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Research paper

An Adaptive Cubature Kalman filter for Target Tracking

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Abstract

Background and Objectives: The target tracking problem is an essential component of many engineering applications.The extended Kalman filter (EKF) is one of the most well-known suboptimal filter to solve target tracking. However, since EKF uses the first-order terms of the Taylor series nonlinear extension functions, it often makes large errors in the estimates of state. As a result, target tracking based on EKF may diverge.
Methods: In this manuscript, an adaptive square root cubature Kalman filter (ASRCKF) is poposed to solve the maneuvering target tracking problem. In the proposed method, the covariance of process and measurement noises is estimated adaptively. Thus, the performance of proposed method does not depend on the noise statistics and its performance is robust with unknown prior knowledge of the noise statistics. Morover, it has a consistently improved numerical stability why the matrices of covariance are guaranteed to remain semi- positive. The performance of the proposed method is compared with EKF, and the unscented Kalman filter (UKF) for target tracking problem.
Results: To evaluate the proposed method, many experiments is performed. The proposed method is evaluated on the non-maneuvering and maneuvering target tracking.
Conclusion: The results show that the proposed method has lower estimation errors with faster convergence rate than other methods. The proposed method can track the tates of moving target effectively and improve the accuracy of the system.

Introduction

The problem of target tracking is a basic problem in the fields of the civil and military. The purpose of target tracking problem is to estimate the velocity and position of a moving target from noisy measurements [1]-[2]. In the target tracking problem, the estimation of state is confronted with two problems: one is that the measurement and process noise cannot be accurately described and are usually not accurate. The second is that the measurement and process noise cannot be accurately described and are usually not accurate the nonlinearity of measurement and motion model [3]-[4]. Various nonlinear Bayesian approach are developed for the problem of target tracking in the literature, which aims to estimate the velocity and position of the target using measurements.

The EKF is a widely used nonlinear filter to target tracking [5]-[6]. An online adaptive Kalman filter is proposed for target tracking with unknown noise statistics in [7].

In this paper, the expectation maximization algorithm is employed to construct the noise can effectively estimate the one-step prediction mean vector, the one-step prediction error covariance matrix. In [8], a robust filter is presented to shape estimation of a maneuvering star-convex extended target based on adaptive extended

Kalman filter. Basically, since EKF uses the Taylor first order approximation for nonlinear functions, it makes large errors [8]-[9]. Therefore, if it is very nonlinear, such as a target tracking problem, the error of estimation can be large or even divergent, and the filter is unstable [9]-[10].

To increase accuracy, UKF based on target tracking is introduced in literatures [11]-[14]. In this method, there is no need to calculate the Jacobin matrix of the nonlinear state and measurement equation [14]-[15]. In [16], target tracking based on square-root unscented Kalman filters is presented. This method, propagate not the covariance matrix itself but its singular value decomposition (SVD) factors instead.

Compared to EKF, UKF has better accuracy. However, UKF does not use for non-Gaussian distributions [17]-[18]. In addition, it's the computation load is heavy for high-dimensional systems such as target tracking consequently thus, the filter can be converged slowly.

In 2009, the cubature Kalman filter (CKF) is proposed [19]-[20].

The CKF creates cubature integral points, and these points are used to calculate the posterior probability of the system. In [21], a Gaussian-sum cubature Kalman filter (GSCKF) is proposed for the problem of tracking and it has excellent performance from the point of view of filter accuracy and consistency. In [22], a strong tracking cubature Kalman filter is proposed for target tracking problem.

A limitation of target tracking based on traditional CKF is that statistical characteristics of noises are assumed to known [23]-[25].

As a result, the development of CKF method is limited. To solve these problems, in this paper, the problem of target tracking based on ASRCKF is proposed. The target tracking based on ASRCKF is updated repeatedly by propagating square root factors of the mean and covariance of the state variable, which ensures the positive semi-definiteness and symmetry of the covariance matrix and thus improves numerical accuracy and stability.

The main contribution of the paper is that proposed adaptive algorithm has good filtering accuracy and strong robustness. This method improves the tracking ability of the SRCKF method for a maneuvering target. The proposed method can prevent potential filter divergence and enhances the numerical stability. Moreover, in the proposed method, the covariance of process and measurement noises is estimated adaptively.

Thus, the performance of proposed approach does not depend on the noise statistics and its performance is robust with unknown prior knowledge of the noise statistics. Simulation results show that the proposed method has a superior tracking performance.

The rest of manuscript is as follows. The target tracking formulation is presented in the second Section. In the next Section, the target tracking based on ASRCKF is proposed.

The results are given in the fourth Section. In the fifth Section, the conclusion is presented.

Target Tracking Formulation

The discrete-time dynamic equation of the target motion is as [26]-[27]:

$$X_k = F^r(X_{k-1})X_{k-1} + G\omega_{k-1}$$

$$G = \begin{bmatrix} \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \frac{T^2}{2} & T \end{bmatrix}^T \quad (1)$$

where G is the input matrix, the (x_k, y_k) is position components, $(\dot{x}_k, \dot{y}_k)$ is velocity components, T is a sampling interval and X_k is as $X_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^T$, ω_k is the process noise with covariance matrices Q_t. Moreover, F^r is transition matrix corresponding to mode r. The transition matrix for non-maneuvring target is as follows:

$$F^1(X_{t-1}) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The coordinated turn model is the most common model for maneuvering targets. In this case, F^r is as [28]-[29]:

$$F^r(X_{t-1}) = \begin{bmatrix} 1 & \frac{\sin\Omega_t^{(r)}T}{\Omega_t^{(r)}} & 0 & \frac{1-\cos\Omega_t^{(r)}T}{\Omega_t^{(r)}} \\ 0 & \cos\Omega_t^{(r)}T & 0 & -\sin\Omega_t^{(r)}T \\ 0 & \frac{1-\cos\Omega_t^{(r)}T}{\Omega_t^{(r)}} & 1 & \frac{\sin\Omega_t^{(r)}T}{\Omega_t^{(r)}} \\ 0 & \sin\Omega_t^{(r)}T & 0 & \cos\Omega_t^{(r)}T \end{bmatrix}, \text{ r=2,3} \quad (3)$$

where $\Omega_t^{(3)} > 0$, $\Omega_t^{(2)} < 0$ are is anticlockwise and clockwise turn maneuver, respectively. Target observation model is as:

$$Z_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \arctan(\frac{y_k}{x_k}) \end{bmatrix} + v_k$$

where_ Z_k denotes the measurement at k^{th} instant and v_k is measurement noise with covariance matrices R_t.

Cubature Rule

Calculating the Gaussian nonlinear transfer density is the most important step for Bayesian filtering in Gaussian domain [18]. It is as follows:

$$I = \int g(x)\mathrm{N}(x, P_x)dx \tag{4}$$

where n_x is dimension of state x, $\mathrm{N}(x,P)$ is the Gaussian prior density of x and g is the nonlinear function.

The third-degree spherical cubature rule to numerically calculate the integral I with $2n_x$ equal weighted cubature points is used in CKF [30]:

$$I = \frac{1}{2n_x}\sum_{j=1}^{2n_x} g(\sqrt{P_x}\xi_j + x) \tag{5}$$

where the cubature point ξ_j is as:

$$\xi_j = \begin{cases} \sqrt{n_x}e_i^T & \mathrm{i}=1,2,\ldots,\mathrm{n}_x \\ -\sqrt{n_x}e_i^T & \mathrm{i}=\mathrm{n}_x+1,\mathrm{n}_x+2,\ldots,2\mathrm{n}_x \end{cases}$$

where $e_i^T \in R^{n_x}$ is the ith column vector of $I_{n_x \times n_x}$.

Target Tracking Based ASRCKF

Assume at time k-1, $s_{K-1|k-1}$ is the square-root of the covariance matrix $P_{K-1|k-1}$, i.e. $P_{K-1|k-1} = s_{K-1|k-1}s_{K-1|k-1}^T$, the current state cubature points are computed as follows:

$$\chi_{k-1|k-1}^i = s_{k-1|k-1}I(i) + \hat{x}_{k-1|k-1} \qquad i=1,\ldots 2n_x \tag{6}$$

where $I(i)$ is as

$$I(i) = \begin{cases} \sqrt{n_x}\,[1]_i\,, & \mathrm{i}=1,\ldots,\mathrm{n}_x \\ -\sqrt{n_x}\,[1]_{i-n_x}\,, & i = n_x+1,\ldots,2n_x \end{cases}$$

With $[1]_i$ is the i-th column vector of the $n \times n$ matrix I. The cubature points are transmitted in state equation as:

$$\chi_{k|k-1}^{i*} = f(\chi_{k-1|k-1}^i) \tag{7}$$

The predicted mean $\hat{x}_{k|k-1}$ is calculated using the transformed cubature points $\chi_{k|k-1}^{*i}$ as follows:

$$\hat{x}_{k|k-1} = \frac{1}{2n_x}\sum_{i=1}^{2n_x}\chi_{k|k-1}^i \tag{8}$$

$$S_{k|k-1} = \mathrm{Tria}\left\{X_{k|k-1}, \sqrt{Q_k}\right\} \tag{9}$$

where $\mathrm{Tria}(.)$ is a general triangularization algorithm and $X_{k|k-1}$ is as:

$$X_{k|k-1} = \frac{1}{\sqrt{2n_x}}\left[\chi_{k|k-1}^{1*} - \hat{x}_{k|k-1} \quad \chi_{k|k-1}^{2*} - \hat{x}_{k|k-1} \quad \ldots \quad \chi_{k|k-1,}^{2n*} - \hat{x}_{k|k-1}\right] \tag{10}$$

When measurement is revisited, the cubature point set is calculated as follows:

$$\chi_{k|k-1}^i = s_{k|k-1}I(i) + \hat{x}_{k|k-1} \qquad i=1,\ldots 2n_x \tag{11}$$

The transformed cubature points are transmitted in measurement equation:

$$\chi_{k|k-1}^{i**} = h(\chi_{k|k-1}^i) \tag{12}$$

The mean values and square-root of the covariance matrix of predicted measurement points are estimated:

$$\hat{z}_{k|k-1} = \frac{1}{2n}\sum_{i=0}^{2n}\chi_{k|k-1}^{**i} \tag{13}$$

$$S_{zz,k|k-1} = \mathrm{Tria}\left\{Z_{K|k-1}, S_R\right\} \tag{14}$$

where $Z_{K|k-1}$ is as:

$$Z_{k|k-1} = \frac{1}{\sqrt{2n}}\left[\chi_{k|k-1}^{1**} - \hat{z}_{k|k-1} \quad \chi_{k|k-1}^{2**} - \hat{z}_{k|k-1} \quad \ldots \quad \chi_{k|k-1}^{2n**} - \hat{z}_{k|k-1}\right]$$

The cross covariance S_{xz} between the states and measurements are:

$$S_{xz,k|k-1} = X_{k|k-1}Z_{k|k-1}^T \tag{15}$$

$$X_{k|k-1} = \frac{1}{\sqrt{2n_x}}\left[\chi_{k|k-1}^1 - \hat{x}_{k|k-1} \quad \chi_{k|k-1}^2 - \hat{x}_{k|k-1} \quad \ldots \quad \chi_{k|k-1,}^{2n} - \hat{x}_{k|k-1}\right] \tag{16}$$

The Kalman gain K_k is calculated by

$$K_k = (S_{xz,k|k-1}/S_{zz,k|k-1}^T)/S_{zz,k|k-1} \tag{17}$$

The updated state $\hat{x}_{k|k}$ and the square-root of covariance $S_{k|k}$ are obtained as follows:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - \hat{z}_{k|k-1}) \tag{18}$$

$$S_{k|k} = Tria([X_{k|k-1} - K_kY_{k|k-1}, K_k\sqrt{R_k}]) \tag{19}$$

According to the equations $S_{k|k-1}$ and $S_{k|k}$ in (9) and (19), respectively, it can be seen that Q_k and R_k have a great impact on their values. However, the noise statistics can change over time in the target tracking application.

The set of unknown statistical of noise needs to estimate with the error covariance and the state of system.

Any mismatch between real noises that affect the system and those that are assumed in SRCKF reduce the performance of SRCKF that can also have divergence. As a result, it is necessary to know that the Q_k and R_k matrices exactly.

In this paper, an adaptive SRCKF is proposed method that solves the SRCKF problems by estimating the Q_k and R_k matrices. Suppose the process and measurement noise are defined as $w_w \sim \mathrm{N}(0, Q_k)$ and $v_k \sim \mathrm{N}(0, R_k)$. To estimate the covariance of process and measurement noises, the function of posteriori density is assumed as follows:

$$J^* = p(X_k, Q_k, R \mid Z_k) \tag{20}$$

where $Z_k = [z_1 \quad z_2 \quad \dots \quad z_k]$ is the measurement vector and $X_k = [x_1 \quad x_2 \quad \dots \quad x_k]$ is the state vector. According to the properties of conditional probability, the J^* function can be written as:

$$J^* = \frac{P(Z_k \mid X_k, Q_k, R_k) P(X_k \mid Q_k, R_k) p(Q_k, R_k)}{P(Z_k)} \tag{21}$$

where $p(Q_k, R_k)$ is depend on with the priori information, which can be considered as constant value. As $p(Z_k)$ is not involved in the problem of optimization. As a result, the function of J^* can be written as follows:

$$J = P(Z_k \mid X_k, Q_k, R_k) P(X_k \mid Q_k, R_k) p(Q_k, R_k) \tag{22}$$

the term $p(Q_k, R_k)$ is a constant value why it calculates based on a priori information. The term $p(X_k \mid Q_k, R_k)$ in (22) can be calculated using the multiplicative theorem of conditional probability as follows.

$$\begin{aligned} p(X_k \mid Q_k, R_k) &= p(x_0) \prod_{j=1}^{k} p(x_j \mid x_{j-1}, Q_k) = \\ & \frac{1}{(2\pi)^{n/2} |P_{0|0}|^{1/2}} \exp(-\frac{1}{2} \|x_0 - \hat{x}_0\|^2_{P_{0|0}^{-1}}) \\ & \prod_{j=1}^{k} \frac{1}{(2\pi)^{n/2} |Q_k|^{1/2}} \exp(-\frac{1}{2} \|x_j - f(x_{j-1})\|^2_{Q_k^{-1}}) = \\ & M_1 |Q_k|^{-k/2} \exp\left\{ -\frac{1}{2} \left[\|x_0 - \hat{x}_0\|^2_{P_{0|0}^{-1}} + \sum_{j=1}^{k} \|x_j - f(x_{j-1})\|^2_{Q_k^{-1}} \right] \right\} \end{aligned} \tag{23}$$

where $M_1 = \dfrac{1}{(2\pi)^{n/2} |P_{0|0}|^{1/2}}$ is a constant, n is the process dimension. Also, the term $p(Z_k \mid X_k, Q_k, R_k)$ can be calculated as follows.

$$\begin{aligned} p(Z_k \mid X_k, Q_k, R_k) &= \prod_{j=1}^{k} p(z_j \mid x_j, R_k) \\ &= \prod_{j=1}^{k} \frac{1}{(2\pi)^{m/2} |R_k|^{1/2}} \exp(-\frac{1}{2} \|z_j - h(x_j)\|^2_{R_k^{-1}}) \\ &= M_2 |R_k|^{-k/2} \exp(-\frac{1}{2} \sum_{j=1}^{k} \|z_j - h(x_j)\|^2_{R_k^{-1}}) \end{aligned} \tag{24}$$

where m represents the measurement dimension, and $M_2 = \dfrac{1}{(2\pi)^{mk/2}}$ is a constant. By considering (23) and (24), the problem of estimation can be reformulated as an optimization problem with the cost function J:

$$\begin{aligned} J &= M_1 M_2 |P_0|^{-1/2} |Q_k|^{-k/2} p(Q_k, R_k) \\ & \exp\left\{ -\frac{1}{2} \left[\|x_0 - \hat{x}_0\|^2_{P_{0|0}^{-1}} + \sum_{j=1}^{k} \|x_j - f(x_{j-1})\|^2_{Q_k^{-1}} + \sum_{i=1}^{k} \|z_j - h(x_j)\|^2_{R_k^{-1}} \right] \right\} \\ &= C |Q_k|^{-k/2} |R_k|^{-k/2} \exp\left\{ -\frac{1}{2} \left[\sum_{i=1}^{k} \|x_j - f(x_{j-1})\|^2_{Q_k^{-1}} + \sum_{j=1}^{k} \|z_j - h(x_j)\|^2_{R_k^{-1}} \right] \right\} \end{aligned} \tag{25}$$

where

$$C = M_1 M_2 |P_{0|0}|^{-1/2} p(Q_k, R_k) \exp\left\{ -\frac{1}{2} \|x_0 - \hat{x}_0\|^2_{P_{0|0}^{-1}} \right\}$$

As the logarithm operation for both sides of (25) cannot change the extreme points of the cast function J, to find the maximized parameter of coast function J, firstly, take a logarithm from both sides of (25):

$$\begin{aligned} \ln J &= -\frac{k}{2} \ln |Q_k| - \frac{k}{2} \ln |R_k| - \\ & \frac{1}{2} \sum_{i=1}^{k} \|x_i - f(x_{i-1})\|^2_{Q_k^{-1}} - \frac{1}{2} \sum_{i=1}^{k} \|z_i - h(x_i)\|^2_{R_k^{-1}} + \ln C \end{aligned} \tag{26}$$

Using the derivative of J relative to Q_k and R_k, the noise covariance values are calculated as:

$$\left. \frac{\partial \ln J}{\partial Q_k} \right|_{Q_k = \hat{Q}_k} = 0, \quad \left. \frac{\partial \ln J}{\partial R_k} \right|_{R_k = \hat{R}_k} = 0 \tag{27}$$

Consequently, covariance values Q_k and R_k can be calculated as:

$$\hat{Q}_k = \frac{1}{k} \sum_{j=1}^{k} \left\{ \left(\hat{x}_j - f(\hat{x}_{j-1}) \right) \left(\hat{x}_j - f(\hat{x}_{j-1}) \right)^T \right\} \tag{28}$$

$$\hat{R}_k = \frac{1}{k} \sum_{j=1}^{k} \left\{ \left(z_j - h(x_j) \right) \left(z_j - h(x_j) \right)^T \right\} \tag{29}$$

The terms $f(\hat{x}_{j-1})$ and $h(\hat{x}_j)$ can be calculated from the SRCKF as follows:

$$f_{j-1}(\hat{x}_{j-1}) = \frac{1}{2n}\sum_{i=1}^{2n} f(\chi_{j-1|j-1}^i) \tag{30}$$

$$h_j(\hat{x}_j) = \frac{1}{2n}\sum_{i=1}^{2n} h(\chi_{j|j-1}^i) \tag{31}$$

By substituting (30) and (31) into (28)-(29), the $\hat{Q}_k$ and $\hat{R}_k$ is obtained as follows:

$$\hat{R}_k = \frac{1}{k}\sum_{j=1}^{k}\left\{\left(z_j - \frac{1}{2n}\sum_{i=1}^{2n} h(\chi_{j|j-1}^i)\right)\left(z_j - \frac{1}{2n}\sum_{i=1}^{2n} h(\chi_{j|j-1}^i)\right)^T\right\} \tag{32}$$

$$\hat{Q}_k = \frac{1}{k}\sum_{j=1}^{k}\left\{\left(\hat{x}_j - \frac{1}{2n}\sum_{i=1}^{2n} f(\chi_{j-1|j-1}^i)\right)\left(\hat{x}_j - \frac{1}{2n}\sum_{i=1}^{2n} f(\chi_{j-1|j-1}^i)\right)^T\right\} \tag{33}$$

Results and Discussion

The proposed method is evaluated on the non-maneuvering and maneuvering target tracking. In simulations, the total number of Monte Carlo runs is 100, the initial state of the target is $x_0 = [0 \quad 1 \quad 0 \quad -0.5]^T$, and the corresponding covariance is $P_0 = diag([0.1 \quad 0.1 \quad 0.1 \quad 0.1])$ the process noise covariance is $Q_k = diag([0.001 \quad 0.001 \quad 0.001 \quad 0.001])$, and the covariance of measurement noise is $R_k = diag([1 \quad 1])$.

Non-Maneuvering Target

For a non-maneuvering target, its course and velocity, both of which are assumed to remain constant throughout the observation duration. In Non-maneuvering target, the target moves with velocity (1m/s, -0.5m/s) starting from the (0m, 0m). T sampling period is T=0.26, The proposed method is compared with that of other methods under different conditional.

Scenario 1: Performance with known statistics noise

First, the proposed method is evaluated under effect of a noise with known statistics and the performance of it is compared with EKF and UKF.

Fig. 1 shows results by EKF, UKF and the proposed method, and Fig. 2 shows the tracking performances of the methods on X and Y. Obviously, the tracking performance of the proposed method is better than that of EKF and UKF. Fig. 1 depicts that EKF and UKF lose the target overtime, and the error of estimation increased mainly.

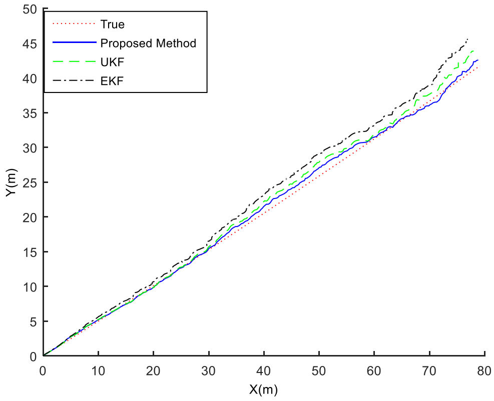


Fig. 1: Results of trajectory.

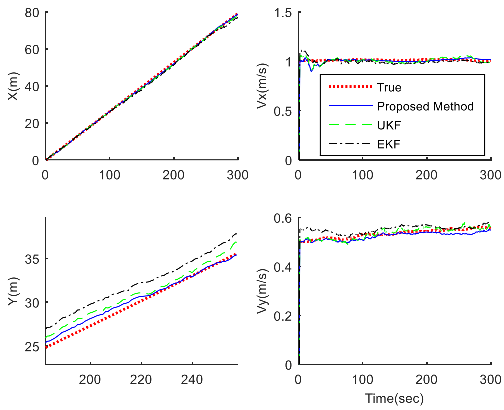


Fig. 2: True states and estimated states values.

Moreover, although EKF and UKF keep the tracking, the error of estimation is large and non-convergent. However, the proposed method follows the target with small estimation error.

Then, in order to more evaluate, the root-mean square error (RMSE) is calculated.

The RMSE of position and velocity for N times simulation is as:

$$RMSE_{pos}(k) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}((x_k^i - \hat{x}_k^i)^2 + (y_k^i - \hat{y}_k^i)^2)} \tag{34}$$

where N is the total number of Monte Carlo simulation, k is the k-th discrete time point of the total simulation time, (x_t, y_t) and $(\hat{x}_t, \hat{y}_t)$ are the true and estimated positions. Similarly, to the RMSE of position, the formulation of RMSE of velocity is as follows:

$$RMSE_{vel}(k) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}((\dot{x}_k^i - \hat{\dot{x}}_k^i)^2 + (\dot{y}_k^i - \hat{\dot{y}}_k^i)^2)} \tag{35}$$

where $(\dot{x}_t, \dot{y}_t)$ and $(\hat{\dot{x}}_t, \hat{\dot{y}}_t)$ are the true and estimated velocities.

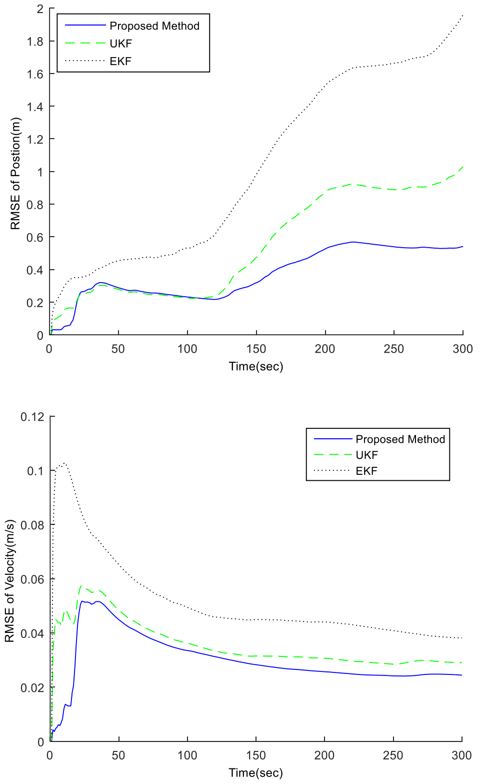


Fig. 3: RMSE over time with known noise statistics.

The RMSE of estimations over time is shown in Fig. 3. Fig. 3 clearly shows that the RMSE of the proposed algorithm are smaller than that of other methods. To further quantify the performance of the methods, the mean and variance of the RMSE algorithms are investigated in Fig. 4. It can be seen that the RMSE in target tracking based on the proposed method is lower than that of EKF and UKF.

Scenario 2: Performance with unknown statistics noise

In this subsection, to illustrate the further benefits of the proposed method, it is assumed that statistics noises are unknown. Assume the initial values of the noise statistics are $Q_k = diag([0.001 \quad 0.001 \quad 0.001 \quad 0.001])$ and $R_k = diag([0.1 \quad 0.1])$, which are different from the true noise covariances. The comparison of methods is shown in Figs. 5-7. The tracking results by EKF, UKF and the proposed method are shown if Fig. 5 and the tracking performances of the methods on X and Y is depicted in Fig. 6. The RMSE of estimation is shown in Fig. 7. It can be observed that the performance of proposed method is almost close to the previous case, while the performance of EKF and UKF is worse than the performance of EKF and UKF in the previous case. The better performance of the proposed method is because that it can estimate the covariance of noises, whereas the other methods depend on the fixed prior knowledge about the process noises.

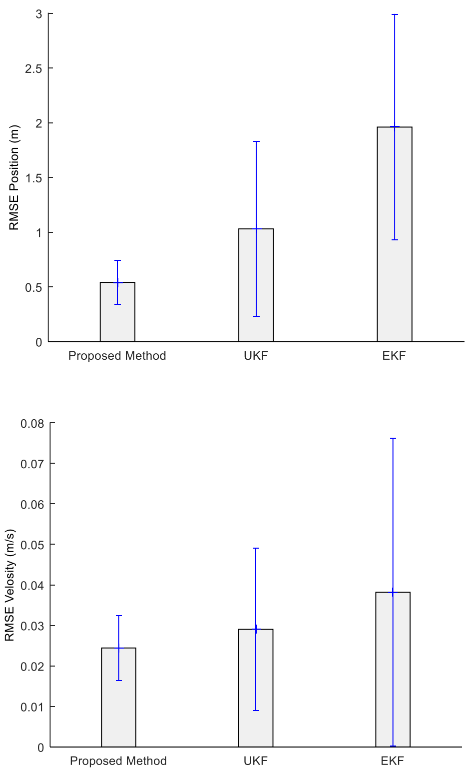


Fig. 4: RMSE of algorithms with known noise statistics.

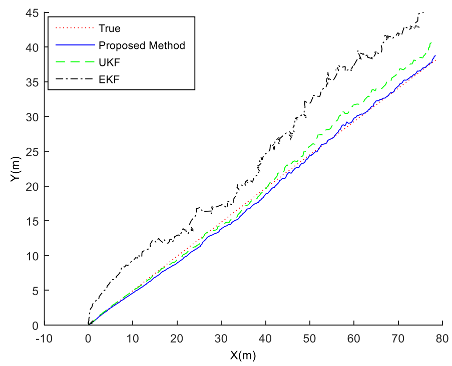


Fig. 5: Results of trajectory.

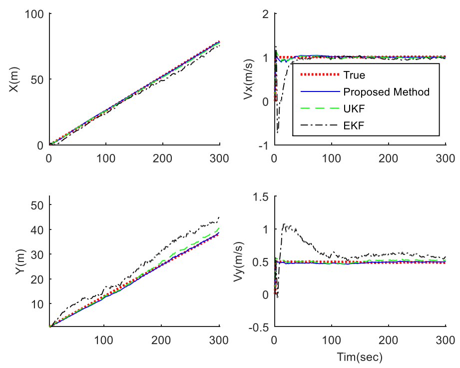


Fig. 6: True states and estimated states values.

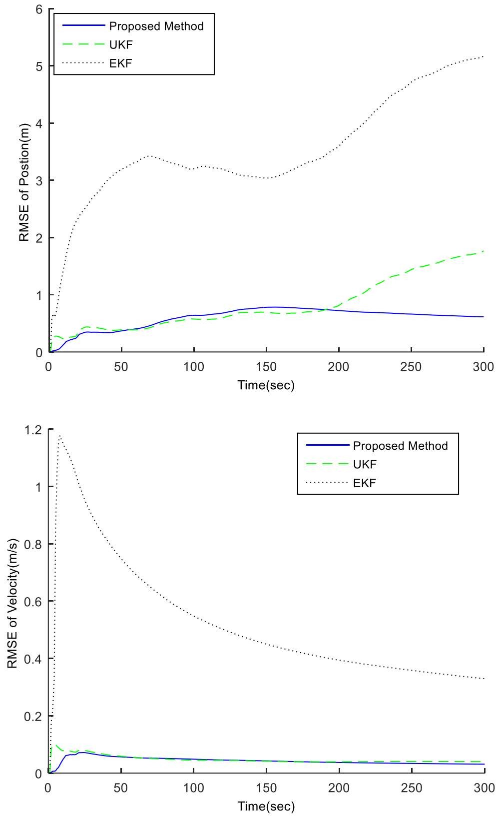


Fig.7: RMSE over time with unknown noise statistics.

Maneuvring Target

The performance of the proposed method is evaluated for tracking of the maneuver target. In maneuvring target, the target moves at an even acceleration and the target motion state will varies, which has to account for the variation of acceleration. In simulations, it is assumed that the target is (0, 0) and the sampling period is T=0.26.

For 100 s, the target starts to make a turn rate of 5°/s. Then it turns for 200s with -8 °/s.

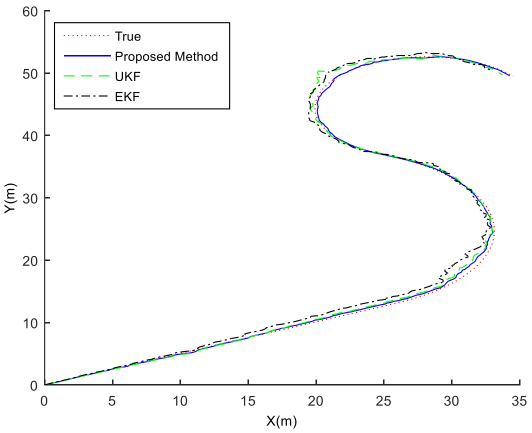


Fig. 8: Results of trajectory.

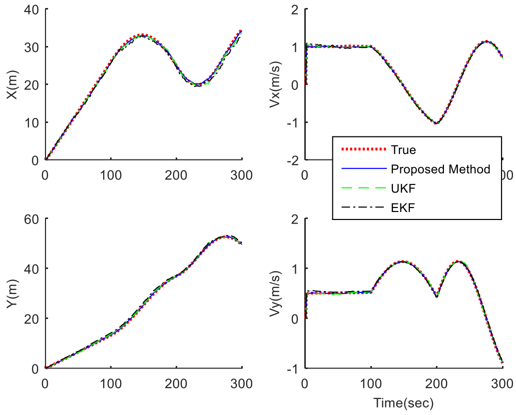


Fig. 9: True states and estimated states values.

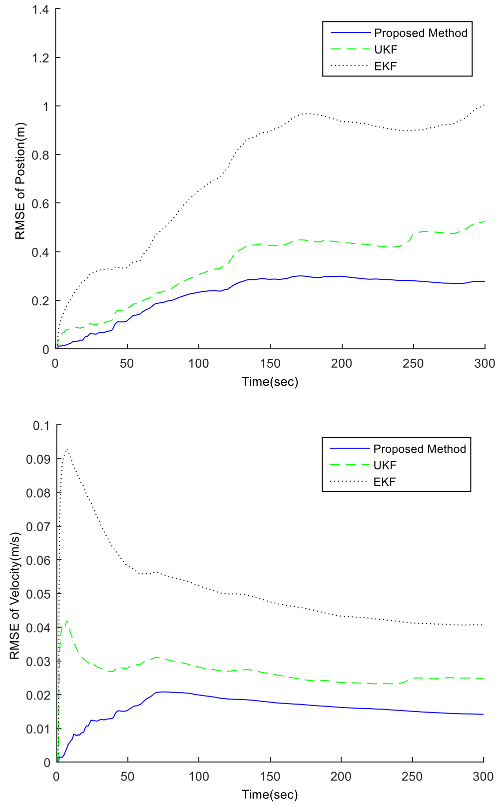


Fig. 10: RMSE of over time with known noise statistics.

Scenario 1: Performance with known statistics noise

In this scenario, first, the performance of the proposed method under the influence of a noise with known statistics is evaluated and compared with performance of UKF and EKF. The estimated trajectories of methods are shown in Fig. 8 and the tracking results by various methods on Y and X are shown in Fig. 9. It observed that tracking accuracy of the proposed method is better than that of EKF and UKF and converges faster. In fact, the estimate value of proposed method is the closest with the true value. The EKF and UKF lose the target and the error of tracking mainly increases.

However, the proposed method traces the target in the whole scenario with small error of estimation. The RMSE of methods are respectively shown in Figs. 10-11. Obviously, the result indicated that the proposed algorithm has better performance in accuracy of estimation compared to EKF and UKF.

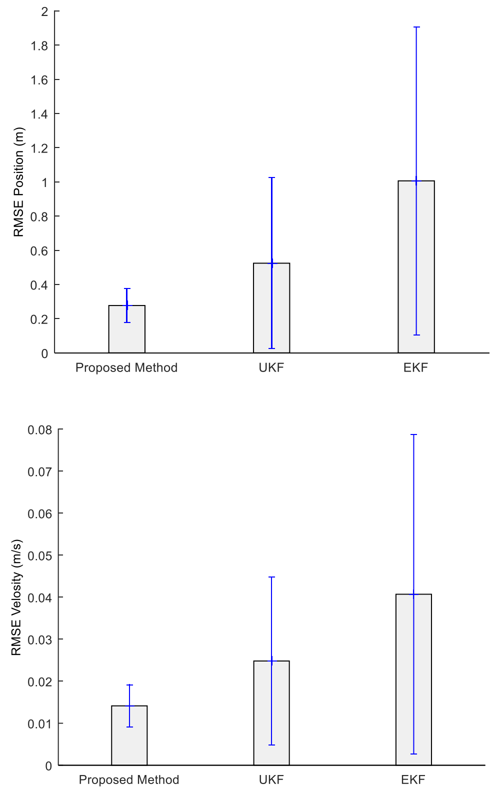


Fig. 11: RMSE of with known noise statistics.

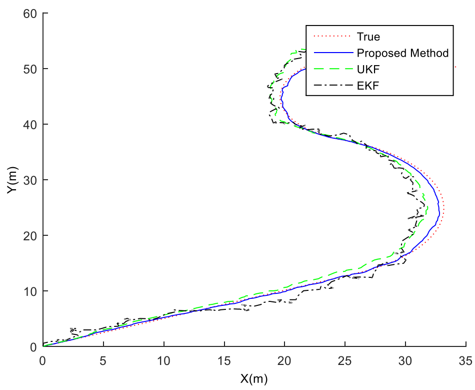


Fig. 12: Results of trajectory.

Scenario 2: Performance with unknown statistics noise

In this sub scenario, the robustness and adaptively of the proposed method is tested when statistics noises are considered wrongly. Figs. 12-14 show the results. Similar to Non-maneuvering case, it observes that the performance of other methods is worse than the performance of them in the previous case, while the performance of proposed algorithm is almost similar to that of the previous case. The RMSE of the position and velocity of the proposed method, UKF and the EKF are shown in Fig. 14.

The proposed method shows the best performance and the range of error is even lower than the UKF and EKF for the RMSE in velocity and position.

The proposed method with noise statistic estimator has very well performance in the complicated conditions, which shows a good adaptability and robustness.

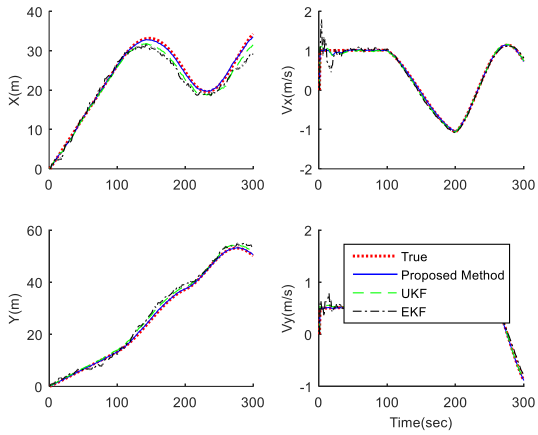


Fig. 13: True states and estimated states values.

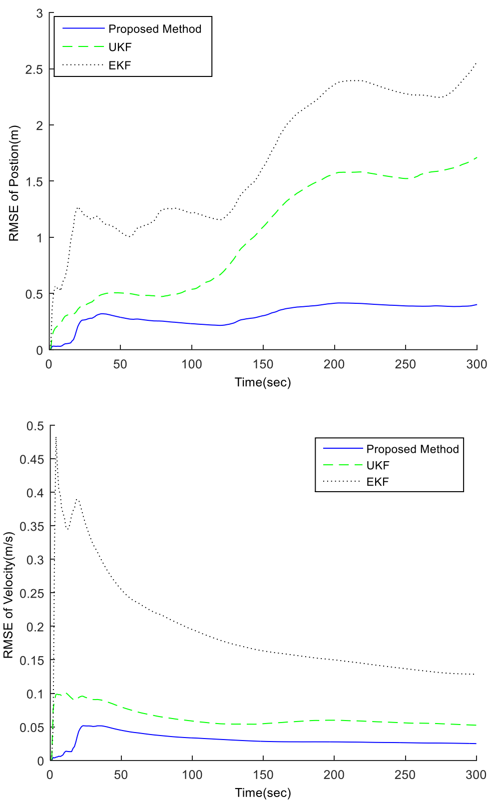


Fig. 14: RMSE of over time with unknown noise statistics.

Table 1 shows the RMSE algorithms. From Table 1, It can be seen that the performance of the proposed method is superior to other methods.

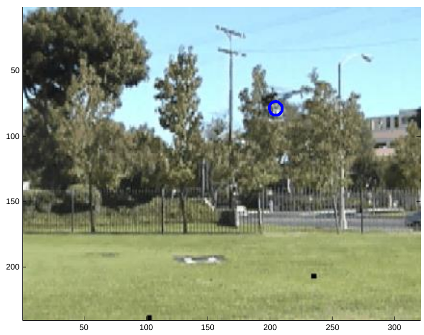
Table 1: Performance of algorithms

Statistics noise	Method	RMSE of Position	RMSE of velocity
Unknown	**Proposed Method**	**0.38**	**0.02**
	CKF	1.6	0.05
	UKF	1.71	0.06
	EKF	2.56	0.13
know	**Proposed Method**	**0.27**	**0.015**
	CKF	0.45	0.21
	UKF	0.52	0.025
	EKF	1	0.045

Helicopter Tracking

For further investigation, the proposed method in tracking the purpose of video sequences is examined.

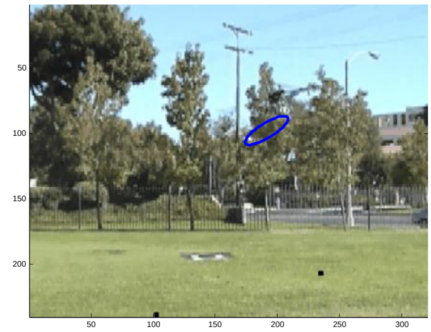
Frame 232



Frame 499

Fig. 15: Tracking by the proposed method.

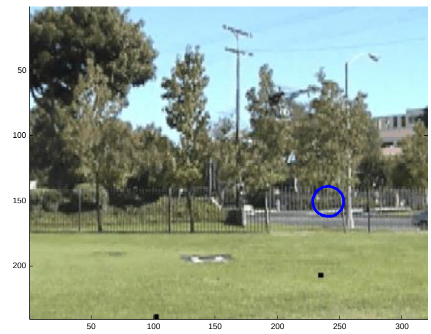
Frame 232



Frame 499

Fig. 16: Tracking by UKF.

Frame 232



Frame 499

Fig. 17: Tracking by EKF.

The results are shown can be seen in Figs. 15-17. The results show the superior performance of proposed method. From the video in Figs. 15-16, it was observed that EKF and UKF can initially successfully track the target. However, when the helicopter moves in front of the leaves, the trackers miss the target. While in the proposed method, the target is successfully tracked during the video.

Conclusion

In this paper, the target tracking based on adaptive square root cubature Kalman filter is proposed. The proposed method does not need to know the statistics of noises, while other traditional cubature Kalman filters need the noise statistics. Moreover, the proposed method has better numerical characteristics and guaranteed positive semi-definiteness of error covariance matrix.

Instead of decomposing Cholesky at each step, the proposed method updates and propagates square-root of the covariance of error. It has a consistently improved numerical stability. The effectiveness and feasibility of the proposed method is evaluated by the Monte Carlo simulations in different scenarios.

The RMSE of the position and velocity have been evaluated using the UKF, EKF, and proposed method. It has been observed that the RMSE of position and velocity are less in the proposed method compared to the EKF and the UKF.

The results imply the superiority of the proposed method compared to the EKF and the UKF. The proposed method provides better performance in tracking accuracy than other methods.

Author Contributions

All the authors participated in the conceptualization, implementation, and writing.

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This work is completely self-supporting, thereby no any financial agency's role is available.

Conflict of Interest

The authors declare no potential conflict of interest regarding the publication of this work. In addition, the ethical issues including plagiarism, informed consent, misconduct, data fabrication and, or falsification, double publication and, or submission, and redundancy have been completely witnessed by the authors.

Abbreviations

ω_k	Process noise
Q_t	Process noise covariance
F^r	Transition matrix corresponding to mode r
Z_k	Measurement at kth instant
v_k	Measurement noise
R_t	Measurement noise covariance
$\sqrt{P_x}$	Square-root factor of P_x
$\chi^{*i}_{k\|k-1}$	Transformed cubature points
$S_{k\|k-1}$	Square root of the covariance matrix
X_k	State vector
S_{xz}	Cross covariance between the states and measurements
J^*	Function of posteriori density
n	Process dimension
$\|A\|$	Determinant of a square matrix A
x_0	Initial state of the target
P_0	Initial covariance
(x_k, y_k)	Position components
$(\dot{x}_k, \dot{y}_k)$	Velocity components
T	Sampling interval
$\Omega_t^{(3)}$	Anticlockwise turn maneuver
$\Omega_t^{(2)}$	Clockwise turn maneuver
$\mathrm{N}(x, P)$	Guassian prior density of x
g	Nonlinear function
n_x	State dimension
ξ_j	Cubature point
e_i^T	The ith column vector of $I_{n_x \times n_x}$
$\mathrm{Tria}(.)$	General triangularization algorithm
$\hat{x}_{k\|k}$	The updated state
N	Total number of Monte Carlo simulation

G	Input matrix
$\hat{x}_{k\|k-1}$	Predicted mean
K_k	Kalman gain
$(\hat{x}_t, \hat{y}_t)$	Estimated positions
$RMSE_{pos}$	RMSE of position
$RMSE_{vel}$	RMSE of velocity

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Biographies

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