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Research paper

Stochastic Block NIHT Algorithm for Adaptive Block-Sparse System Identification

Z. Habibi[1,*], H. Zayyani[2], M.S.E Abadi[3]

[1]Research Institute for Information and Communications Technologies, Academic Center for Education, Culture and Research, Tehran, Iran.

[2]Faculty of Electrical and Computer Engineering, Qom University of Technology, Qom, Iran.

[3]Electronics Engineering Department, Faculty of Electrical Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran.

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*Corresponding Author's Email Address:
zhabibi9819@gmail.com

Abstract

Background and Objectives: Compressive sensing (CS) theory has been widely used in various fields, such as wireless communications. One of the main issues in the wireless communication field in recent years is how to identify block-sparse systems. We can follow this issue, by using CS theory and block-sparse signal recovery algorithms.

Methods: This paper presents a new block-sparse signal recovery algorithm for the adaptive block-sparse system identification scenario, named stochastic block normalized iterative hard thresholding (SBNIHT) algorithm. The proposed algorithm is a new block version of the SSR normalized iterative hard thresholding (NIHT) algorithm with an adaptive filter framework. It uses a search method to identify the blocks of the impulse response of the unknown block-sparse system that we wish to estimate. In addition, the necessary condition to guarantee the convergence for this algorithm is derived in this paper.

Results: Simulation results show that the proposed SBNIHT algorithm has a better performance than other algorithms in the literature with respect to the convergence and tracking capability.

Conclusion: In this study, one new greedy algorithm is suggested for the block-sparse system identification scenario. Although the proposed SBNIHT algorithm is more complex than other competing algorithms but has better convergence and tracking capability performance.

Introduction

Compressive sensing (CS) theory has been widely used in various fields, such as mathematics, signal processing, and wireless communications [1]-[3]. One of the main issues in the wireless communication field in recent years is how to identify block-sparse systems. According to [4]-[8], by using the CS theory and proposing block-sparse signal recovery algorithms, we can follow this issue. In the sparse systems, the impulse response of the system has a small number of non-zero coefficients. In some practical cases, the impulse response of the system may include a small number of subspaces (block) (see Fig. 3), which is known as a block-sparse system. With this description, we can model a block-sparse system with impulse response $\mathbf{h} \in R^N$ as

$$\mathbf{h} = [\underbrace{h_1,\ldots,h_d}_{\mathbf{h}^T[1]}, \underbrace{h_{d+1},\ldots,h_{2d}}_{\mathbf{h}^T[2]},\ldots,\underbrace{h_{N-d+1},\ldots,h_N}_{\mathbf{h}^T[L]}]^T \quad (1)$$

where $\mathbf{h}^T[i]\ (i \in \{1,2,...,L\})$, indicates the *i*-th block of the **h**, *d* is the block length, and *N* = *dL*. The **h** vector is termed *K* block-sparsity, where $K \in \{1,2,...,L\}$, is the maximum number of blocks involving nonzero components.

Three major approaches sparse signal recovery (SSR) algorithms including convex optimization algorithms [4], [9], greedy pursuit algorithms [5], [8], and stochastic gradient descent (SGD) algorithms [10] have been recently developed to implement the block-sparse signal reconstruction.

In order to reconstruct block-sparse signals, in the convex optimization class, in [4], [9], the basis pursuit (BP) algorithm has been extended the l_1-minimization to a mixed l_2/l_1-norm minimization in the recovery algorithm. Also an improved algorithm, named dynamic recovery of block-sparse signal (D-BSS) has been proposed in [11]. In [12], the proposed algorithm benefits from $l_{0(}l_{2)}$ penalty to reconstruct the block-sparse signal. However the mentioned algorithms show high computational cost, and so they are not suitable for the large-scale scenario [13].

Recently, a block version of the greedy pursuit algorithms such as matching pursuit (MP), orthogonal MP (OMP), stagewise OMP (StOMP), and iterative hard thresholding (IHT), named BMP, BOMP [4], BStOMP [8], and BIHT [5] respectively, have been introduced that show a better performance than their original versions for the block-sparse signal recovery scenario. Moreover, the extended version of the compressive sampling matching pursuit (CoSaMP) algorithm has been presented in [5]. Also, the block normalized iterative hard thresholding (BNIHT) algorithm, has been proposed in [14]. Although the greedy pursuit class algorithms have low computational complexity, its performance degrades under strong background noise [13].

The SGD-based sparse adaptive filtering algorithms such as zero-attracting least-mean-square (ZA-LMS) algorithm and l_0 -norm least mean square (l_0-LMS) algorithms in [15], which have shown moderate computational complexity and robustness against noise [13], prove a better performance than other the types of the SSR algorithms in the sparse signal reconstruction process. Using this approach, several SGD-based algorithms have been proposed for the block-sparse signal reconstruction in recent years. In the block-sparse LMS (BS-LMS) algorithm [16], and block zero-point attracting projection (BZAP) [17], a penalty of block sparsity as a mixed norm of the adaptive tap-weights as $l_{2,0}$-norm in [16], and a mixed $l_{1,0}$-norm in [17] have been added to the cost function of the LMS and ZAP algorithms respectively. Moreover, block zero attracting LMS (BZA-LMS) and the block l_0-norm LMS (Bl_0-LMS) algorithms which can sense the block-sparse structure information of the block-structured sparse signal, have been presented in [18].

In order to improve the convergence performance of the solution of the block-sparse system identification problem, this paper presents a new block version of the NIHT algorithm with an adaptive filter framework, named stochastic block normalized iterative hard thresholding (SBNIHT) algorithm. In this work, we use a search method to identify the blocks of the impulse response of the block-sparse system that we wish to estimate. Simulation results show that the proposed algorithm has a faster convergence rate and a better tracking capability than other the competing algorithms in the literature.

Adaptive Filter Framework for Sparse Signal Recovery

Based on the CS theory, by using the proper dictionary matrix $\mathbf{A} \in R^{M \times N}\ (M << N)$ that satisfies the restricted isometry property (RIP), we can compress the sparse signal $\mathbf{s} \in R^N$ to a down-sampling signal $\mathbf{q} \in R^M$ [19]. To achieve more accurate results, the unavoidable background noise **v** e.g., additive white Gaussian noise (AWGN) can be considered as an additional term in the SSR equation. Therefore, an SSR problem is formulated by the following underdetermined equation

$$\mathbf{q} = \mathbf{A}\mathbf{s} + \mathbf{v} \tag{2}$$

In order to achieve less complexity cost and robustness against background noise, we can use an adaptive signal processing framework to solve the SSR problem [20].

In the sparse systems identification scenario which can be considered as an SSR problem, the desired signal with adaptive filtering framework is achieved as

$$d(n) = \mathbf{u}^T(n)\mathbf{h}_o + \eta(n) \tag{3}$$

where $\mathbf{u}(n) = [u(n), u(n-1), ..., u(n-M+1)]^T$, is the input signal vector, $(.)^T$ denotes the transpose, $\mathbf{h}_o$ is the impulse response of the unknown system with the length *M* that we wish to estimate, and $\eta(n)$ indicates the additive white Gaussian noise. Also, the recursion error is obtained as

$$e(n) = d(n) - \mathbf{u}^T(n)\mathbf{h}(n) \tag{4}$$

where $\mathbf{h}(n) = [h_1(n), h_2(n), ..., h_M(n)]^T$, denotes the iterative reconstruction impulse response vector. In Table 1 and Fig. 1, we can see the SSR problem that is effectively solved by the adaptive framework, while each row vector $\mathbf{a}_j$ in the dictionary matrix **A** in (2) plays the role of $\mathbf{u}^T(n)$ in the adaptive framework in (3), and the components of the compressed measurement vector **q** in (2) that is introduced as q_j, is regarded as $d(n)$ in (3) [13].

Table 1: Corresponding variables between CS problem and adaptive framework [13].

CS Problem	Adaptive Framework
$\mathbf{a}_j, j \in \{1,2,\dots,M\}$	$\mathbf{u}^T(n)$
$\mathbf{s}(n)$	$\mathbf{h}(n)$
$q_j = \mathbf{a}_j\mathbf{s} + v_j$	$d(n) = \mathbf{u}^T(n)\mathbf{h}_o + \eta(n)$

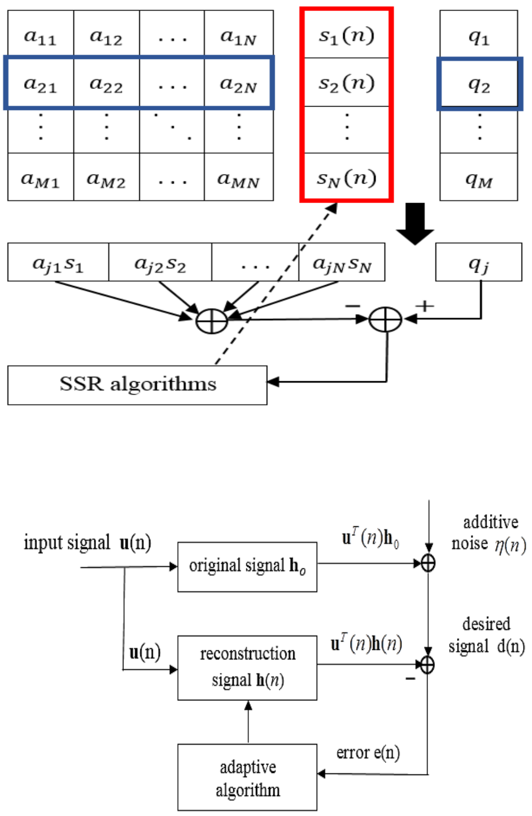


Fig. 1: Adaptive framework for CS problem [13].

Proposed Algorithm

To improve the convergence performance of the solution of the block-sparse system identification problem, this paper by considering this problem as an SSR problem, presents a new block version of the greedy NIHT algorithm with an adaptive filter framework. The proposed stochastic block normalized iterative hard thresholding (SBNIHT) algorithm uses a search method to identify the blocks of the impulse response of the unknown the block-sparse system.

To proceed, we make the following assumptions:

A1: The impulse response of the unknown block-sparse system has at most *K* blocks of nonzero coefficients;

A2: The maximum length of the blocks including nonzero coefficients is L;

A3: The numbers of zero coefficients between two adjacent blocks are at least L.

The proposed algorithm benefits from a cost function as:

$$J = \frac{1}{2}E\{|e(n)|^2\} \tag{5}$$

where $E\{.\}$ denotes the expectation, and $e(n)$ is the recursion error which is obtained as (4).

Based on (4), and dropping the time index for simplicity, i.e., $\mathbf{R} \equiv \mathbf{R}(n)$, we can expand (5) for a time-varying system as

$$J(\mathbf{h},\mathbf{R},\mathbf{r}) = \frac{1}{2}\mathrm{E}\{|d - \mathbf{u}^T\mathbf{h}|^2\}$$

$$= \frac{1}{2}(\mathrm{E}\{d^2\} - 2\mathbf{h}^T\mathbf{r} + \mathbf{h}^T\mathbf{R}\mathbf{h}) \tag{6}$$

where $\mathbf{R} = E\{\mathbf{u}(n)\mathbf{u}(n)^T\}$ is the autocorrelation matrix of the input vector **u**(n), and $\mathbf{r} = E\{\mathbf{u}(n)d(n)\}$ is the cross-correlation of the input vector **u**(n) and the desired signal $d(n)$.

Assuming that the unknown system has a time-varying nature, we use an exponentially time-averaged window to obtain the **R** matrix and the **r** vector

$$\mathbf{R}(n) = \lambda\mathbf{R}(n-1) + \mathbf{u}(n)\mathbf{u}(n)^T \tag{7}$$

$$\mathbf{r}(n) = \lambda\mathbf{r}(n-1) + \mathbf{u}(n)d(n)^* \tag{8}$$

where $\lambda \in (0,1]$ is the forgetting factor, and $(.)^*$ denotes the complex conjugate. It is noteworthy that we consider the term *stochastic* for the proposed algorithm because the **R** matrix and **r** vector in (7) and (8), are stochastic quantities.

In the following, we obtain the negative gradient of (6) at the n-th iteration as

$$-\nabla_{\mathbf{h}}J(\mathbf{h},\mathbf{R},\mathbf{r}) = \mathbf{r} - \mathbf{R}\mathbf{h} \tag{9}$$

By using the steepest descent principle, we have

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \mu(n)\nabla_{\mathbf{h}}J(\mathbf{h},\mathbf{R},\mathbf{r}), \tag{10}$$

where $\mu(n)$ denotes the step-size of the n-th time iteration. By substituting (9) into (10), we can rewrite (10) as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu(n)(\mathbf{r}(n) - \mathbf{R}(n)\mathbf{h}(n)) \tag{11}$$

In the following to make a better estimate of the unknown block-sparse system, we apply a block-sparsity constraint as a hard thresholding operator $H_k(.)$ (similar to the NIHT algorithm [21]) to the right-hand site of (11)

$$\mathbf{h}(n+1) = H_k(\mathbf{h}(n) + \mu(n)(\mathbf{r}(n) - \mathbf{R}(n)\mathbf{h}(n))) \tag{12}$$

where, by defining

$$\bar{\mathbf{h}}(n) = H_k(\mathbf{h}(n)) \tag{13}$$

and

$$\begin{aligned}\mathbf{p}(n) &= H_k(\mathbf{r}(n) - \mathbf{R}(n)\mathbf{h}(n)) \\ &= \mathbf{r}(n) - \mathbf{R}(n)\bar{\mathbf{h}}(n)\end{aligned} \tag{14}$$

and substituting (13) and (14) into (12), we can rewrite the update formula of tap-weights for the proposed SBNIHT algorithm as

$$\mathbf{h}(n+1) = \bar{\mathbf{h}}(n) + \mu(n)\mathbf{p}(n) \tag{15}$$

where the vector $\bar{\mathbf{h}}(n)$ in (13-15) is an estimated impulse response vector of the adaptation step *n,* (**h**(*n*)), which its components that do not belong to the support set $\Lambda^{(n)}$, have been zeroed. In here, we consider the support set $\Lambda^{(n)}$ as the indexes sets of the *K* blocks with the length *L* involving nonzero components in the estimated vector **h**(*n*). By using the line search optimization method, we can consider the step-size of the SBNIHT algorithm in (15) as

$$\mu(n) = \frac{\mathbf{p}_{|\Lambda^{(n)}}(n)^T \mathbf{p}_{|\Lambda^{(n)}}(n)}{\mathbf{p}_{|\Lambda^{(n)}}(n)^T \mathbf{R}_{|\Lambda^{(n)}}(n)\mathbf{p}_{|\Lambda^{(n)}}(n)} \tag{16}$$

where $\mathbf{p}_{|\Lambda^{(n)}}(n)$ is a sub-vector of the **p**(*n*) vector that contains only the elements with the indexes sets of the support set $\Lambda^{(n)}$ and $\mathbf{R}_{|\Lambda^{(n)}}(n)$ is the sub-matrix of the **R** matrix that includes corresponding columns and rows, indexed by support set $\Lambda^{(n)}$.

A. *Identify the Support Set* Λ

In the following we present a new method with a search approach to identify the support set Λ at each iteration. To achieve this purpose, we define at first, a new vector named $\tilde{\mathbf{h}}$, equal to the iterative reconstruction **h** vector

$$\tilde{\mathbf{h}} = \mathbf{h} \tag{17}$$

Then, according to Fig. 2, we find the index of the component with the largest absolute value of the magnitude in the vector $\tilde{\mathbf{h}}$

$$\Omega = \arg\max(|\tilde{\mathbf{h}}|, 1) \tag{18}$$

Then, in the vector $\tilde{\mathbf{h}}$, between the *L* blocks with the length *L* that contain the obtained index set Ω (see Fig. 2), we find the index set of the block with the largest l_2-norm value

$$\Delta = \arg\max(\{\|\tilde{\mathbf{h}}_{[1]}\|_2, \|\tilde{\mathbf{h}}_{[2]}\|_2, \ldots, \|\tilde{\mathbf{h}}_{[L]}\|_2\}, 1) \tag{19}$$

Then, we change the magnitude of the components of the obtained block $\tilde{\mathbf{h}}_{[\Delta]}$ as

$$\tilde{\mathbf{h}}_{[\Delta]} = \{\tilde{\mathbf{h}}_{(\Omega+\Delta-L)}, \tilde{\mathbf{h}}_{(\Omega+\Delta-L+1)}, \ldots, \tilde{\mathbf{h}}_{(\Omega+\Delta-1)}\} \tag{20}$$

into zero value

$$\tilde{\mathbf{h}}_{|\Psi_m} = 0 \tag{21}$$

where $m \in \{1, 2, \ldots, K\}$, and

$$\Psi_m = \{\Omega+\Delta-L, \Omega+\Delta-L+1, \ldots, \Omega+\Delta-1\} \tag{22}$$

includes the indexes sets of the vector $\tilde{\mathbf{h}}_{[\Delta]}$. In order to find the support set Λ at each time iteration, we repeat the stages (18-22), until the finding *K* blocks. Therefore, the support set Λ at each time iteration is obtained by merging indexes sets which are resulted in each stage as the subspace Ψ_m where $m \in \{1, 2, \ldots, K\}$

$$\Lambda^{(n)} = \Psi_1 \cup \Psi_2 \cup \ldots \cup \Psi_K \tag{23}$$

Then, the block sparse vector $\bar{\mathbf{h}}(n+1)$ can be obtained by pruning the elements which do not belong to the support set $\Lambda^{(n)}$

$$\bar{\mathbf{h}}_{|\Lambda(n)}(n+1) = \mathbf{h}_{|\Lambda(n)}(n+1), \quad \bar{\mathbf{h}}_{|\Lambda(n)^C}(n+1) = 0 \tag{24}$$

According to the above stages, we can summarize the steps of the proposed the algorithm at each time instant as:

Step 1) Update the proxy vector via (14);

Step 2) Define the step-size value by using the line search optimization method as (16), and estimate the unknown vector **h** via the gradient update equation (15);

Step 3) Identify the support set Λ that shows the set of the positions of the nonzero components corresponding to the *K* blocks with the length *L* of the estimated vector **h**;

Step 4) Prune the elements which do not belong to the support set Λ, for the estimated vector **h**.

The steps of the SBNIHT algorithm for the *n*-th time iteration are summarized in Table 2.

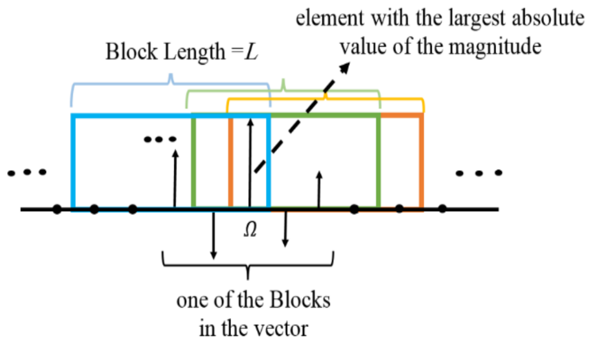


Fig. 2: Method of the finding a block in the block-sparse vector via the proposed algorithm.

Convergence Analysis

To see how the recursive update **h**(n) converges toward $\mathbf{h}_o$, we rewrite (20) as

$$\mathbf{h}(n+1) = (\mathbf{I} - \mu(n)\mathbf{R}(n))\bar{\mathbf{h}}(n) + \mu(n)\mathbf{r}(n) \tag{25}$$

where **I** is the M-by-M identity matrix. Next, from (15) and by subtracting $\mathbf{h}_o$ from both sides of (25), we can rewrite (29) as

$$\mathbf{h}(n+1) - \mathbf{h}_o = (\mathbf{I} - \mu(n)\mathbf{R}(n))(\bar{\mathbf{h}}(n) - \mathbf{h}_o) \tag{26}$$

the $\bar{\mathbf{h}}(n+1)$ is the best approximation to **h**(n+1), if we have

$$\begin{aligned} \|\bar{\mathbf{h}}(n+1) - \mathbf{h}_o\|_2 &\le \|\mathbf{h}(n+1) - \mathbf{h}_o\|_2 \\ &= \|(\mathbf{I} - \mu(n)\mathbf{R}(n))(\bar{\mathbf{h}}(n) - \mathbf{h}_o)\|_2 \end{aligned} \tag{27}$$

Defining the vector **v**(n) as

$$\mathbf{v}(n) = \bar{\mathbf{h}}(n) - \mathbf{h}_o \tag{28}$$

and substituting this in (27), we obtain

$$\begin{aligned} \|\mathbf{v}(n+1)\|_2 &\le \|(\mathbf{I} - \mu(n)\mathbf{R}(n))\mathbf{v}(n)\|_2 \\ &\le \|\mathbf{I} - \mu(n)\mathbf{R}(n)\|_2 \|\mathbf{v}(n)\|_2 \end{aligned} \tag{29}$$

where $\|\mathbf{I} - \mu(n)\mathbf{R}(n)\|_2$ denotes the spectral norm of $\mathbf{I} - \mu(n)\mathbf{R}(n)$ matrix, and defined as

$$\begin{aligned} \|\mathbf{I} - \mu(n)\mathbf{R}(n)\|_2 &= \sqrt{\lambda_{\max}[(\mathbf{I}-\mu(n)\mathbf{R}(n))(\mathbf{I}-\mu(n)\mathbf{R}(n))^*]} \\ &= \sqrt{\lambda_{\max}[\mathbf{I}-\mu^2(n)\mathbf{R}(n)\mathbf{R}^*(n)-2\mu(n)\mathbf{R}(n)]} \\ &= \sqrt{1+\mu^2(n)\lambda^2_{\max}[\mathbf{R}(n)]-2\mu(n)\lambda_{\max}[\mathbf{R}(n)]} \\ &= \sqrt{(1-\mu(n)\lambda_{\max}[\mathbf{R}(n)])^2} \\ &= |1-\mu(n)\lambda_{\max}[\mathbf{R}(n)]| \end{aligned} \tag{30}$$

The proposed algorithm converges in mean square sense, if we have in (29).

Table 2: SBNIHT algorithm for the n-th time iteration

Input: Maximum block length L; length of unknown system M; maximum number of blocks involving nonzero coefficients K; Input vector $\mathbf{u} \in R^{M\times 1}$; desired signal d.

Initialize: $\bar{\mathbf{h}} = 0 \in R^{M\times 1}$, $\mathbf{R}(0) = 0 \in R^{M\times M}$, $\mathbf{r}(0) = 0 \in R^{M\times 1}$, $\Lambda^{(1)} = [1:KL]$

Step 1: *gradient update*

$$\mathbf{R}(n) = \lambda\mathbf{R}(n-1) + \mathbf{u}(n)\mathbf{u}(n)^T$$

$$\mathbf{r}(n) = \lambda\mathbf{r}(n-1) + \mathbf{u}(n)d(n)^*$$

$$\mathbf{p}(n) = \mathbf{r}(n) - \mathbf{R}(n)\bar{\mathbf{h}}(n-1)$$

Step 2: *line search optimization*

$$\mu(n) = \frac{\mathbf{p}_{|\Lambda^{(n)}}(n)^T \mathbf{p}_{|\Lambda^{(n)}}(n)}{\mathbf{p}_{|\Lambda^{(n)}}(n)^T \mathbf{R}_{|\Lambda^{(n)}}(n)\mathbf{p}_{|\Lambda^{(n)}}(n)}$$

$$\mathbf{h}(n) = \bar{\mathbf{h}}(n-1) + \mu(n)\mathbf{p}(n)$$

Step 3: *support set update*

$$\tilde{\mathbf{h}} = \mathbf{h}(n+1);$$

for m=1:K

$$\Omega = \arg\max(|\tilde{\mathbf{h}}|, 1)$$

for $i = 1, 2, \ldots, L$

$$\tilde{\mathbf{h}}_{[i]} = \{\tilde{\mathbf{h}}_{(\Omega+i-L)}, \tilde{\mathbf{h}}_{(\Omega+i-L+1)}, \ldots, \tilde{\mathbf{h}}_{(\Omega+i-1)}\}$$

$$\|\tilde{\mathbf{h}}_{[i]}\|_2 = \sqrt{\sum_{j=i-L}^{i-1} \tilde{\mathbf{h}}^2_{(\Omega+j)}}$$

end

$$\Delta = \arg\max(\{\|\tilde{\mathbf{h}}_{[1]}\|_2, \|\tilde{\mathbf{h}}_{[2]}\|_2, \ldots, \|\tilde{\mathbf{h}}_{[L]}\|_2\}, 1)$$

$$\Psi_m = \{\Omega+\Delta-L, \Omega+\Delta-L+1, \ldots, \Omega+\Delta-1\}$$

$$\tilde{\mathbf{h}}_{|\Psi_m} = 0,$$

end

$$\Lambda^{(n)} = \Psi_1 \cup \Psi_2 \cup \ldots \cup \Psi_K$$

Step 4: *pruning*

$$\bar{\mathbf{h}}_{|\Lambda(n)}(n) = \mathbf{h}_{|\Lambda(n)}(n), \quad \bar{\mathbf{h}}_{|\Lambda(n)^C}(n) = 0$$

$$\frac{\|\mathbf{v}(n+1)\|_2}{\|\mathbf{v}(n)\|_2} < 1 \tag{31}$$

Therefore, based on (29) and (30), a necessary mean-square convergence condition obtained as

$$\|\mathbf{I} - \mu(n)\mathbf{R}(n)\|_2 = |1 - \mu(n)\lambda_{\max}[\mathbf{R}(n)]| < 1 \tag{32}$$

The inequalities (36) can be expanded as

$$-1 < 1 - \mu(n)\lambda_{\max}[\mathbf{R}(n)] < 1 \tag{33}$$

Therefore, based on (33), a necessary mean-square convergence condition obtained as

$$0 < \mu(n) < \frac{2}{\lambda_{\max}[\mathbf{R}(n)]} \quad (34)$$

Complexity

In Table 3, the computational complexity of the proposed algorithm is compared with the other state-of-the-art algorithms in the literature. The comparison is carried out in terms of multiplications per adaptation step of the algorithm.

For the proposed SBNIHT algorithm, we have an increase in complexity compared with the other competing algorithms including BZA-LMS, [18] BS-PNLMS [22], BS-MPNLMS [23], and $l_{2,0}$-SMPNLMS [24], due to the existence of the gradient **p**(*n*) and vector product $\mathbf{R}_{|\Lambda}(n)\mathbf{p}_{|\Lambda}(n)$.

But instead of this increasing complexity, the proposed algorithm can provide a much better solution for the block-sparse system identification problem compared to other competing algorithms.

Table 3: Comparison in terms of multiplications

Adaptive Algorithm	Complexity Order per Adaptation Step
BZA-LMS	*O*(M)
BS-PNLMS	*O*(M)
BS-MPNLMS	*O*(M)
$l_{2,0}$-SMPNLMS	*O*(M)
BNIHT	*O*(MKL)
SBNIHT	*O*(MKL)

Results and Discussion

In this section, the proposed algorithm is compared with the algorithms BZA-LMS, BS-PNLMS, BS-MPNLMS, $l_{2,0}$-SMPNLMS, and BNIHT in the application of block-sparse system identification.

The unknown system is a network echo path with the length *M=512*, and the adaptive filter has the same length.

In order to evaluate the tracking capability, in all simulations, we switch the echo path from the one-cluster in Fig. 3(a) to the two-clusters in Fig. 3(b) in iteration 5000.

The input vector is a white Gaussian sequence or an AR(1) signal with a pole at 0.9 or a speech signal. The background noise $\eta(n)$ is a white Gaussian process with a signal-to-noise ratio (SNR) of 30 dB.

All the results are averaged over 30 independent trials.

The normalized mean square deviation (NMSD), which is used to compare the convergence and the tracking capability performance, is defined as

$$NMSD_{(dB)} = 10\log_{10}(\|\mathbf{h}(k) - \mathbf{h}_0\|^2 / \|\mathbf{h}_0\|^2) \quad (35)$$

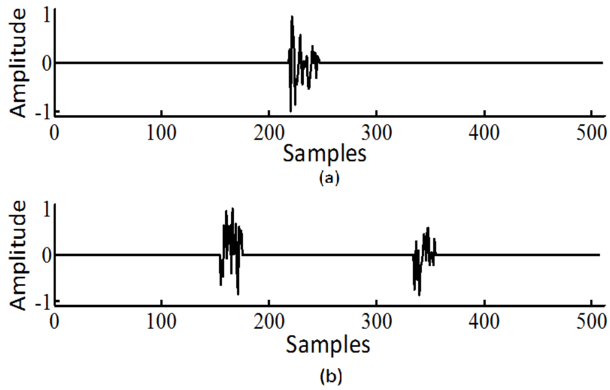
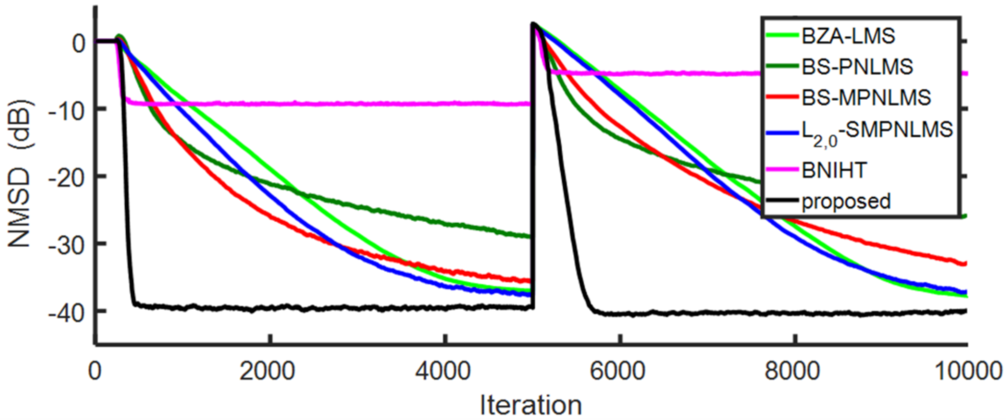


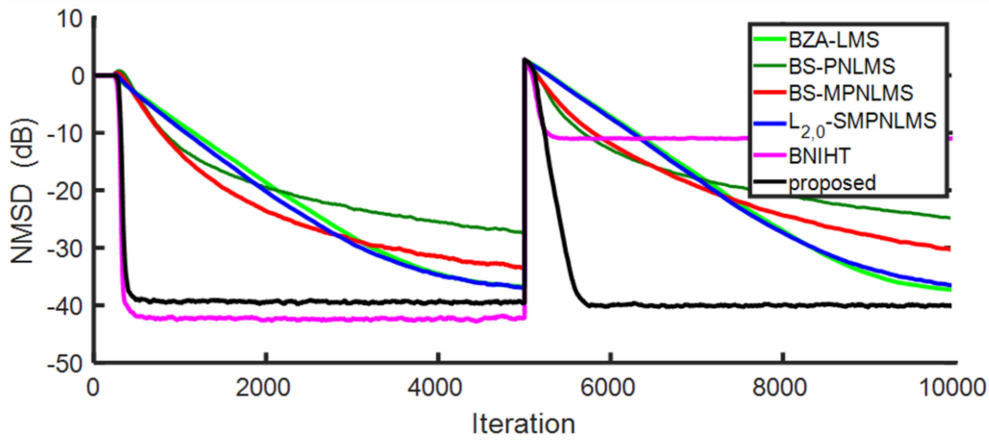
Fig. 3: Two types of measured acoustic-echo-channels as the unknown block-sparse systems, (a) one cluster, (b) two clusters.

In the following, we assume that the impulse response of the unknown system has a maximum of 2 blocks with nonzero coefficients where the maximum block length of these 2 blocks is 32, and the number of zero coefficients between two adjacent blocks are at least 32. According to the mentioned system, we consider in all simulations, our system as Fig. 3, which is one-cluster or two-clusters with a maximum block length 28. By using the above assumptions, we consider for the proposed SBNIHT algorithm, the maximum number of the blocks as *K*=2, and the maximum block length as *L*=32. Also for a better comparison, the number of the blocks for the BNIHT algorithm is considered as 2, as same as the SBNIHT algorithm. In addition, we chose the value of the parameter of the competing algorithms in all the simulations, according to the best values in their references, in such a way that all the algorithms have the same steady-state NMSD with the maximum convergence speed in achieving such a steady-state level.

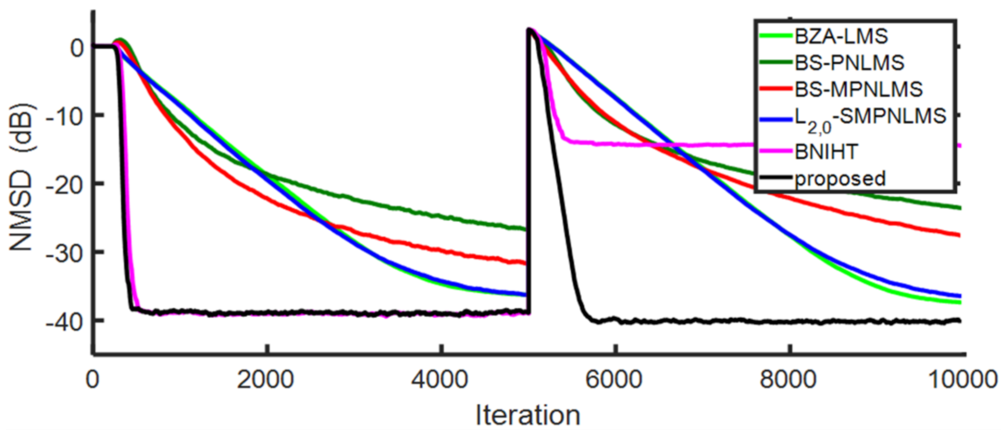
Figure 4 shows the NMSD curves of the BZA-LMS, BS-PNLMS, BS-MPNLMS, $l_{2,0}$-SMPNLMS, BNIHT, and the proposed SBNIHT algorithms for a white Gaussian sequence input signal, by considering the different block lengths $L \in \{8,16,32\}$ for the BZA-LMS, BS-PNLMS, BS-MPNLMS, BNIHT and $l_{2,0}$-SMPNLMS algorithms. For the fair comparisons, the value of the algorithm parameters in Fig. 4 and Fig. 5 are selected as: BZA-LMS ($\mu = 0.002$, $\delta = 0.8, \lambda = 1\times10^{-7}$), BS-PNLMS ($\mu = 0.8, q = 0.01$), BS-MPNLMS ($\mu = 0.02, \beta = 0.5, \delta = 0.1$), and $l_{2,0}$-SMPNLMS ($\beta = 20, \delta = 0.01, \kappa = 0.01$). It is observed that the performance of the proposed algorithm is evidently better than other competing algorithms, in terms of convergence and tracking capability.



(a)

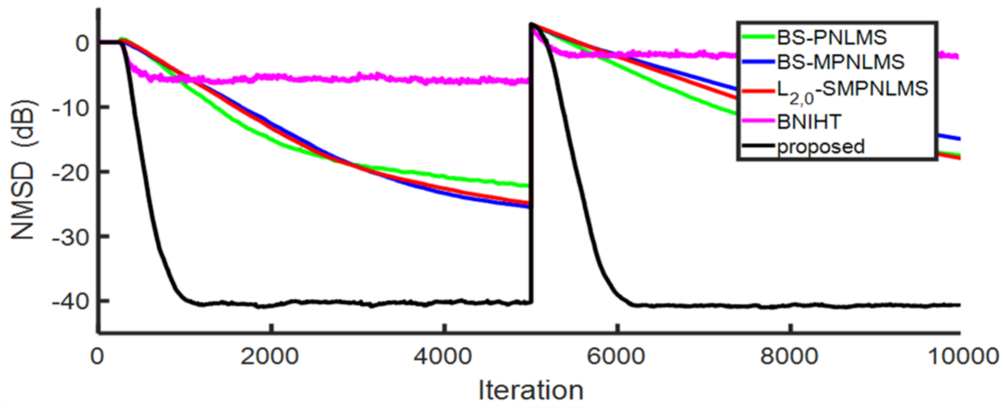


(b)

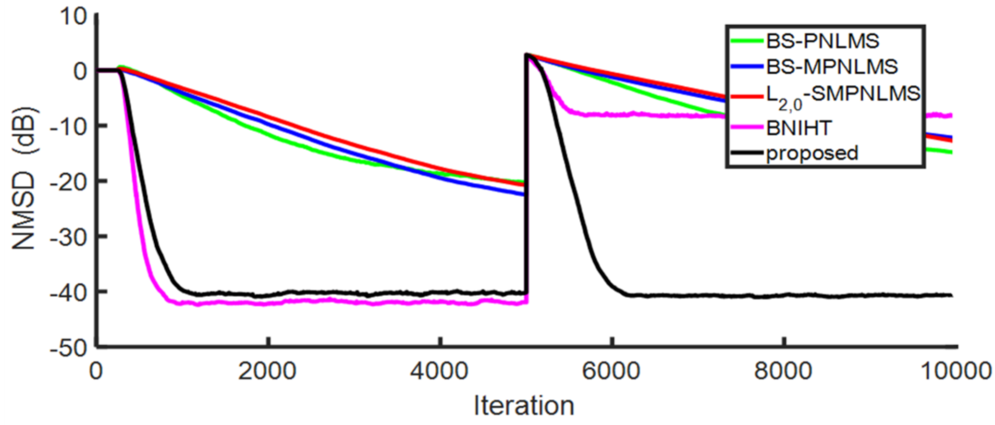


(c)

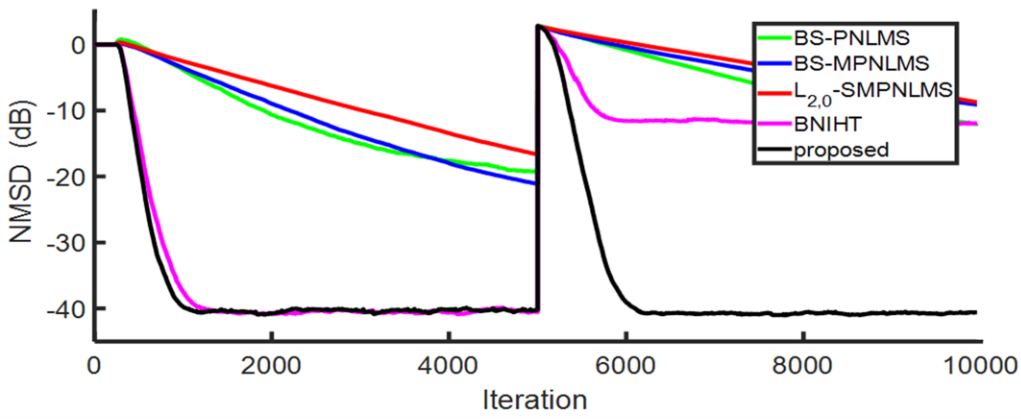
Fig. 4: NMSD learning curves of the several algorithms and the proposed algorithm with a WGN input signal for the different block length of competing algorithms BZA-LMS, BS-PNLMS, BS-MPNLMS , l2,0-SMPNLMS and BNIHT, (a) L=8, (b) L=16, (C) L=32.



(a)

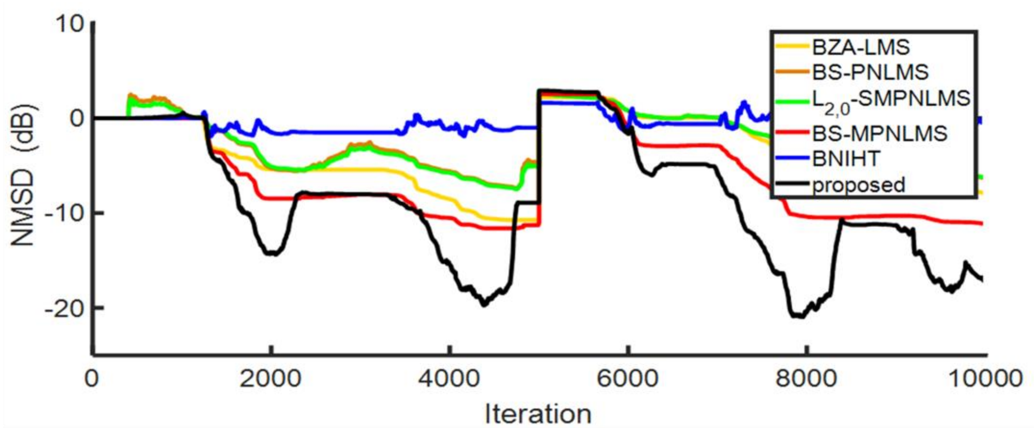


(b)

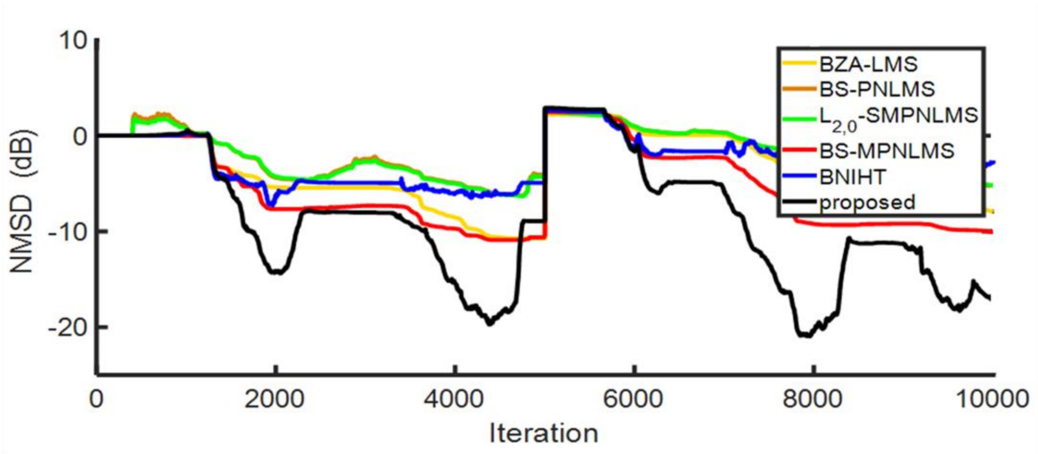


(c)

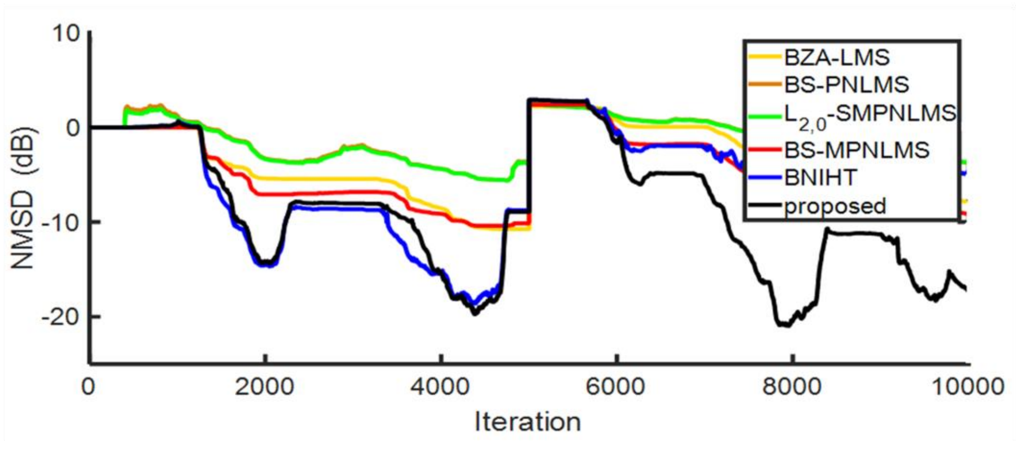
Fig. 5: NMSD learning curves of the several algorithms and the proposed algorithm with an AR(1) input signal for the different block length of competing algorithms BS-PNLMS, BS-MPNLMS, l2,0-SMPNLMS and BNIHT, (a) L=8, (b) L=16, (C) L=32.



(a)



(b)



(c)

Fig. 6: NMSD learning curves of the several algorithms and the proposed algorithm with a speech input signal for the different block length of competing algorithms BZA-LMS, BS-PNLMS, BS-MPNLMS, l2,0-SMPNLMS and BNIHT, (a) L=8, (b) L=16, (C) L=32.

Figure 5 shows the NMSD curves of the BS-PNLMS, BS-MPNLMS, $l_{2,0}$-SMPNLMS, BNIHT and the proposed SBNIHT algorithms for an AR(1) input signal generated by filtering white Gaussian noise using a first-order $H(z) = 1/(1-0.9z^{-1})$ system.

As same as Fig. 4, we consider the different block lengths $L \in \{8,16,32\}$ for the BS-PNLMS, BS-MPNLMS, BNIHT, and $l_{2,0}$-SMPNLMS algorithms. As same as the white input signal in Fig. 4, we can see that for the colored AR(1) input signal, the performance of the proposed algorithm is evidently better than other competing algorithms, in terms of convergence and tracking capability.

We do not consider the BZA-LMS algorithm in Fig. 5, because it shows a bad convergence performance for the colored AR(1) input signal.

Figure 6 shows the NMSD curves of the BZA-LMS, BS-PNLMS, BS-MPNLMS, BNIHT and SBNIHT algorithms for a speech input signal, by considering the different block lengths $L \in \{8,16,32\}$ for the BZA-LMS, BS-PNLMS, BNIHT, and BS-MPNLMS algorithms. For fair comparisons, the value of the algorithm parameters in Fig. 6 are selected as: BZA-LMS $(\mu = 0.1, \delta = 0.8, \lambda = 1\times 10^{-7})$, BS-PNLMS $(\mu = 0.1, q = 0.01)$, BS-MPNLMS $(\mu = 0.9, \beta = 0.1, \delta = 0.1)$. It is also observed that the performance of the proposed algorithm is evidently better than other competing algorithms, in terms of convergence and tracking capability.

Also, we can see that for all the input signals, the BNIHT algorithm shows a bad tracking capability.

Conclusion

Adaptive filter algorithms have been widely used in various fields, such as system identification, channel equalization, and noise cancellation.

In many system identification scenarios for example acoustic echo path, the impulse response of the system is block-sparse.

In the block-sparse systems, the coefficients of the system are in the form of a single cluster or multi-cluster, wherein a cluster is a gathering of nonzero coefficients. To solve the block-sparse system identification problem, we use the normalized iterative hard thresholding (NIHT) algorithm that is one of the effective algorithms in the compressive sensing (CS) field, with an adaptive filter framework as a basis of the our work in this paper. The proposed algorithm named stochastic block normalized iterative hard thresholding (SBNIHT) algorithm is a new block version of the greedy NIHT algorithm with an adaptive filter framework.

The SBNIHT algorithm uses a new search method to identify the blocks of the impulse response of the unknown block-sparse system. In addition, in this paper, the necessary condition to guarantee the convergence of SBNIHT is derived. Although the proposed SBNIHT algorithm is more complex than other state-of-the-art algorithms in the literature, but Simulation results demonstrate that the proposed algorithm has better convergence and tracking capability.

Author Contributions

Z. Habibi proposed algorithm, did the simulations, interpreted the results, and wrote the manuscript. H. Zayyani corrected the proofing of the article. M.S.E. Abadi supported the article.

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Conflict of Interest

The authors declare no potential conflict of interest regarding the publication of this work. In addition, the ethical issues including plagiarism, informed consent, misconduct, data fabrication and, or falsification, double publication and, or submission, and redundancy have been completely witnessed by the authors.

Abbreviations

CS	Compress Sensing
IHT	Iterative Hard Thresholding
NIHT	Normalized IHT
BIHT	Block-IHT
BNIHT	Block-NIHT
BS	Block-Sparse
SSR	Sparse Signal Recovery
SGD	Stochastic Gradient Descent
BP	Basis Pursuit
MP	Matching Pursuit
OMP	Orthogonal MP
StOMP	Stagewise OMP
CoSaMP	Compressive Sampling Matching Pursuit
ZA-LMS	Zero-Attracting Least-Mean-Square
l_0-LMS	l_0-norm Least Mean Square
BS-LMS	Block-Sparse LMS
ZAP	Zero-point Attracting Projection
BZAP	Block-ZAP
BZA-LMS	Block Zero Attracting LMS
Bl_0-LMS	Block l_0-norm LMS
RIP	Restricted Isometry Property

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Biographies

ZAHRA Habibi was born in 1984. She received her B.S. degree from Amirkabir University of Technology, Theran, Iran, in 2007 and her M.S. degree from Malek Ashtar University of Technology, Tehran, Iran, in 2011. She is currently pursuing her Ph.D. degree with the Reserch Institute for Information and Communications Technologies, Academic Center for Education, Culture and Research, Tehran, Iran. Her research interests include adaptive filters, sparse systems identification.

HADI Zayyani was born in 1978. He received his B.Sc., M.Sc., and Ph.D. degrees from Sharif University of Technology, Tehran, Iran. He is currently with the Faculty of Electrical and Computer Engineering, Qom University of Technology, Qom, Iran. His research interests include statistical signal processing, sparse signal processing, compressed sensing, adaptive filters and their applications.

Mohammad Shams Esfand Abadi was born in 1978. He received his B.S. degree from Mazandaran University, Mazandaran, Iran and his M.S. degree from Tarbiat Modares University, Tehran, Iran in 2000 and 2002, respectively, and his Ph.D. degree from Tarbiat Modares University, Tehran, Iran in 2007. He is currently with the faculty of Electrical and Computer Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran. His research interests include digital filter theory and adaptive signal processing algorithms.

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