. Here, a flat rate for electricity price is assumed at wholesale market place and then the rates of selling energy to consumers in retail market on the form of TOU contracts is determined at all grid buses. In this regard, demand curve is divided into three integral intervals on the time axis to discriminate peak, off-peak, and low periods. To each segment, a rate on which the electricity is going to be sold is then considered. Thus, customers’ benefit function at for instance bus \( b \) can be formulated as follows:

\[
CB_b = \left( \rho_b^0 \times \sum_t d_{bt}^0 \right) - \left( \rho_b^{LTP} \times \sum_{t \in LTP} d_{bt} + \rho_b^{OTP} \times \sum_{t \in OTP} d_{bt} \right) - \left( \rho_b^{PTP} \times \sum_{t \in PTP} d_{bt} \right)
\] (6)

where

\[
d_{bt} = d_{bt}^0 + \Delta d_{bt}
\] (7)

\[
\rho_b^{LTP} = \rho_b^0 + \Delta \rho_b^{LTP}
\] (8)

\[
\rho_b^{OTP} = \rho_b^0 + \Delta \rho_b^{OTP}
\] (9)

\[
\rho_b^{PTP} = \rho_b^0 + \Delta \rho_b^{PTP}
\] (10)

Replacing (7)-(10) in (6) gives:

\[
CB_b = \left( \rho_b^0 \times \sum_t d_{bt}^0 \right) - \left( \rho_b^0 + \Delta \rho_b^{LTP} \times \sum_{t \in LTP} \left( d_{bt}^0 + \Delta d_{bt} \right) \right) - \left( \rho_b^0 + \Delta \rho_b^{OTP} \times \sum_{t \in OTP} \left( d_{bt}^0 + \Delta d_{bt} \right) \right) - \left( \rho_b^0 + \Delta \rho_b^{PTP} \times \sum_{t \in PTP} \left( d_{bt}^0 + \Delta d_{bt} \right) \right)
\] (11)

and then,

\[
CB_b = \left( \rho_b^0 \times \sum_t d_{bt}^0 \right) - \left( \sum_{t \in LTP} \left( \rho_b^0 \times d_{bt}^0 + \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{LTP} \right) + \sum_{t \in OTP} \left( \rho_b^0 \times d_{bt}^0 + \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{OTP} \right) + \sum_{t \in PTP} \left( \rho_b^0 \times d_{bt}^0 + \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{PTP} \right) \right)
\] (12)

Rearrangement of (12) will be yielded the following:

\[
CB_b = - \left( \sum_{t \in LTP} \left( \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{LTP} \right) + \sum_{t \in OTP} \left( \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{OTP} \right) + \sum_{t \in PTP} \left( \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{PTP} \right) \right)
\] (13)

Ignoring the terms of \( \Delta d_{bt}, \Delta \rho_b^{LTP}, \Delta d_{bt}, \Delta \rho_b^{OTP}, \), and \( \Delta d_{bt}, \Delta \rho_b^{PTP} \) we can be resulted in:

\[
CB_b = - \left( \sum_{t \in LTP} \left( \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{LTP} \right) + \sum_{t \in OTP} \left( \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{OTP} \right) + \sum_{t \in PTP} \left( \rho_b^0 \times \Delta d_{bt} + d_{bt}^0 \times \Delta \rho_b^{PTP} \right) \right)
\] (14)

where, it will be used as a linear approximation of customer benefit function into the developed mathematical model.

B. Problem formulation

The proposed SCUC formulation for pricing TOU program is given in this section. The proposed SCUC model is to determine the TOU rates at each load bus such that these rates cause to minimize the operating costs and maximize the customers’ benefit function while satisfying the technical constraints including DC load flow equations, upper and lower limitations of generation units, minimum up and down time constraints, ramp rate limits and line flows over a NT-
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In addition, other constraints from load modeling are also considered. They consist of equality and non-equality constraints associated with linear TOU program model, minimum requirements of energy consumption, DR potential limitation and pricing limits. The corresponding mathematical formulation has been expressed in the following:

- **Objective function:**
  
  The objective function comprises generation costs of active power and customers' benefit achieved by running TOU program:
  
  \[
  \text{Min} \sum_{i=1}^{NT} \sum_{t=1}^{NG} \left[ GC_i(P_{it}) + (SR_{it}^{up} \times q_{it}^{up} + SR_{it}^{down} \times q_{it}^{down}) \right] - \sum_{b=1}^{CB_b} C_{bb}
  \]
  
  (15)

  The hourly operation cost of units is assumed to be a quadratic function of active power generation, as follows:
  
  \[
  GC_i(P_{it}) = a_i \times P_{it} + b_i \times (P_{it})^2 + c_{oi}
  \]
  
  (16)

  To linearize the objective function, equation (16) can be well represented by a set of line segments as shown in Fig. 1.

  ![Fig. 1: Cost function of generating units with piece-wise linear approximation.](image)

  This linearization is given as follows:
  
  \[
  GC_i(P_{it}) = GC_i(P_{it}^{min}) \times u_{it} + \sum_{m=1}^{NL} \delta_{it}^m \times P_{it}^{min} + \sum_{m=1}^{NL} \pi_i^m \times \delta_{it}^m
  \]
  
  (17)

  \[
  P_{it} = \sum_{m=1}^{NL} \delta_{it}^m + P_{it}^{min} \times u_{it}
  \]
  
  (18)

  \[
  \delta_{it}^1 \leq T_i^1 - P_{it}^{min}
  \]
  
  (19)

  \[
  \delta_{it}^m \leq T_i^m - T_i^{m-1}
  \]
  
  (20)

  \[
  \delta_{it}^{NL} \leq P_i^{max} - T_i^{NL-1}
  \]
  
  (21)

  where, \( \pi_i^m \) represents the slope of the block \( m \) in the cost function curve.

  Technical constraints should be considered along with the objective function in order to characterize the system operational restrictions as well. The main constraints are:

  - **DC power flow equation:**
    
    \[
    \sum_{b=1}^{NG} \sum_{t=1}^{TN} \left[ P_{it} - (d_{it}^0 + \Delta d_{it}) = \sum_{b=1}^{NG} \left( \theta_{bt} - \theta_{bt} \right) \frac{X_{bb}}{X_{bb}} \right]
    \]
    
    (22)

    This implies that the total generation at any bus \( b \) has to be balanced with the total load at bus \( b \) even when the demand responds to the price.

  - **Upper and lower limits on generation:**
    
    \[
    P_{it} \leq P_{i}^{max} \times u_{it} - SR_{it}^{up}
    \]
    
    (23)

    \[
    P_{it} \geq P_{i}^{min} \times u_{it} + SR_{it}^{down}
    \]
    
    (24)

  - **Up- and down-spinning reserve limit:**
    
    \[
    0 \leq SR_{it}^{up} \leq RR_{i}^{up} \times \tau \times u_{it}
    \]
    
    (25)

    \[
    0 \leq SR_{it}^{down} \leq RR_{i}^{down} \times \tau \times u_{it}
    \]
    
    (26)

  - **Deterministic up- and down-spinning reserve criteria:**
    
    \[
    CAP_t \leq \sum_{b=1}^{NG} SR_{it}^{up}
    \]
    
    (27)

    \[
    CAP_t \leq \sum_{b=1}^{NG} SR_{it}^{down}
    \]
    
    (28)

  - **Minimum up and down time constraints:**
    
    \[
    [X_{it}^{on} - T_i^{on}] \times (u_{i(t-1)} - u_{it}) \geq 0
    \]
    
    (29)

    \[
    [X_{it}^{off} - T_i^{off}] \times (u_{it} - u_{i(t-1)}) \geq 0
    \]
    
    (30)

  - **Ramping up and down constraints:**
    
    \[
    P_{it} - P_{i(t-1)} \leq RR_{i}^{up} \times 60
    \]
    
    (31)

    \[
    P_{i(t-1)} - P_{it} \leq RR_{i}^{down} \times 60
    \]
    
    (32)

  - **Thermal line flow constraints:**
    
    \[
    F_{i}^{min} \leq F_{it} \leq F_{i}^{max} \quad \forall l, \forall t
    \]
    
    (33)

  - **TOU demand response program constraints:**
    
    \[
    \Delta d_{it} = d_{it}^0 \times \left\{ \sum_{i \in \text{E-LTP}} e_t \times \frac{(\Delta \rho_{b}^{LTP} - \rho_{b}^{0})}{\rho_{b}^{0}} \right\}
    \]
    
    (34)

  Based on the DR model presented in section 2, the previous equation models TOU program in which three time-periods as peak, off-peak and low have been considered.
• Energy consumption requirement:
\[
\sum_{t=1}^{NT} \Delta d_{bt} = 0 \quad \forall b
\]  
(35)
this enforces that the total energy consumption remain unchanged at each bus over the scheduling horizon.
• DR potential limits:
\[
\Delta d_{bt} \leq DRP_{b}^{up} \times d_{bt}^0
\]  
(36)
\[
\Delta d_{bt} \geq -DRP_{b}^{down} \times d_{bt}^0
\]  
(37)
Where DRP_{b}^{up}, DRP_{b}^{down} are scalars in the range of [0, 1] and indicate the maximum and minimum percent of load in each hour that customers at bus b are willing to reduce or increase. The larger amount means the more willingness to shift consumption from peak hours to off-peak hours.
• TOU pricing limits:
\[
\Delta \rho_{b}^{LTP} \leq 0
\]  
(38)
\[
\Delta \rho_{b}^{LTP} \leq \Delta \rho_{b}^{OTP}
\]  
(39)
\[
\Delta \rho_{b}^{OTP} \leq \Delta \rho_{b}^{PTP}
\]  
(40)
\[
0 \leq \Delta \rho_{b}^{PTP}
\]  
(41)
\forall b
The set of equations (38)-(41) attempts to well implement the pricing limits for a TOU program in the proposed mixed-integer linear programming.

Results and Discussion

The proposed approach has been modeled in GAMS and solved using well-known CPLEX as a MILP solver [25]. The method is examined on the IEEE 24-bus test system as shown in Fig. 2 [26].

Five scenarios are considered to demonstrate different aspects of the problem, which aims at determining efficient rates for the executed TOU program:

Scenario 1: In this scenario, as the base case, a flat rate for electricity price is offered to each customer independent of where it is connected to the grid. This is known as “flat rate contract” in the literature. Under such condition, customers are charged with a fixed rate proportional to the amount of energy they are using at any time of a day. To implement this scenario, ordinary form of SCUC formulation is sufficient to be used to achieve hourly marginal costs of the sold energy at every bus. The average hourly marginal cost in this case has been calculated 26.6 $/MWh which is assigned to the parameter \( \rho_{b}^{0} \) when other scenarios are examined.

Scenarios 2 to 5: In these scenarios, same values for DRP_{b}^{up} and DRP_{b}^{down} are considered but in each scenario the amount is different and varies in steps of 5%; say 5%, 10%, 15% and 20% in scenarios 2 to 5, respectively. The proposed TOU rates based on the

### Table 1: Daily peak load for buses

<table>
<thead>
<tr>
<th>Bus number</th>
<th>Daily peak load (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>108</td>
</tr>
<tr>
<td>2</td>
<td>97</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>74</td>
</tr>
<tr>
<td>5</td>
<td>71</td>
</tr>
<tr>
<td>6</td>
<td>136</td>
</tr>
<tr>
<td>7</td>
<td>125</td>
</tr>
<tr>
<td>8</td>
<td>171</td>
</tr>
<tr>
<td>9</td>
<td>175</td>
</tr>
<tr>
<td>10</td>
<td>195</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>265</td>
</tr>
<tr>
<td>14</td>
<td>194</td>
</tr>
<tr>
<td>15</td>
<td>317</td>
</tr>
<tr>
<td>16</td>
<td>100</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>333</td>
</tr>
<tr>
<td>19</td>
<td>181</td>
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<tr>
<td>20</td>
<td>128</td>
</tr>
<tr>
<td>21</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 2: Single line diagram of the RTS.

The test system includes 17 bulk load point customers. The parameters of a, b and c0 in equation (16) are adopted from [27] and the generation cost curve is approximated linearly by 3 segments. The daily peak loads as well as the hourly peak loads as a percentage of the daily peak for all buses have been listed in Tables 1 and 2, respectively.
suggested framework are computed and then offered to the customers which connected in different buses. Finally, their responses will be determined by means of the TOU linear model expressed in equation (34).

Table 2: Hourly peak load in percent of daily peak for buses

<table>
<thead>
<tr>
<th>Hour</th>
<th>Hourly peak load (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-1 am</td>
<td>78</td>
</tr>
<tr>
<td>1-2</td>
<td>72</td>
</tr>
<tr>
<td>2-3</td>
<td>68</td>
</tr>
<tr>
<td>3-4</td>
<td>66</td>
</tr>
<tr>
<td>4-5</td>
<td>64</td>
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<tr>
<td>5-6</td>
<td>65</td>
</tr>
<tr>
<td>6-7</td>
<td>66</td>
</tr>
<tr>
<td>7-8</td>
<td>70</td>
</tr>
<tr>
<td>8-9</td>
<td>80</td>
</tr>
<tr>
<td>9-10</td>
<td>88</td>
</tr>
<tr>
<td>10-11</td>
<td>90</td>
</tr>
<tr>
<td>11-Noon</td>
<td>91</td>
</tr>
<tr>
<td>Noon-1 pm</td>
<td>90</td>
</tr>
<tr>
<td>1-2</td>
<td>88</td>
</tr>
<tr>
<td>2-3</td>
<td>87</td>
</tr>
<tr>
<td>3-4</td>
<td>87</td>
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<td>100</td>
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<td>9-10</td>
<td>92</td>
</tr>
<tr>
<td>10-11</td>
<td>87</td>
</tr>
<tr>
<td>11-12</td>
<td>81</td>
</tr>
</tbody>
</table>

The load curve is divided into three discrete intervals: the first interval includes Low load period that is determined from 24:00 p.m. to 8:00 a.m., second one includes peak period from 17:00 p.m. to 24:00 p.m. and finally the other hours is defined as the off-peak period. Also, the corresponding values for self-elasticities and cross-elasticities are considered as reported in Table 3. As seen, self-elasticities and cross-elasticities, respectively, have been highlighted with yellow and green colors.

It is assumed that the elasticities coefficients as well as DR potentials values are the same for all buses of the test system and the largest capacity of generating unit is assigned for provision of spinning reserve at any hour of operation. Here, its value for all considered scenarios is 400 MW.

Table 3: Coefficient of self and cross elasticities

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Off-peak</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>-0.10</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td>Off-peak</td>
<td>0.014</td>
<td>-0.10</td>
<td>0.012</td>
</tr>
<tr>
<td>Peak</td>
<td>0.016</td>
<td>0.012</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

It is worth to be mentioned that the main goal of scenarios 2 to 5 is to pursue the optimum solution of the proposed SCUC problem by minimizing the operation costs while simultaneously maximizing the amount of customers’ benefit subjected to TOU-DR which are modeled by presumptions of known demand elasticity and DR potentials values.

The process of determining TOU rates in scenarios 2 to 5 has been shown in Fig. 3.

The system load demand curves for all explored scenarios have been plotted in Fig. 4. As observed from Fig. 4, in reaction to the computed TOU rates, the customers have shifted their active power consumptions from peak interval and off-peak period to low load periods in scenarios 2-5. Furthermore, this desired outcome has been strengthened by increasing DR potential. Lower active power consumption in peak periods will decrease the operation costs, since it is not compulsory that the expensive generating units to be committed only for supplying the peak load. Consequently, the cost of power generation diminishes beside the electricity energy prices are held down. Therefore, the customers can gain more benefit since they spend less payment for their bills.

Table 4: Customers economical characteristics ($/day)

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>Bus No. 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>57197.4</td>
<td>56939.2</td>
<td>56563.5</td>
<td>56070.4</td>
<td>55323.5</td>
</tr>
<tr>
<td>2</td>
<td>51371.8</td>
<td>51139.8</td>
<td>50802.4</td>
<td>50359.5</td>
<td>49688.7</td>
</tr>
<tr>
<td>3</td>
<td>95329.1</td>
<td>95184.5</td>
<td>94272.5</td>
<td>93450.6</td>
<td>92205.9</td>
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<tr>
<td>4</td>
<td>39190.8</td>
<td>39013.9</td>
<td>38756.5</td>
<td>38418.6</td>
<td>37906.9</td>
</tr>
<tr>
<td>5</td>
<td>37602</td>
<td>37432.3</td>
<td>37185.3</td>
<td>36861.1</td>
<td>36370.1</td>
</tr>
<tr>
<td>6</td>
<td>72026.4</td>
<td>71701.2</td>
<td>71228.1</td>
<td>70607.1</td>
<td>69666.6</td>
</tr>
<tr>
<td>7</td>
<td>66200.8</td>
<td>65901.9</td>
<td>65467</td>
<td>64896.2</td>
<td>64031.8</td>
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<tr>
<td>8</td>
<td>90562.6</td>
<td>90153.8</td>
<td>89558.9</td>
<td>88778.1</td>
<td>87595.6</td>
</tr>
<tr>
<td>9</td>
<td>92681.1</td>
<td>92262.6</td>
<td>91653.8</td>
<td>90854.7</td>
<td>89644.6</td>
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<td>103273.2</td>
<td>103191.4</td>
<td>102128.6</td>
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<td>11</td>
<td>140345.6</td>
<td>140234.5</td>
<td>138790.1</td>
<td>137580</td>
<td>135747.5</td>
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<tr>
<td>12</td>
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<td>102662.2</td>
<td>101604.8</td>
<td>100719</td>
<td>99377.4</td>
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<tr>
<td>13</td>
<td>167885.1</td>
<td>167752.2</td>
<td>166024.4</td>
<td>164576.9</td>
<td>162520.5</td>
</tr>
<tr>
<td>14</td>
<td>52960.6</td>
<td>52721.5</td>
<td>52373.6</td>
<td>51917</td>
<td>51225.5</td>
</tr>
<tr>
<td>15</td>
<td>176358.8</td>
<td>176219.2</td>
<td>174404.2</td>
<td>172883.6</td>
<td>170580.8</td>
</tr>
<tr>
<td>16</td>
<td>95858.7</td>
<td>95782.8</td>
<td>94796.3</td>
<td>93969.8</td>
<td>92718.1</td>
</tr>
<tr>
<td>Total Bill</td>
<td>1509377</td>
<td>1505777</td>
<td>1492684</td>
<td>1479635</td>
<td>1460062</td>
</tr>
</tbody>
</table>

Bill Savings Percent of Bill Saving (%)

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3600.7</td>
<td>16729</td>
<td>29742.7</td>
<td>49315.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5%</td>
<td>1.1%</td>
<td>2%</td>
<td>3%</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 3: Methodology to determine TOU rates.

**Scenarios 2 - 5**

Fig. 4: The impact of adopting different scenarios on system load profile.

Table 5: System economical characteristics ($/day)

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production cost</td>
<td>916397.9</td>
<td>905416.8</td>
<td>901581.3</td>
<td>891943</td>
<td>890845.1</td>
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<tr>
<td>Reserve cost</td>
<td>199669.7</td>
<td>198227.1</td>
<td>194822</td>
<td>199837.7</td>
<td>199741.9</td>
</tr>
<tr>
<td>Sum (Operation cost)</td>
<td>1116067.6</td>
<td>1103643.9</td>
<td>1096403.3</td>
<td>1091780.6</td>
<td>1090587</td>
</tr>
</tbody>
</table>
Responsive Load Model Integration with SCUC to Design Time-of-Use Program

Table 4 reports different economical features of customers including electricity bill for all load buses, total bill and the attained profit. From Table 4, it can be concluded that more participation in the DR program brings to further bill reduction for customers of all buses. Therefore, the total bill which becomes 1509377 $/day in scenario 1 will tend to 1460062 as a minimum attainable value among other scenario, by employing scenario 5.

The values of bill saving in scenarios 2 to 5 in comparison with scenario 1 have been listed in the last two rows of Table 4. It can be observed that customers' profit will increase with more participation in the proposed TOU program.

The maximum customers' profit becomes 3% for 20% DR potential, and accordingly this TOU designing leads to more satisfaction for all customers. Economic aspects of the system operation have been brought in Table 5 which show (guaranty) the operation cost reduction by calculated TOU rates. These results show that the increase of DR potential reduces operation cost. As a thumb rule, the more increase in DR potential causes the more decrease in operation cost, since the load profile is approaching to a flat profile.

Table 6: Electricity prices in different time periods ($/MWh)

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>Low</th>
<th>Off-peak</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.6</td>
<td>25.07</td>
<td>23.54</td>
</tr>
<tr>
<td>2</td>
<td>26.6</td>
<td>27.05</td>
<td>27.05</td>
</tr>
<tr>
<td>3</td>
<td>26.6</td>
<td>27.05</td>
<td>27.50</td>
</tr>
<tr>
<td>4</td>
<td>26.6</td>
<td>27.05</td>
<td>27.96</td>
</tr>
<tr>
<td>5</td>
<td>26.6</td>
<td>27.05</td>
<td>28.41</td>
</tr>
</tbody>
</table>

Table 6 shows the electricity prices that are identical for different scenarios at all buses due to no congestion in transmission lines.

In order to probe the effect of the transmission system congestion, the TOU rates have been calculated assuming that one line with a high flow capacity is congested at all hours of day. In this regard, in scenario 3 when DR potential is 10%, we reduce the flow capacity of line between bus 3 and 24 from 400 MW to 150 MW. This also limits the flow capacity between bus 15 and 24 because it is in series connection with the congested line.

In this condition, the customers’ bill saving at each load bus is calculated as 1.1% except that this amount is 0.6% and 0.8% at buses 3 and 15, respectively. Thus the total bill saving is reduced to 1% from 1.1% in case without congestion (according to Table 4). This means that lines congestion reduces profit of customers who is served at load points connected to the congested lines.

Furthermore, the operating cost increases from 1096403.3 ($/day) in case without congestion to 1129333.6 ($/day) in case with congestion. The calculated TOU rates have been reported in Table 7.

Table 7: Electricity prices in ($/MWh) in different time-periods in case of line congestion between bus 3 and bus 24

<table>
<thead>
<tr>
<th>Low</th>
<th>Off-peak</th>
<th>Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 3</td>
<td>23.74</td>
<td>26.67</td>
</tr>
<tr>
<td>Bus 15</td>
<td>23.58</td>
<td>26.99</td>
</tr>
<tr>
<td>The other load buses</td>
<td>23.54</td>
<td>27.50</td>
</tr>
</tbody>
</table>

Conclusion

In this paper, the response of loads to TOU program are reviewed based on a linear model. The TOU model has been formulated and included as a constraint to SCUC problem.

The proposed TOU-SCUC optimization framework has been linearized and programmed in GAMS environment and solved using CPLEX in one-step. While considering different DR potential levels, optimal TOU rates at load buses have been determined based on the proposed TOU-SCUC optimization model. The TOU-SCUC optimization model has been applied to the IEEE 24-bus test system.

It has been shown that offering the calculated TOU rates to customers could improve some customers’ economical characteristics such as electricity bill and profit and the best result was related to the highest participation level of customers.

The simulation demonstrates that the TOU rates leads to financial profit for all customers, reduction of peak load as well as the operation cost while 24-hour energy consumptions of customers at load buses have been fulfilled.

Furthermore, the operation cost decreases gradually by attaining more flat load profile.

In addition, the effect of lines congestion on the proposed method has been investigated and it has been shown that lines congestion leads to profit reduction of customers at load points connected to the congested lines. The presented methodology in this paper can be utilized by ISO to scheme a suitable TOU plan and attain a flat and optimum load curve in which an appropriate trade-off between operation cost and customers’ benefit is achieved owing to the objective function implemented in this paper.

Author Contributions

M. Nikzad and A. Samimi provided problem modeling and mathematical formulation of the study. M. Nikzad collected the data, performed the simulations in GAMS software and supplied the drafting of manuscript. M. Nikzad and A. Samimi interpreted the results and wrote the manuscript. A. Samimi revised the article critically for
important intellectual content and gave final approval of
the version to be submitted

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Conflict of Interest
All authors have no conflict of interest to report.

Abbreviation

- **0**: Initial state index (Superscript)
- **m**: Block index of cost function (superscript)
- **LTP**: Low time period index (superscript)
- **OTP**: Off-peak time period index (superscript)
- **PTP**: Peak time period index (superscript)
- **t, t̂**: Time period indices (subscript)
- **b, b̂**: Bus indices (subscript)
- **i**: Generation unit index (subscript)
- **l**: Line index (subscript)
- **NG**: Number of generating units
- **NG_b**: Number of generating units connected to bus b
- **NL**: Number of blocks of the piecewise linearization of the cost function
- **NN**: Number of buses
- **NT**: Number of scheduling hours
- **d**: Demand (MW)
- **ρ**: Price ($/MWh)
- **Δρ**: ($/MWh)
- **e_t**: Self elasticity
- **e_t̂**: Cross elasticity
- **E**: Elasticity matrix
- **CAP_t**: Total amount of considered reserve capacity in megawatt at time t
- **CB_b**: Customer's benefit at bus b
- **GC_i**: Generation cost (k$/h)
- **P_{it}**: Real power generation of unit i at time t (MW)
- **\( p_i^{\text{max}}, p_i^{\text{min}} \)**: Maximum and minimum real power generation (MW)
- **\( u_{it} \)**: Commitment state of unit i at time t
- **\( \theta \)**: Slope of segment m of the piecewise linear cost function of unit i
- **\( \pi_i^m \)**: Real power generation of unit i in segment m at time t
- **X_{bb}**: Bus angle
- **\( X_{bn}, X_{nb} \)**: Reactance of line between bus b and b̂
- **\( X_{it}, X_{ot} \)**: On time of unit i at time t
- **\( X_{it}^{\text{off}}, X_{ot}^{\text{off}} \)**: Off time of unit i at time t
- **\( T_{it}^{\text{on}}, T_{it}^{\text{off}} \)**: Minimum up time of unit i
- **\( T_{it}^{\text{on}}, T_{it}^{\text{off}} \)**: Minimum down time of unit i
- **\( RR_{it}^{\text{up}}, RR_{it}^{\text{down}} \)**: Ramp-up limit of unit i (MW/min).
- **\( RR_{it}^{\text{up}}, RR_{it}^{\text{down}} \)**: Ramp-down limit of unit i (MW/min).
- **\( SR_{it}^{\text{up}}, SR_{it}^{\text{down}} \)**: Deployed up- and down-spinning reserve of unit i at time t
- **\( q_{it}^{\text{up}}, q_{it}^{\text{down}} \)**: Offered energy cost of unit i for providing up- and down-spinning reserve at time t
- **\( F_{it} \)**: Real power flow of line l at time t
- **\( p_{it}^{\text{max}}, p_{it}^{\text{min}} \)**: Maximum and minimum line flow limits of line l at time t
- **\( \rho_b^{\text{LTP}} \)**: TOU rate for low time periods at bus b ($/MWh)
- **\( \rho_b^{\text{OTP}} \)**: TOU rate for off-peak time periods at bus b ($/MWh)
- **\( \rho_b^{\text{PTP}} \)**: TOU rate for peak time periods at bus b ($/MWh)
- **\( \Delta \rho_b^{\text{LTP}} \)**: Rate change of low time periods at bus b ($/MWh)
- **\( \Delta \rho_b^{\text{OTP}} \)**: Rate change of off-peak time periods at bus b ($/MWh)
- **\( \Delta \rho_b^{\text{PTP}} \)**: Rate change of peak time periods at bus b ($/MWh)
- **\( DR_{it}^{\text{up}}, DR_{it}^{\text{down}} \)**: Demand response potential for increasing load at bus b
- **\( DR_{it}^{\text{up}}, DR_{it}^{\text{down}} \)**: Demand response potential for reducing load at bus b
- **\( \tau \)**: Spinning reserve market lead time (minute)
References


Biographies

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