Journal of Electrical and Computer Engineering Innovations (JECEI)
Journal homepage: http://www.jecei.sru.ac.ir

Research Paper

1WQC Pattern Scheduling to Minimize the Number of Physical Qubits

E. Nikahd[1,]*, M. Houshmand[2], M. Houshmand[3]

[1]*Computer Engineering Department, Shahid Rajaee Teacher Training University, Tehran, Iran.*

[2]*Department of Computer Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran.*

[3]*Department of Electrical Engineering, Imam Reza International University, Mashhad, Iran.*

Article Info

Article History:
Received 24 November 2022
Reviewed 12 January 2023
Revised 26 February 2023
Accepted 11 March 2023

Keywords:
One-way quantum computing model
Measurement-based quantum computation
Scheduling
Integer-linear programming

*Corresponding Author's Email Address: *nikahd@sru.ac.ir*

Abstract

Background and Objectives: One of the quantum computing models without a direct classical counterpart is one-way quantum computing (1WQC). The computations are represented by measurement patterns in this model. One of the main downsides of the 1WQC model is the much larger number of qubits in a measurement pattern, compared to its equivalent in the circuit model. Therefore, proposing a method for optimally using the physical qubits to implement a measurement pattern is of interest,

Methods: In a measurement pattern, despite a large number of qubits, the measured qubit is not needed after each measurement and can be used as another logical qubit. In this study, by using this feature and presenting an integer linear programming (ILP) model to change the ordering of a standard measurement pattern actions, the number of physical qubits required to implement that measurement pattern is minimized.

Results: In the proposed method, compared to the scheduling based on the standard pattern, the number of required physical qubits on benchmark circuits is reduced by 56.7% on average. Although the proposed method produces the optimal solution, one of the most important limitations of that and ILP-based methods, in general, is their high execution time and memory requirements, which grow exponentially with the increase of the problem size.

Conclusions: In this study, an ILP model is proposed to minimize the number of physical qubits used to realize a measurement pattern by efficiently scheduling the operations and reusing the physical qubits. Due to its exponential complexity, the proposed method cannot be used for large measurement patterns whose solution can be conspired as future works.

Introduction

Quantum computing is a branch of information processing, which is a combination of three sciences, namely, computer science, information theory, and quantum physics [1]. This science is of great interest due to reaching the end of CMOS technology advancement, its high computing power, and also the significant role it plays in the secure transmission of information [1]-[3].

The most famous model of quantum computing is the quantum circuit model which is an analogy to the common classical computation model composed of a network of logic gates [1], [4]. One-way quantum computing model (1WQC), first proposed by Raussendorf and Briegel in 2001 [5], is a practically and conceptually different alternative model [5], [6]. This model utilizes unique features of quantum mechanics such as entanglement and measurement and hence it has no classical analogue. In this model, qubits are initialized in a special highly entangled resource state (namely a cluster or graph state), and the universal quantum computation is driven by performing a sequence of single-qubit measurements in certain basis and post-measurement corrections. Calculations in 1WQC are shown in the form of measurement patterns consisting of four types of instructions: qubits preparation (N), entanglement (E), measurement (M) and correction (C) [7], [8].

1WQC is one of the measurement-based quantum computing models (MBQC) which is promising for physical implementation and has attracted the attention of researchers [9]-[11]. Despite the advantages that this model has [12], [13], one of the main down sides of that is the much larger number of qubits in a measurement pattern, compared to its equivalent in the circuit model. On the other hand, one of the limitations of physical construction, especially in ion-trap technology, is the number of available physical qubits. Therefore, 1WQC will be hard to realize due to its large number of required qubits [9].

Despite the large number of qubits in a measurement pattern, there is no need to construct the whole graph state at the beginning and it is possible to extend it on the fly by reordering the measurement pattern [14]-[16]. Furthermore, after measuring a logical qubit, there is no more required action on it and it can be removed from the computation space. The measured physical qubit can also be reused as another logical qubit to extend the graph state on the fly if there is not a limitation in underlying technology. As a result, by a proper reordering the actions of a pattern, it is possible to minimize the number of required physical qubits to realize a 1WQC measurement pattern. Now, a question is remained to answer: what is the best order of a measurement pattern that minimizes the number of necessary physical qubits and which physical qubit is allocated to a logical qubit? Focusing on this issue, in this paper an integer-linear programing (ILP) model is proposed to schedule a measurement pattern targeting to minimize the number of required physical qubits.

The rest of this study is organized as follows. Section 2 covers basic concepts related to the 1WQC model. Related work is reviewed in Section 3. The proposed approach is described in Section 4. In Section 5, the proposed method is evaluated by some measurement patterns and finally Section 6 concludes the paper.

Background

Computations in the 1WQC model are shown by a measurement pattern which is defined by the set $P = (V, I, O, A)$ [7], [8]. V is the set of all qubits, $I \subseteq V$ is the set of input qubits, $O \subseteq V$ is the set of output qubits, and A is the set of actions that act on V. The pattern is written as a sequence of actions that includes four different types: preparing qubits (N), entanglement (E), measurement (M) and correction (C) that are applied from right to left as determined in the following.

- **Qubit Preparation N_u:** prepares a qubit *u* in the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Normally this action is applied to all of the non-input qubits.
- **Entanglement action E(u,v):** entangles the qubits *u* and *v* by applying CZ gate on them. To visualize a pattern, qubits can be shown by vertices of a graph, namely entanglement graph, where the entanglement between the qubits is represented by the edges of the graph.
- **Single-qubit measurement M_u^α:** measures the qubit *u* in the orthonormal basis of:

$$|\pm\alpha\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\alpha}|1\rangle) \quad (1)$$

where, $\alpha \in [0, 2\pi]$ is the measurement angle. Normally all of the non-output qubits will be measured. The measurement result applied to a qubit *u* is denoted by $s_u \in \mathbb{Z}_2$. If *u* collapses into the $|+\alpha\rangle$ after the measurement, then $s_u = 1$ and otherwise if it collapses into the $|-\alpha\rangle$, then $s_u = 0$. The measurement outcomes can be summed module 2 to generate a signal. In general, a measurement angle may depend on the other ones through two signals s and t as:

$$^t[M_u^\alpha]^s = M_u^{(-1)^s\alpha + t\pi} \quad (2)$$

A measurement that depends on the signals *s* and *t* can be done if all the measurement results appeared in *s* and *t* are known. That means all those measurements must be done beforehand.

- **Pauli correction X_u^s and Z_u^s:** apply the Pauli X and Z gates on the qubit *u*, respectively, if s=1 and do nothing if s=0.

A pattern is called a standard pattern, if the order of actions appeared in it is preparation, entanglement, measurement, and finally corrections, respectively [7]. In a 1WQC pattern, a qubit can be removed from the computation space only after measuring it [15], [16]. Therefore, if all preparation and entanglement operations are performed first, as in a standard pattern, it is necessary to allocate a physical qubit for each qubit of the pattern. This means that the number of physical qubits required to implement a standard pattern will be equal to the number of qubits of that pattern.

Definition 1 [7]: An open graph (G, I, O) has flow if and only if there exists a map $f: O^c \to I^c$ and a strict partial order $\prec_f$over V
such that all of the following conditions hold for all $i \in O^c$

- $i \prec_f f(i)$
- if $j \in N(f(i))$, then $j = i$ or $i \prec_f j$, where $N(v)$ contains adjacent vertices of v in G

- $i \in N(f(i))$

In this case, $(f, \prec_f)$ is called a flow on (G, I, O).

Definition 2 [8]: An open graph (G, I, O) has generalized flow (gflow) if and only if there exists a map $g: O^c \rightarrow P^{I^c}$ (the set of all subsets of vertices in I^c) and a strict partial order $\prec_g$ over V such that all of the following conditions hold for all $i \in O^c$.

- if $j \in g(i)$ then $i \prec_g j$,
- if $j \in Odd(g(i))$, then $j = i$ or $i \prec_g j$, where $Odd(K) = \{k||N(k) \cap K| = 1 \bmod 2\}$,
- $i \in Odd(g(i))$.

In this case, $(g, \prec_g)$ is called a gflow on (G, I, O).

Related Work

The unique features of the 1WQC model has drawn the researchers' attention in many studies after its first proposal in 2001. A number of studies have focused on the fast simulate of the 1WQC model on the classic computers [16], [17].

One of the important applications of the 1WQC model is blind quantum computation [18]-[20]. Blind quantum computation allows a client with limited quantum capabilities to delegate his computational problem to a remote quantum server such that the client's input, output, and algorithm are kept private from the server.

Most of physical design and scheduling work done in quantum computing has been focused on the quantum circuit model [21]-[28]. While, there is only a few researches focused on the physical design of 1WQC [29]-[32]. The studies done in the 1WQC model assume all preparations and entanglements are first done and after that computation is pursued by only single-qubit measurements and post-measurement corrections, as in the standard pattern. For example, [33] proposed a design flow to directly map a 1WQC pattern to a 2D nearest-neighbor architecture, without trying to reduce the number of physical qubits.

The most related work to our study is the work done in [15]. In that study, the minimal number of physical qubits that must be present in a system to directly implement a given measurement pattern has theoretically been proven. It hast been shown that to realize a measurement pattern $P = (V, I, O, A)$ with flow [7], the minimum number of physical qubits is $min(|O| + 1, |V|)$, while for measurement patterns with only gflow[1] [8], the number of needed qubits may be as high as $|V| - 2$. However, that approach does not provide a practical way to reach this minimal number of the physical qubits which is the main concern of this study.

Proposed Approach

In a standard pattern, all preparation operations followed by entanglement are performed first. Therefore, the number of physical qubits needed to realize a standard pattern is equal to the number of total qubits of the pattern, i.e., $|V|$. However, it is possible to reorder the operations of the pattern to minimize the required number of physical qubits [15]. Indeed, we can extent the graph state on the fly by reusing a physical qubit after measuring it as another logical qubit. The problem of finding the best order of operations that minimizes the number of required physical qubits is the subject of this paper. To do so, an ILP model is proposed to schedule a measurement pattern in such a way that it minimizes the needed physical qubits by maximizing qubit reusing. Our approach works for all of the patterns with flow or only gflow.

Theorem 1 [14]: In a measurement pattern, a qubit *u* can be measured if its dependencies to the other measurement have been resolved and also its entanglements with its neighbor qubits have been applied.

Therefore, based on this theorem, one can select a logical qubit *u* from the list of qubits with resolved dependencies for measurement. Then, one can allocate physical qubits to it and its neighbor qubits and perform the entanglements between them. After that, the qubit *u* is measured. Finally, the allocated physical qubit to *u* is released and can be reused as another logical qubit.

To illustrate the method, an example is provided. Fig. 1 shows the entanglement graph of the SWAP gate. Each node represents a qubit and each edge is an entanglement operation between the corresponding qubits. $\{q1, q2\}$ and $\{q6, q8\}$ are the input and output qubits, respectively. We suppose that the input qubits already exist i.e., physical qubits with proper states have been allocated to them beforehand or may feed into the circuit from outside. All of the non-output qubits must be measured and, in this case, there is no dependency between them. Therefore, in the first step all non-output qubits are candidates to be chosen for measurement. Intuitively, input qubits are the best choices for the measurement in the first step, as they already exist.

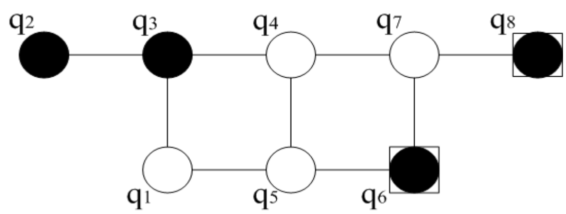

Fig. 1: Graph state of SWAP gate.

[1] Generalized flow

Table 1 shows the best order of measurements that reduces the number of physical qubits from 8 to 3 in comparison with the standard pattern. In this table, logical and physical qubits are denoted by lq and pq respectively and $(lq{:}\,pq)$ means that a physical qubit pq is assigned to a logical qubit lq.

0) Physical qubits are allocated to input qubits.
1) $lq2$ is selected for measurement. Therefore, the entanglement between $lq2$ and its neighbors, i.e. $lq3$, must be performed before measuring $lq2$. As there is no physical qubit allocated to $lq3$ and there is no free physical qubit, we need a new one, i.e. $pq3$ to be allocated to $lq3$. Now we are ready to perform $E(lq2, lq3)$ and then measure $lq2$. After measuring $lq2$, $pq2$ is released and can be reused as another logical qubit.
2) $lq3$ is chosen for measurement. So, $E(lq3, lq1)$ and $E(lq3, lq4)$ must be done first. To perform $E(lq3, lq4)$ it is needed to allocate a physical qubit to $lq4$. To do so, $pq2$ (that was released in the previous step) is used, i.e. $(lq4{:}\,pq2)$. After that, $lq3$ is measured and its corresponding physical qubit, i.e. $pq3$, is released.
3) to 6) This process will be continued until all of the measurements are performed using only three physical qubits $pq1$, $pq2$ and $pq3$.
7) Finally, the output qubits are $lq6$ and $lq8$ corresponding to $pq2$ and $pq3$, respectively.

Table 1: an example of optimal solution for scheduling of SWAP pattern

step	Measurement order	Qubit allocation (Logical Qubit: Physical Qubit)
0	-	{(lq1:pq1), (lq2:pq2), (lq3:-), (lq4:-), (lq5:-), (lq6:-), (lq7:-), (lq8:-)}
1	lq2	{(lq1:pq1), (;q2:pq2), (lq3:pq3), (lq4:-), (lq5:-), (lq6:-), (lq7:-), (lq8:-)}
2	lq3	{(lq1:pq1), (lq3:pq3), (lq4:pq2), (lq5:-), (lq6:-), (lq7:-), (lq8:-)}
3	lq1	{(lq1:pq1), (lq4:pq2), (lq5:pq3), (lq6:-), (lq7:-), (lq8:-)}
4	lq4	{(lq4:pq2), (lq5:pq3), (lq6:-), (lq7:1), (lq8:-)}
5	lq5	{(lq5:pq3), (lq6:pq2), (lq7:pq1), (lq8:-)}
6	lq7	{(lq6:pq2), (lq7:pq1), (lq8:pq3)}
7	-	{(lq6:pq2), (lq8:pq3)}

Note that there is no dependent measurement in the SWAP pattern. In general case with dependent measurement, a qubit can be selected for measurement only if its dependencies have been resolved.

A. ILP Model

In this section, an ILP model is proposed to schedule a measurement pattern $P = (V, I, O, A)$ targeting to minimize the number of required physical qubits to realize that pattern.

I) Parameters

$\boldsymbol{lq} \in \mathbb{N}$**:** an index to identify a logical qubit

$\boldsymbol{pq} \in \mathbb{N}$**:** an index to identify a physical qubit

$\boldsymbol{N} \in \mathbb{N}$**:** the number of non-output qubits of the pattern P, i.e. $|V| - |O|$

$\boldsymbol{E(lq)}$**:** the set of neighbor qubits of lq along with lq itself

$\boldsymbol{D(lq)}$**:** the set of lq dependencies, i.e. the set of qubits on which the measurement of lq depends

II) Variables

$\mathbf{TA(lq)} \in \mathbb{N}$**:** the time (step) that a physical qubit is allocated to the logical qubit lq

$\mathbf{TM(lq)} \in \mathbb{N}$**:** the time (step) of measuring the logical qubit lq

$\boldsymbol{u(pq)} \in \mathbb{Z}_2$**:** a binary variable to determine whether the physical qubit pq is used or not, defined as (3):

$$(pq) = \begin{cases} 1 & if\ physical\ qubit\ pq\ is\ used \\ 0 & o.w. \end{cases} \tag{3}$$

$\boldsymbol{x(lq,t)} \in \mathbb{Z}_2$**:** a binary variable to determine whether qubit lq is measured at time t or not, shown in (4):

$$x(lq,t) = \begin{cases} 1 & if\ lq\ is\ measured\ in\ time\ t \\ 0 & o.w. \end{cases} \tag{4}$$

$\boldsymbol{y(lq,pq)} \in \mathbb{Z}_2$**:** a binary variable to determine whether the physical qubit pq is assigned to the logical qubit lq or not, provided by (5):

$$y(lq,pq) = \begin{cases} 1 & if\ pq\ is\ allocated\ to\ lq \\ 0 & o.w. \end{cases} \tag{5}$$

$\boldsymbol{z(lq,t)} \in \mathbb{Z}_2$**:** a binary variable to determine whether at time *t* a physical qubit is assigned to the logical qubit lq but has not yet been measured or not:

$$z(lq,t) = \begin{cases} 1 & TA(lq) \le t \le TM(lq) \\ 0 & o.w. \end{cases} \tag{6}$$

To set this variable, two auxiliary variables are defined as follows, where $z(lq, t)$ will be equal to their logical *AND* operation:

$\boldsymbol{z1(lq,t)} \in \mathbb{Z}_2$**:** a binary variable to determine whether the logical qubit lq is measured after time t or not, defined as (7):

$$z1(lq,t) = \begin{cases} 1 & t \le TM(lq) \\ 0 & o.w. \end{cases} \quad (7)$$

$\boldsymbol{z2(lq,t)} \in \mathbb{Z}_2$**:** a binary variable to determine whether a physical qubit as allocated to the logical qubit lq before time t or not, shown in (8):

$$z2(lq,t) = \begin{cases} 1 & TA(lq) \le t \\ 0 & o.w. \end{cases} \quad (8)$$

III) Objective Function

The objective function is minimizing the number of used physical qubits, as provided by (9):

$$min \sum_{pq} u(pq) \quad (9)$$

IV) Constraints

1) If a physical qubit pq is assigned to any logical qubit, that physical qubit must exist:

$$y(lq,pq) \le u(pq) \qquad \forall lq, \forall pq \quad (10)$$

2) Each logical qubit is selected and measured only once:

$$\sum_{t=1}^{N} x(lq,t) = 1 \qquad \forall\ lq \backslash O \quad (11)$$

3) Only one logical qubit can be selected at any time for measurement:

$$\sum_{lq} x(lq,t) = 1 \qquad \forall t \quad (12)$$

4) Exactly one physical qubit must be allocated to each logical qubit:

$$\sum_{pq} y(lq,pq) = 1 \qquad \forall lq \quad (13)$$

5) Calculating the measurement time of a logical qubit:

$$TM(lq) = \sum_{t=1}^{N} t * x(lq,t) \quad (14)$$

6) If a logical qubit lq is measured after time t, $z1(lq,t)$ must be set:

$$TM(lq) - t < z1(lq,t) * BigM \quad (15)$$

7) If a physical qubit is assigned to the logical qubit lq before time t, $z2(lq,t)$ must be set:

$$t - TA(lq) < z2(lq,t) * BigM \quad (16)$$

8) Before measuring a logical qubit lq, physical qubits must be assigned to it and its neighbors:

$$TM(lq) \ge TA(lq') \qquad \forall lq \text{ and } \forall lq' \epsilon E(lq) \quad (17)$$

9) Determining whether at time t the physical qubit is assigned to the logical qubit lq or not:

$$z(lq,t) = z1(lq,t) \wedge z2(lq,t) \quad (18)$$

10) In each time, a physical qubit can be assigned to at most one logical qubit:

$$\sum_{lq} y(lq,pq) \wedge z(lq,t) \le 1 \qquad \forall pq, \quad \forall t \quad (19)$$

11) A logical qubit lq can be selected for measurement if and only if its dependencies have been resolved:

$$TM(lq) > TM(lq') \qquad \forall lq \text{ and } \forall lq' \epsilon D(lq) \quad (20)$$

There is an implementation note in this model: the maximum and the minimum number of physical qubits needed to realize a pattern $P = (V, I, O, A)$ is equal to $|V|$, and $|O| + 1$ [15], respectively. One can assume that the number of available physical qubits is equal to $|V|$ and finally the solution determines whether they are used or not. However, the only metric that is minimized in the objective function (9) is the number of used physical qubits and it is not important that which of them are used. This feature causes a large number of optimal solutions, which enlarges the solution space and reduces the convergence speed of the model. Indeed, any composition of the minimum required qubits from $|V|$ is a candidate solution. To limit the solution space, one can set the number of available physical qubits to $|O| + 1$ and increase it one by one until an optimal solution is found.

As an example, the optimal solution of the proposed ILP model for SWAP pattern is shown in Table 2.

Table 2: ILP result for SWAP pattern

Parameter	Value
N	6
lq	{1,2,3, …, 8}
pq	{1,2,3, …, 8}
E(lq)	E[1]={1, 3, 5}, E[2]={2, 3} E[3]={1, 2, 3, 4}, E[4]={3, 4, 5, 7} E[5]={1, 4, 5, 6}, E[6]={5, 6, 7} E[7]={4, 6, 7, 8}, E[8]={7, 8}
D(lq)	D[1], D[2], …, D[8] = {}
Variable	**Value**
TA(lq)	TA[1]=1, TA[2]=1, TA[3]=1, TA[4]=2, TA[5]=3, TA[6]=5, TA[7]=1, TA[8]=6
TM(lq)	TM[1]=3, TM[2]=1, TM[3]=2, TM[4]=4, TM[5]=5, TM[7]=6
u(pq)	u[1], u[2], u[3]
x(lq, t)	x[1,3], x[2,1], x[3,2], x[4,4], x[5,5], x[7,6]
y(lq,t)	y[1,3], y[2,1], y[3,2], y[4,1], y[5,2], y[6,1], y[7,3], y[8,2]
z(lq, t)	z[1,1], z[1,2], z[1,3], z[2,1], z[3,1], z[3,2], z[4,2], z[4,3], z[4,4], z[5,3], z[5,4], z[5,5], z[6,5], z[6,6], z[7,4], z[7,5], z[7,6], z[8,6]
z1(lq, t)	z1[1,1], z1[1,2], z1[1,3], z1[2,1], z1[3,1], z1[3,2], z1[4,1], z1[4,2], z1[4,3], z1[4,4], z1[5,1], z1[5,2], z1[5,3], z1[5,4], z1[5,5], z1[6,1], z1[6,2], z1[6,3], z1[6,4], z1[6,5], z1[6,6], z1[7,1], z1[7,2], z1[7,3], z1[7,4], z1[7,5], z1[7,6], z1[8,1], z1[8,2], z1[8,3], z1[8,4], z1[8,5], z1[8,6]
z2(lq, t)	z2[1,1], z2[1,2], z2[1,3], z2[1,4], z2[1,5], z2[1,6], z2[2,1], z2[2,2], z2[2,3], z2[2,4], z2[2,5], z2[2,6], z2[3,1], z2[3,2], z2[3,3], z2[3,4], z2[3,5], z2[3,6], z2[4,2], z2[4,3], z2[4,4], z2[4,5], z2[4,6], z2[5,3], z2[5,4], z2[5,5], z2[5,6], z2[6,5], z2[6,6], z2[7,4], z2[7,5], z2[7,6], z2[8,6]
Objective Function	**Value**
$\min \sum_{pq} u[pq]$	3

Indeed, ILP solver has found the values of variables based on the input model and parameters in such a way that it minimizes the objective function, while it satisfies constraints. One can simply verify the result using the information of this table. Note that, for the binary variables, only variables with a value of the unity are shown.

Results and Discussion

The proposed model was implemented using SCIP solver [33] and was run on a Core-i7 CPU operating at 2.4 GHz with 8 GB of memory. To evaluate the model, we applied it to some benchmark circuits form [16], [34], [35]. To generate the equivalent measurement patterns of the benchmark circuit, they were decomposed into CZ and J(α) gates. Then, the approach presented in [36], [37] was applied in order to produce the corresponding pattern. The optimizations which include standardization, signal shifting and Pauli simplifications [36] were also performed on the patterns.

The runtime of the proposed method as well as the obtained optimal solutions are given in Table 3. As this table shows, the number of the required qubits in the proposed approach (|O|+1) is the same as the theoretically proven minimal number of qubits in [16]. It should be recalled that the number of used physical qubits in a standard model is equal to $|V|$. Based on the obtained results, our approach (which find the optimal solution) reduces the number of physical qubits by 56.7% on average.

As shown in Table 3, for small patterns with less than 14 qubits, the model obtains the answer in a few seconds. However, with the increase in the pattern size, the run time grew exponentially and took more than 6 hours for GHZ_23 with 45 qubits. For larger patterns, e.g. GHZ_25, ILP solver was unable to find the answer for up to 12 hours.

Conclusion

1WQC is one of the measurement-based quantum computing models that presents a different approach to build quantum computers and is one of the most promising models for physical realization. However, the number of qubits in a 1WQC measurement pattern is much more than its number in the equivalent circuit model, and this issue makes this model hard to implement.

In this study, an ILP model is proposed to minimize the number of used physical qubits to realize a measurement pattern by efficiently scheduling the operations and reusing the physical qubits. The proposed method is able to find the optimal solution for both patterns with flow or only gflow.

Although the proposed method produces the optimal solution, one of the most important limitations of the proposed method and ILP-based methods in general is their high execution time and memory requirements, which grows exponentially with the increase of the problem size. For this reason, the proposed method cannot be used for large measurement patterns. Providing a suitable heuristic method to solve this problem will be pursued as future works.

Table 3: The runtime (in second) of the proposed ILP model and comparison of the obtained result with the standard pattern

Measurement pattern	|V|	|O|	The runtime of the proposed method (s)	The number of required physical qubits	Improvement %
CNOT	4	2	1	3	25.0
SWAP	8	2	1	3	62.5
Toffoli	17	3	5	4	76.5
QECC2_0_2	5	2	1	3	40.0
QECC3_0_2	6	3	1	4	33.3
QECC4_0_2	10	4	3	5	50.0
QECC4_1_2	9	4	3	5	44.4
QECC4_2_2	15	4	720	5	66.6
QECC6_2_2	14	6	405	7	50.0
QFT2	12	2	2	3	75.0
QFT3	30	3	537	4	86.6
Dusch10	29	10	232	11	62.0
Grover3	29	3	897	4	86.2
GHZ_20	39	20	8496	21	46.1
GHZ_23	45	23	21968	24	46.6
GHZ_25	49	25	N/A	NA	NA
Avg. Improvement	-	-	-	-	**56.7**

Author Contributions

Eesa Nikahd proposed the algorithm and designed the experiments. He also carried out the data analysis. Mahboobeh Houshmand and Monireh Houshmand collected the data. All of the authors interpreted the results and contributed to the writing of the manuscript.

Acknowledgment

The authors are grateful to anonymous reviewers for their valuable and constructive comments on an earlier version of this manuscript.

Conflict of Interest

The authors declare no potential conflict of interest regarding the publication of this work. In addition, the ethical issues including plagiarism, informed consent, misconduct, data fabrication and, or falsification, double publication and, or submission, and redundancy have been completely witnessed by the authors.

Abbreviations

1WQC	One-Way Quantum Computing
gflow	Generalized Flow
ILP	Integer Linear Programming
MBQC	Measurement-Based Quantum Computing

References

[1] M. A. Nielsen, I. L. Chuang, Quantum Computation and Quantum Information, 10th Anniversary Edition. Cambridge: Cambridge University Press, 2010.

[2] M. Nakahara, Quantum Computing: From Linear Algebra to Physical Realizations. CRC press, 2008.

[3] G. Benenti, G. Casati, G. Strini, Principles of Quantum Computation and Information-Volume I: Basic Concepts. World scientific, 2004.

[4] D. McMahon, Quantum Computing Explained. John Wiley & Sons, 2007.

[5] R. Raussendorf, H. J. Briegel, "A one-way quantum computer," Phys. Rev. lett., 86(22): 5188, 2001.

[6] R. Jozsa, An Introduction to Measurement Based Quantum Computation, NATO Science Series, III: Computer and Systems Sciences. Quantum Information Processing from Theory to Experiment, 199: 137-158, 2006.

[7] V. Danos, E. Kashefi, P. Panangaden, S. Perdrix, Extended Measurement Calculus, Semantic Techniques in Quantum Computation, 235-310, 2009.

[8] D. E. Browne, E. Kashefi, M. Mhalla, S. Perdrix, "Generalized flow and determinism in measurement-based quantum computation," New J. Phys. 9(8): 250, 2007.

[9] B. Lanyon, P. Jurcevic, M. Zwerger, C. Hempel, E. Martinez, W. D¨ur, H. Briegel, R. Blatt, C. F. Roos, "Measurement-based quantum computation with trapped ions," Phys. Rev. lett., 111(21): 210501, 2013.

[10] J. E. Bourassa, R. N. Alexander, M. Vasmer, A. Patil, I. Tzitrin, T. Matsuura, D. Su, B. Q. Baragiola, S. Guha, G. Dauphinais, et al., "Blueprint for a scalable photonic fault-tolerant quantum computer," Quantum, 5: 392, 2021.

[11] C. Reimer, S. Sciara, P. Roztocki, M. Islam, L. Romero Cort´es, Y. Zhang, B. Fischer, S. Loranger, R. Kashyap, A. Cino, et al., "High-dimensional one-way quantum processing implemented on d-level cluster states," Nat. Phys., 15(2): 148-153, 2019.

[12] M. Zwerger, H. Briegel, W. D¨ur, "Hybrid architecture for encoded measurement-based quantum computation," Sci. Rep., 4(1): 1-5, 2014.

[13] M. Zwerger, H. Briegel, W. D¨ur, "Measurement-based quantum communication," Appl. Phys. B, 122(50): 1-15, 2016.

[14] E. Nikahd, M. Houshmand, M. S. Zamani, M. Sedighi, "One-way quantum computer simulation," Microprocess. Microsyst., 39(3): 210-222, 2015.

[15] M. Houshmand, M. Houshmand, J. F. Fitzsimons, "Minimal qubit resources for the realization of measurement-based quantum computation," Phys. Rev. A, 98(1): 012318, 2018.

[16] E. Nikahd, M. Houshmand, M. S. Zamani, M. Sedighi, "OWQS: one-way quantum computation simulator," in Proc. 15th Euromicro Conference on Digital System Design: 98–104, 2012.

[17] E. Nikahd, M. Houshmand, M. S. Zamani, M. Sedighi, "GOWQS: Graph-based one-way quantum computation simulator," in Proc. 24th Iranian Conference on Electrical Engineering (ICEE): 738-744, 2016.

[18] A. Broadbent, J. Fitzsimons, E. Kashefi, "Universal blind quantum computation," in Proc. 50th Annual IEEE Symposium on Foundations of Computer Science: 517-526, 2009.

[19] J. Fitzsimons, E. Kashefi, "Unconditionally verifiable blind quantum computation," Phys. Rev. A, 96(1): 012303, 2017.

[20] M. Houshmand, M. Houshmand, S. Tan, J. Fitzsimons, "Composable secure multi-client delegated quantum computation," arXiv preprint arXiv:1811.11929, 2018.

[21] A. Farghadan, N. Mohammadzadeh, "Quantum circuit physical design flow for 2D nearest-neighbor architectures," Int. J. Circuit Theory Appl., 45(7): 989-1000, 2017.

[22] A. Farghadan, N. Mohammadzadeh, "Mapping quantum circuits on 3D nearest-neighbor architectures," Quantum Sci. Technol., 4(3): 035001, 2019.

[23] A. M. Childs, E. Schoute, C. M. Unsal, "Circuit transformations for quantum architectures," arXiv preprint arXiv:1902.09102, 2019.

[24] G. Li, Y. Ding, Y. Xie, "Tackling the qubit mapping problem for NISQ-era quantum devices," in Proc. the 24th International Conference on Architectural Support for Programming Languages and Operating Systems: 1001-1014, 2019.

[25] A. Zulehner, A. Paler, R. Wille, "An efficient methodology for mapping quantum circuits to the IBM QX architectures," IEEE Trans. Comput. Aided Des. Integr. Circuits Syst., 38(7): 1226-1236, 2018.

[26] R. Wille, L. Burgholzer, A. Zulehner, "Mapping quantum circuits to IBM QX architectures using the minimal number of SWAP and H operations," in Proc. 56th ACM/IEEE Design Automation Conference (DAC): 1-6, 2019.

[27] P. Murali, J. M. Baker, A. Javadi-Abhari, F. T. Chong, M. Martonosi, "Noise-adaptive compiler mappings for noisy intermediate-scale quantum computers," in Proc. the 24th International Conference on Architectural Support for Programming Languages and Operating Systems: 1015-1029, 2019.

[28] E. Nikahd, N. Mohammadzadeh, M. Sedighi, M. S. Zamani, "Automated window-based partitioning of quantum circuits," Phys. Scr., 96(3): 035102, 2021.

[29] E. T. Campbell, J. Fitzsimons, "An introduction to one-way quantum computing in distributed architectures," Int. J. Quantum Inf., 8(1): 219-258, 2010.

[30] S. C. Benjamin, J. Eisert, T. M. Stace, "Optical generation of matter qubit graph states," New J. Phys., 7(1): 194, 2005.

[31] J. Chen, L. Wang, E. Charbon, B. Wang, "Programmable architecture for quantum computing," Phys. Rev. A, 88(2): 022311, 2013.

[32] S. Sanaei, N. Mohammadzadeh, "Qubit mapping of one-way quantum computation patterns onto 2D nearest-neighbor architectures," Quantum Inf. Process., 18: 1-19, 2019.

[33] A. Gleixner, M. Bastubbe, L. Eifler, T. Gally, G. Gamrath, R. L. Gottwald, G. Hendel, C. Hojny, T. Koch, M. E. L¨ubbecke, S. J. Maher, M. Miltenberger, B. M¨uller, M. E. Pfetsch, C. Puchert, D. Rehfeldt, F. Schl¨osser, C. Schubert, F. Serrano, Y. Shinano, J. M. Viernickel, M. Walter, F. Wegscheider, J. T. Witt, J. Witzig, The SCIP Optimization Suite 8.0, Technical Report (Optimization Online, 2021).

[34] V. V. Albert & P. Faist, eds., The Error Correction Zoo, 2022.

[35] D. Maslov, Reversible logic synthesis Benchmarks page, 2021. https://reversiblebenchmarks.github.io

[36] A. Broadbent, E. Kashefi, "Parallelizing quantum circuits," Theor. Comput. Sci., 410(26): 2489-2510, 2009.

[37] M. Houshmand, M. H. Samavatian, M. S. Zamani, M. Sedighi, "Extracting one-way quantum computation patterns from quantum circuits," in Proc. The 16th CSI International Symposium on Computer Architecture and Digital Systems (CADS 2012): 64–69, 2012.

Biographies

Eesa Nikahd received his B.Sc. degree in Computer Engineering from Shiraz University, Shiraz, Iran in 2010. He received his M.Sc. and Ph.D. degrees from Amirkabir University of Technology, Tehran, Iran in 2012 and 2018, respectively. In 2021, he joined the department of computer engineering, Shahid Rajaee Teacher Training University as an assistant professor. His current research interests include fault-tolerant quantum computing, quantum physical design and quantum machine learning.

- Email: nikahd@sru.ac.ir
- ORCID: 0000-0001-5112-1695
- Web of Science Researcher ID: HNJ-2096-2023
- Scopus Author ID: 55570206700
- Homepage: https://www.sru.ac.ir/nikahd/

Mahboobeh Houshmand received her B.Sc. and M.Sc. in computer engineering, majoring in software, from Ferdowsi University of Mashhad in 1386 and 1389, respectively, and her Ph.D. in computer engineering, majoring in computer architecture from Amirkabir University of Technology in 1393. From the end of the summer of 2015 to the end of the summer of 2016, she was a post-doctoral researcher in the field of quantum cryptography jointly at the National University of Singapore and Sigapore University of Technology and Design. Dr. Houshmand is currently an assistant professor in the computer engineering department of Islamic Azad University of Mashhad. Her research interests include quantum information theory and quantum computing, cryptography, multi-agent systems and data mining.

- Email: ma.houshmand@iau.ac.ir
- ORCID: 0000-0003-2017-4369
- Web of Science Researcher ID: AAA-0000-0000
- Scopus Author ID: 36080655500
- Homepage: http://ceit.aut.ac.ir/~houshmand/

Monireh Houshmand received the B.S., M.S. and Ph.D. degrees in Electrical Engineering from Ferdowsi University of Mashhad, in 2005, 2007 and 2011, respectively. In 2011, she joined the department of electrical engineering, Imam Reza International University as an assistant professor. She became an associate professor in 2019. Her research interests include quantum cryptography, quantum error correction, quantum logic synthesis and artificial intelligence.

- Email: m.hooshmmmand@imamreza.ac.ir
- ORCID: 0000-0001-9215-1532
- Web of Science Researcher ID: ABD-4497-2021
- Scopus Author ID: 35752876400
- Homepage: https://scholar.google.com.sg/citations?user=K5bMQeQAAAAJ&hl=en

How to cite this paper:
E. Nikahd, M. Houshmand, M. Houshmand, “1WQC pattern scheduling to minimize the number of physical qubits,” J. Electr. Comput. Eng. Innovations, 11(2): 391-398, 2023.

DOI: 10.22061/jecei.2023.9374.613

URL: https://jecei.sru.ac.ir/article_1849.html