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Research paper

Response Surface Methodology for Behavior Analysis and Performance Improvement of Gravitational Search Algorithm

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Abstract

Background and Objectives: In recent years, various metaheuristic algorithms have gained popularity due to their effectiveness in solving complex optimization problems across diverse domains. These algorithms are now being utilized for an ever-increasing number of real-world applications. However, two important factors that significantly impact the performance of metaheuristic algorithms are understanding their behavior and fine-tuning their parameters. Deep understanding of an algorithm behavior assists in improving efficiency, while meticulous parameter calibration enhances optimization capability.
Methods: In this study, a response surface methodology-based approach is proposed to analyze the behavior of optimization algorithms. This approach constructs a comprehensive model to determine parameter importance and interaction effects. Although applied to the Gravitational Search Algorithm, this methodology can serve as a generally applicable strategy to gain insights into any metaheuristic algorithm's functionality through quantitative and visual analysis.
Results: Evaluation using 23 test functions exhibited that the technique identifies ideal parameter values and their comparative importance and interplays, enabling superior comprehension.
Conclusion: The proposed framework utilizes informative modeling and multi-faceted analysis to elucidate algorithm mechanics for more targeted calibration, thereby enhancing optimization performance.

Introduction

Metaheuristic approaches help in solving optimization problems; therefore, optimizing their performance is crucial, given their extensive use in a wide array of scientific and engineering problems. Different variants of these approaches have been proposed in recent years [1]-[4]; however, the primary challenge persists in ensuring the efficiency and stability of these algorithms.

The performance of these algorithms is heavily dependent on finding the right set of parameter values. However, navigating the high-dimensional space of possible parameter combinations can be computationally prohibitive. Parameters can be tuned using two different strategies: offline parameter initialization and online parameter tuning. In offline methods, parameters are initialized and fixed before the execution of the algorithm whereas online strategies dynamically and adaptively tune parameters during running time [5], [6]. On the other hand, online approaches are very complicated and time consuming, also their usage is not possible in all scenarios.

A simple method in offline category is the Design of

Experiments (DOE) method that provides a systematic framework for analyzing parameter effects and interactions. It can be very useful for gaining deeper insight into optimization algorithms and improving their performance. DOE establishes a structured approach for determining the relative importance of parameters, unveiling complex interactions, and finding optimal values. By strategically selecting experimental designs, informative models can be constructed to clarify the behavior of algorithms with minimal runs [7].

Two recently proposed methods [8], [9] employed a two-stage algorithm based on DOE to optimize the objective function and maximize the efficiency of the problem. Gunawan and Lau [10] presented an experimental sequential approach to determine the important parameters and fine-tune their values in two stages. The proposed framework relies on design of experiments (DOE) methods, where it employs factorial design in the first phase to screen parameter effects and extract key parameters. In second phase, it estimates the range of these parameters using the surface coverage method.

Response surface methodology (RSM) have gained significant traction across various scientific fields in recent times. [11] applies multi-objective heuristic optimization to generate ensemble classifiers and then utilizes factorial design and response surface methodology to study interactions between algorithm factors and performance, characterizing ensemble reliability. Actually, this paper has focused on the stability analysis of the multi-objective optimization-based classifier. The authors in present [12] an integrated approach combining Taguchi experimental design, response surface modeling, and multi-objective optimization to balance energy and time in machining. Key parameters are identified then surfaces constructed to characterize interactions, enabling navigation of tradeoffs between efficiency and rate through Pareto-based techniques.

GSA (Gravitational Search Algorithm) [13], inspired by the Newtonian gravity and the laws of motion, stands out as a powerful metaheuristic algorithm. This algorithm has shown satisfactory results in solving not only benchmark optimization problems but also in tackling diverse real-world problems. While some variants of this algorithm have been proposed in single objective, binary, and multimodal domains, there remains considerable potential for future exploration.

Particularly, the fine-tuning of the GSA algorithm has not been explored through offline strategies until our previous research [14]. Previously, initial parameter values were chosen based on the recognition of the problem space and trial-and-error mechanisms. In the mentioned study [14], a simple and systematic approach is introduced to fine-tune the parameters of the GSA algorithm using Taguchi method, which is one of existing DOE methods [15], [16]. To the best of our knowledge, there is no publication addressing the parameters' importance and the effects of their interactions. Therefore, contributions and achievements of this research are as follows:

1- Determining the importance and effect of each metaheuristic algorithms' parameter on the outcome using a systematic and analytical approach.
2- Analyzing the interaction effects between parameters.
3- Fine-tuning the parameters as a secondary objective.

Examining the behavior of the GSA can be generalized through testing across various problems. To achieve this, 23 benchmark test functions were carefully selected, and a comprehensive set of quantitative and visual analyses were conducted. The insights gained from these analyses offer valuable guidance for researchers aiming to enhance the performance of the GSA algorithm.

The remaining sections of this paper are structured as follows: Section 2 introduces basic concepts, including GSA and RSM (Response Surface Methodology). Section 3 describes the process of the proposed approach. A comprehensive analysis of 23 test functions and a detailed description of GSA's behavior are presented in section 4. The final section provides conclusions and outlines future directions for research.

Related Work

Previous works can be categorized into two groups. The first category aims at fine-tuning the parameters, while the second category focuses on understanding the parameters and investigating their effects.

In the first category, parameters have been tuned using Design of Experiments (DOE) methods [17]-[20]. Adenso-Diaz and Laguna [21] have presented a framework called CALIBRA, which uses Taguchi method and a local search procedure to find appropriate parameter settings. However, this approach is limited to handling only five parameters and focuses solely on the main effects, neglecting interaction effects between them. Hutter *et al.* [22] employed Sequential Parameter Optimization (SPO) algorithm and DOE techniques to construct a model for optimization problems, exploring the problem space in order to tune the parameters. Akbaripour and Masehian [23] introduced an approach to find the best initial values for optimization algorithms using DOE, Signal to Noise (S/N) ratio, Shannon entropy, and VIKRO methods. This paper considered both the quality of the solution and the running time of algorithm.

Gunawan and Lau [10] proposed a two phase sequential experimental method to determine important parameters and fine-tune them. They used a framework based on DoE, employing the Factorial method to assess the parameters' importance and extract effective parameters in the first step. In the second step, they used Response Surface Model (RSM) [19] to estimate the promising initial value range for the important parameters. In a recent work by Pereira *et al.* [24], the Lichtenberg algorithm, a metaheuristic algorithm, was tuned and accelerated using RSM methodology and chaos theory. Comparisons on benchmark functions revealed that their proposed chaotic Lichtenberg algorithm achieved superior accuracy, lower cost, and stability compared to genetic algorithms and other bio-inspired metaheuristics. This demonstrates that carefully tuning parameters and integrating acceleration techniques can substantially enhance metaheuristic performance.

The limited research in the second category has specifically explored the effects of parameters using statistical methods. Some of these studies, as a secondary purpose, have delved into the effects of one or two parameters. For example, Kapoor *et al.* [25] has evaluated the effect of the mutation operator of the genetic algorithm (GA) and its importance on both simple and complicated problems. Haines *et al.* [26], proposed an approach to determine the relative importance of GA parameters using Fractional factorial as a DOE method. After running the experiments, the optimal (best) parameter values are provided, and the statistical significance for each parameter is calculated to determine and rank its importance. Arenas et al. [27] has selected four parameters of the genetic considered different values for them, and designed several experiments covering all combinations of parameter values. The proposed approach did not use any DOE methods, making it less suitable, especially when the number of parameters and their values are increased. This paper utilized analysis of variance table (ANOVA) to determine the importance of the selected parameters.

Background

A. Gravitational Search Algorithm (GSA)

One of the most fundamental laws in physics is Newton's Law of Gravity, which states: "every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.". Based on this law, the Gravitational Search Algorithm (GSA), as a meta-heuristic optimization algorithm, is formulated [13]. This algorithm is applicable to many optimization problems [28]-[31], and the obtained results confirm its performance.

Agents are considered as objects, and their fitness is measured based on their masses. According to the force of gravity, all agents attract each other, resulting in a global movement of all objects towards agents with heavier masses. The heavier masses signify the good solutions to the problem.

The GSA algorithm can be described as follows: Consider a system with N masses (agents), in which the position of the i^{th} mass is defined as follows:

$$X_i = (x_i^1, ..., x_i^d, ..., x_i^n) \quad for\ i = 1, 2, ..., N \tag{1}$$

where x_i^d presents the position of ith agent in the dth dimension where n is dimension of the search space. It's important to note that the positions of the masses correspond to the solutions of the problem. The mass of each agent is calculated after computing the current fitness of the population as follows:

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{2}$$

$$M_i(t) = \frac{q_i(t)}{\sum_{j=1}^{N} q_j(t)} \tag{3}$$

where $fit_i(t)$ represents the fitness value of the agent i in tth iteration and $worst(t)$ and $best(t)$ are defined as follows (for a maximization problem):

$$best(t) = \min_{j \in \{1,..,N\}} fit_j(t) \tag{4}$$

$$worst(t) = \max_{j \in \{1,..,N\}} fit_j(t) \tag{5}$$

At a specific time "t", the force acting on mass " i " from mass " j " is defined as follows:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}^{rpower}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{6}$$

To calculate the distance between ith and jth agent, "Rpower" plays an important role, which was set to one in the original version. To compute the acceleration of an agent, the total forces from a set of heavier masses that apply to it should be considered based on law of gravity (13), followed by the calculation of the agent's acceleration using the law of motion (14).

$$F_i^d(t) = \sum_{j \in Kbest, j \neq i} rand_j F_{ij}^d(t) \tag{7}$$

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{8}$$

Afterward, the next velocity of an agent is calculated as a fraction of its current velocity added to its acceleration (15). Then, its next position could be calculated using (16).

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{9}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (10)$$

where $rand_i$ and $rand_j$ are two uniformly distributed random numbers in the interval [0, 1], and e is a small constant. $G(t)$ is the gravitational constant at time t, which is a decreasing function of time; it will take an initial value G_0, and it will be reduced over time. "kbest" is a function of time, initialized with K_0 at the beginning and is decreased with time, and $R_{ij}(t)$ is the Euclidean distance between two agents i and j:

$$R_{ij}(t) = \left\| X_i(t), X_j(t) \right\|_2 \quad (11)$$

Fig. 1 shows the steps of GSA.

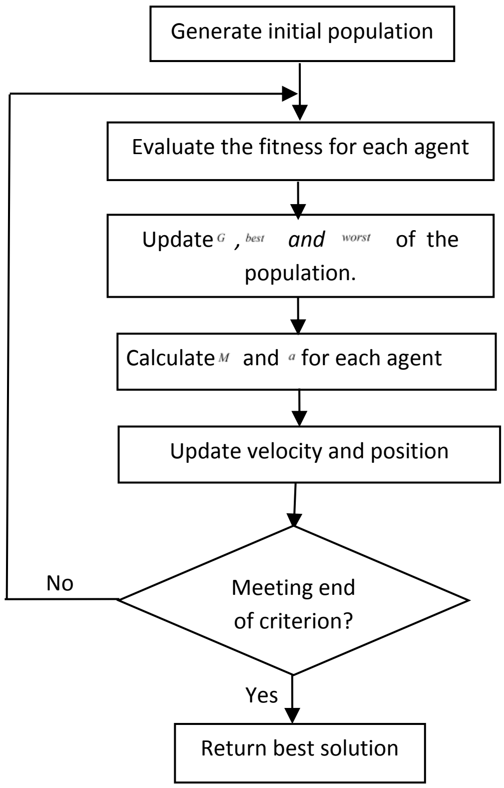

Fig. 1: Flowchart of GSA.

B. *Identifying GSA Parameters*

The gravitational constant plays a crucial role in maintaining an appropriate balance between exploration and exploitation. Large values of G represent high power attraction between the masses that causes more movement and complexity. Over time, G should be decreased in order to find the optima around a good solution. Therefore, the value of G significantly influences the algorithm's performance and should be carefully controlled. According to (12), G is dependent on G0 and alpha.

$$G(t) = G_0 e^{-\alpha \frac{t}{T}} \quad (12)$$

Kbest is used to control the active agents. In the early iterations, almost all agents attract others to prevent GSA from getting trapped in local optima. This behavior guarantees the exploration in the search process of GSA. In the next iterations, the value of Kbest should be decreased to emphasize on exploitation instead of exploration and improve the convergence rate. It is noteworthy that in the final iteration, only one agent attracts the others.

As mentioned in our previous research [14], a large value for Kbest decreases the convergence rate and provides the algorithm with a chance to explore the search space more thoroughly, preventing it from getting trapped in local optima. Nevertheless, with an increase in Kbest, the computational time and complexity also increase. Therefore, (10) is designed to control the value of Kbest during the progress of the algorithm.

$$Kbest = \beta * \left(\frac{(1-p)}{T} * t + 1 \right), \quad (13)$$

where t is the current iteration, T is the maximum number of iterations, and p is the fraction of total agents in the last iteration. In this equation 0.2<β≤1, whereas in the original version of GSA the value of β is equal to one. When the value of β is 0.5, only half of the agents that are applied in the original GSA will be considered. This reduction decreases the complexity of GSA. Excessively low beta values reduced randomness and exploration causing premature convergence to poor local optima. Excessively high values introduced randomness that hindered convergence, with searches failing to refine and exploit detected promising areas. An intermediate balanced beta enables sufficient diversification to escape local traps yet allows adequate intensification when promising regions are uncovered to drive solutions towards global optimality.

C. *Response Surface Methodology (RSM)*

In experiments and simulations, many influential factors exist, and their influences should be examined. Moreover, each factor can be initialized with a different variety of values. Therefore, deliberate factor investigation and initialization is very expensive and time consuming. However, it becomes even more complex and challenging when the number of factors and their possible values is increase. In recent years, some automated approaches have been proposed, such as Design of Experiments (DOE), which is an approach for systematizing the process of designing experiments [16].

To Design experiments using DOE, three elements should be addressed: the factors to be tested, the levels of those factors, and the plan of the experiments. Factors represent the parameters of the problem, and levels are different values for them. The plan of the experiments is a series of tests or runs, selected in a way that the effect of different levels of each factor on the response will be investigated [19].

Factorial design [16] and Taguchi design [32] are two well-known approaches of DOE. The factorial design method tests all possible factor-level combinations, but it becomes inefficient in terms of time and cost when many factors and levels exist in the problem. On the other hand, the Taguchi method designs a small number of experiments, which is not proficient in analyzing the factors' effect.

Response Surface Methodology (RSM) is a model-based approach of DOE. RSM constructs a surface model from the designed experiments to extrapolate previously-unseen regions of the factor space and analyze the relations between input factors and responses [19]. Fig. 2 illustrates the process of RSM.

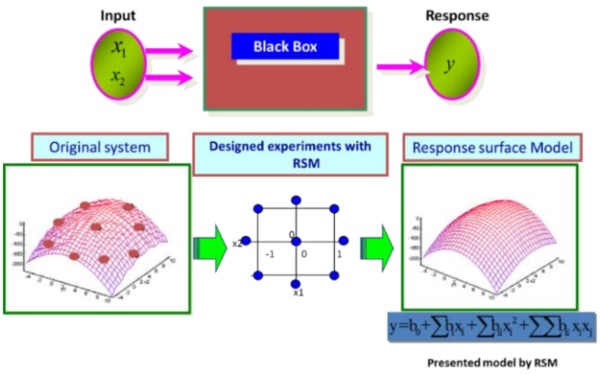

Fig. 2: The RSM method.

Some important advantages of RSM supporting its usage are:

1- Simplifying the problem by providing the surface model.
2- Determining the sensitivity of each factor.
3- Analyzing continuous variables.

Despite these advantages, RSM has a prediction error, as depicted in Fig. 3.

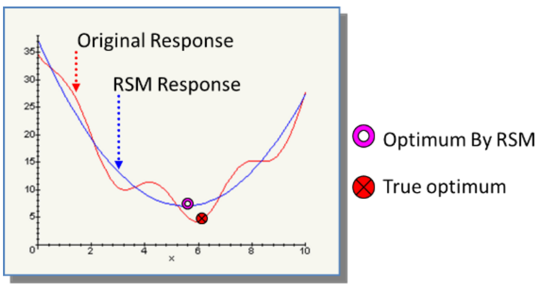

Fig. 3: Original response vs. RSM optimum response.

Two designs based on RSM are presented: central composite design (CCD) and Box-Behnken design (BBD). CCD is routable and popular, providing three to five levels for each factor. In BBD, factors can have only three levels and the provided model is not always routable. After designing and running the experiments, all the results are used to construct the statistical model.

Fig. 4 shows the designed experiments by CCD. There are three factors, x1, x2 and x3, each of them can be initiated with 5 levels, 1, -1, 0, α, -α. CCD includes three groups of experiments:

1- Factorial points: all levels of factors are 1 or -1.
2- Axial points: the value of one factor is α or -α, and other factors are initiated whit 0.
3- Central points: the values of all factors are zero. Additional center points are considered to construct an appropriate surface model.

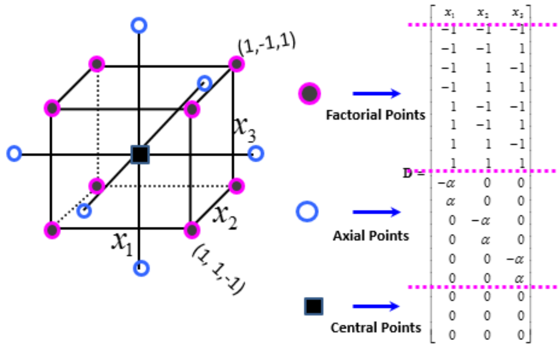

Fig. 4: Designed experiments by CCD.

The current level's values are called coded values, which can be replaced with real values of factors in the problem space. Alpha is distance of each axial point from the center in a central composite design.

RSM is supported by popular statistical tools such as SAS and Minitab. These tools analyze the results and provide plots that are very proficient in determining the impact of each factor on response.

Proposed Method

This section describes the proposed approach, employing Design of Experiments (DOE) to evaluate the parameters' importance of GSA and understand their interaction effects. The objective is to comprehend the behavior of GSA.

Initially, Key parameters of GSA (as factors) and different ranges of their values (as levels) are identified. The parameters we chose to focus on - population size (N), power of the gravitational constant (Rpower), alpha (α), initial gravitational constant (G0), and beta (β) - directly control the major components of how GSA operates. More specifically:

- Population size (N) determines the number of agent solutions in each iteration of the algorithm. This impacts the exploration of the search space, with larger N enabling sampling from more diverse areas.

- Rpower controls the rate of decay of the gravitational constant over iterations. This decay rate affects the exploration-exploitation tradeoff, with higher Rpower providing more thorough exploration initially.

- Alpha (α) controls the sharing of information between agents, with higher alpha meaning that poorer solutions receive stronger gravitational forces from better solutions. This guides the search direction.

- G0 sets the starting gravitational constant, determining the initial search scope and exploration range. Larger G0 leads to wider exploration early on.

- Beta (β) controls the stochastic contribution for moving agents. This introduces randomness to avoid local optima traps.

In this way, N, Rpower, α, G0, and β together determine the exploration-exploitation tradeoff, sharing of information, escape from local optima, and other core components of GSA operation. We selected these parameters specifically because they offer comprehensive control over the search behavior, convergence, and solution quality obtained by the algorithm. Adjusting these parameters provides insight into improving GSA's usage across problem domains.

Subsequently, experiments are designed. The next step involves the selection of an optimization problem and conducting experiments to generate results. The final step includes constructing a model to analyze the parameters' importance and their interactions. All of the abovementioned steps are thoroughly described in the following subsections.

A comprehensive analysis of an algorithm's behavior is only possible when its performance is observed across a diverse set of optimization problems. To achieve this goal, 23 standard functions were meticulously chosen, and the described proposed method was applied to each. The following section provides a summary of GSA's behavior based on the comprehensive analysis conducted.

A. Factor Identification and Level Selection

Studying GSA, independent parameters are extracted and listed in Table 1. Along parameter selection, determining their range value is important. Proper identification of parameter ranges leads to a more comprehensive statistical model and, consequently, better results. Each parameter's value range is chosen and presented in Table 1 based to a literature review, algorithm's behavior, and the role of each parameter. Hereafter, parameters and their value ranges will be referred to as Factor and Level, respectively, following the Design of Experiments (DoE) literature.

Table 1: Selected Factors and corresponding Levels for GSA parameters

Factor name	Level 1	Level 2	Level 3	Level 4	Level 5
N (Population Size)	20	30	40	50	60
Rpower	0.25	0.5	0.75	1	1.25
α (Alpha)	10	20	30	40	50
G0	10	40	70	100	130
β (Beta)	0.2	0.4	0.6	0.8	1

B. Design of Experiments Using RSM

RSM optimization navigates to optimal areas rapidly through sequential experimentation leveraging model predictions. This facilitates optimization with reduced sampling requirements versus. Additionally, RSM visualizations unlock intuitive comprehension of performance drivers, guiding reasoning about parameter tuning. The interpolated response surfaces expose sensitivity not discernible from individual DOE samples. In summary, RSM facilitates optimized enhancement of GSA through revealing performance interactions plus efficiently focusing sampling to navigate towards improved solutions. The combination of modeling, visualization, and sequential experimentation in RSM surpasses fixed sampling techniques like DOE by enabling more insightful navigation of high-dimensional spaces.

Designing appropriate experiments is crucial for creating an accurate analytical model. Considering the mentioned advantages of Response Surface Methodology (RSM) and the objectives of this research, RSM has been chosen to design experiments. Since Central Composite Design (CCD) offers five levels for each factor, it can explore more situations compared to Box-Behnken Design (BBD). Therefore, CCD design has been selected.

Configuring CCD parameters, Alpha is set to 2; therefore, five levels' values will be adjusted to -2, -1, 0, 1, and 2. In this situation, 52 experiments are designed. Having more replicas for each experiment in CCD enhances the model's accuracy, leading to better analysis. Considering the stochastic nature of the gravitational algorithm, 15 replicas for each experiment are taken into account.

Considering the number of experiments and replicas, a total of 780 experiments are designed, which are wisely selected by the model. All Steps were conducted in Minitab and Table 2 represents the first ten of these designed experiments.

Table 2: Initial ten experiments designed by RSM

EXP number	N	Rpower	A	G0	B
1	30	0.75	40	100	0.8
2	40	1	30	70	0.2
3	40	1.5	30	70	0.6
4	40	1	30	70	0.6
5	50	0.75	40	100	0.4
6	50	1.25	20	100	0.8
7	40	1	30	70	0.6
8	30	1.25	40	40	0.4
9	40	1	30	10	0.6
10	40	1	30	70	0.6

C. Selection of Optimization Problems and Experiment Execution

In the initial step, an optimization problem needs to be chosen. In this paper, the Ackley function is chosen from benchmark functions outlined in Appendix 1 of [13]. Denoted as F10, the Ackley function represents one of the multimodal functions and is illustrated in Fig. 5.

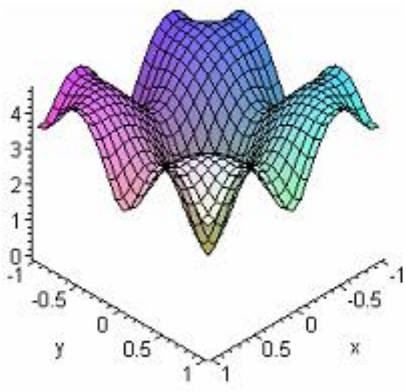

Fig. 5: The Ackley Function plot.

In the next step, parameters of GSA need to be initialized according to Table 2. In addition, the number of iterations of GSA will be set to 500.

During the execution phase, all 780 designed experiments are executed in Matlab, and the obtained results are subsequently imported back into Minitab.

D. Experimental Analysis of the Selected Function

This section outlines the study and analysis of the parameters' effect on the Ackley function.

I) Optimal Parameter Setting

A straightforward analysis involves identifying the optimal values for the parameters, a task facilitated by examining the results of experiments. From the outcomes of the 780 experiments conducted on the Ackley function, it was determined that experiment no. 141 yielded the best (optimal) solution. The parameter values for this particular experiment are presented in Table 3. The initial outcome from applying the proposed method involves fine-tuning the gravitational search algorithm's parameters, resulting in improved performance responses on the test functions.

Table 3: Response Surface Methodology (RSM) results for the Ackley function

		Tuned GSA with RSM	GSA[13]
Result		4.44×10^{-15}	6.9×10^{-6}
Parameters	Population size	50	50
	Rpower	0.75	1
	Alpha	40	20
	G_0	100	100
	Beta	0.8	-
Experiment number		141	-

II) Parameter Importance Analysis

In this paper, a two-way ANOVA is employed to determine the statistical significance of each main effect and interaction effect. The p-value serves as an indicator of whether a factor has a significant impact on the response. If the p-value is less than 0.001, the factor is considered effective; otherwise, it is deemed not effective.

Assessing the effect size of a significant factor is of great importance. In statistics, an effect size serves as a quantitative measure of the strength of a phenomenon [33]. Relative Importance and Absolute Importance measures are employed for this purpose [26]. Absolute importance of a parameter quantifies the difference between the highest and lowest responses achieved through the tuning of that parameter.

Initially, the average of responses ($\bar{y}_{ij}$) resulting from adjusting x_i in level L_i is calculated according to (14). For problem modeling, assume set S consists of all combinations of factors based on the RSM model; therefore, $S_{ij} = \{x \in s \mid x_i = l_j\}$.

$$\bar{y}_{ij} = \frac{\sum_{d \in s_{ij}} y_d}{|s_{ij}|} \tag{14}$$

Absolute Importance and Relative Importance are calculated based on (15) and (16), respectively.

$$E_i = \max\{\bar{y}_{ij}\} - \min\{\bar{y}_{ij}\} \tag{15}$$

$$RE_i = 100.E_i / \bar{\bar{y}} \tag{16}$$

In these equations, $\bar{\bar{y}}$ is calculated according to (17):

$$\bar{\bar{y}} = \frac{\sum_{d=1}^{n} y_d}{n} \tag{17}$$

Finally, the Absolute Importance, Relative Importance, and p-values of the parameters of the Ackley function are presented in Table 4.

Table 4: Absolute Importance, Relative Importance, and p-values of the Ackley function

Factor	P-value	E_i	RE_i	rate
population size	0.000	3.29	139.35	5
$Rpower$	0.000	14.01	592.97	1
α	0.000	4.34	183.69	4
G_0	0.000	11.08	469.17	2
β	0.000	3.91	165.70	3

In Table 4, the P-value column reveals that all five factors significantly impact the GSA results. A comparison between E_i and RE_i makes it clear that Rpower is the most effective factor, with G0 i closely following as the second most impactful factor with a slight difference. This analysis, applied to problems similar to the Ackley function, helps identify the focal factors that contribute to more effective results.

In addition to these findings, Fig. 6 illustrates the main effect graph of the Ackley function generated in Minitab. The graph reveals the following observations:

- **Effect size of a factor on response.** A factor is more effective if it induces significant changes in the response. For instance, Rpower led to a wide range of responses, establishing it as the most influential factor, while G0 holds the second position.
- **Identifying the optimal value for each parameter and observing the behavior of the function around it.** The optimal value for Rpower is 0.8, and for G0, it is 100, leading to improved responses.
- **The direction of parameters changes.** Rpower and G0 exhibit changes in the same direction, resembling a quadratic curve. Consequently, determining values for these parameters requires more sensitivity. On the other hand, Population and alpha follow linear detrimental and incremental trends, making tuning for these parameters simpler.

III) Analysis of Interaction Effects

Analyzing the interplay between parameters, referred to as the interaction effect, is valuable. When the impact of one factor is contingent on the level of another factor, an interaction plot can provide insights. This graph illustrates possible interactions, which consists of the average of responses. The graph is constructed by fixing the first parameter and plotting the changes in the second parameter against the response. This process is then repeated for different levels of the first parameter. The interaction plot for each pair of parameters and their effects on the response of the Ackley function is illustrated in Fig. 7.

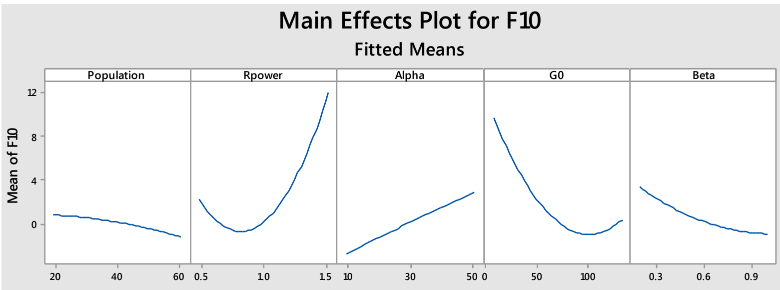

Fig. 6: main effect plot for the Ackley function.

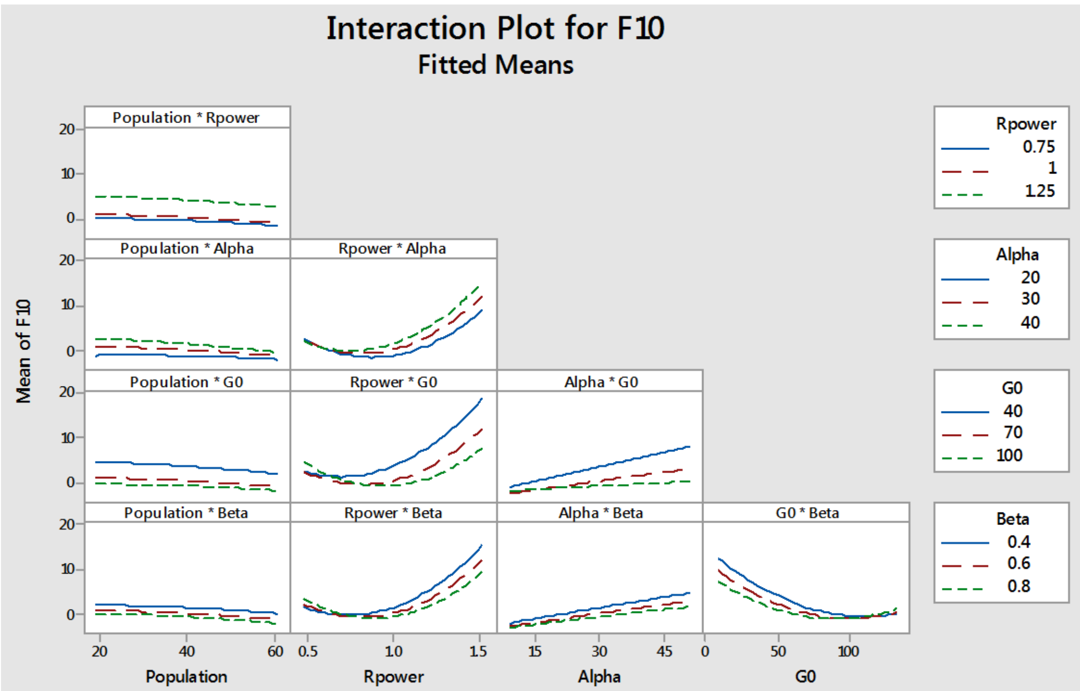

Fig. 7: Interaction plot of each parameter’s pair and their effect on the Ackley function.

Deviation from parallel mode and the extent of this deviation respectively indicate interaction effects and the degree of this interaction. The second column of Fig. 7 indicates that Rpower exhibits interaction effects with all other factors, although the magnitude of this interaction is not significant. Additionally, the G0 and Beta pair (G0*Beta) also shows a slight interaction at their ends. The Population factor shows no sign of interaction with other factors.

Moreover, the P-value measure of paired factors is utilized to determine whether the interaction effects are significant or not. This P-value is extracted from ANOVA, confirming the analysis of the aforementioned plot. For instance, ANOVA reported the interaction between Rpower and Alpha (Rpower*Alpha) is significant at the 0.001 level.

Other types of plots, specifically contour and surface plots, prove valuable in analyzing parameters' interactions. Fig. 8 and Fig. 9 illustrate these plots for the Rpower and G0 parameter pair in the context of the Ackley function. According to these plots, the worst response is obtained when Rpower is set to the maximum and G0 is set to the minimum. The best response occurs under conditions where Rpower is low and G0 is high.

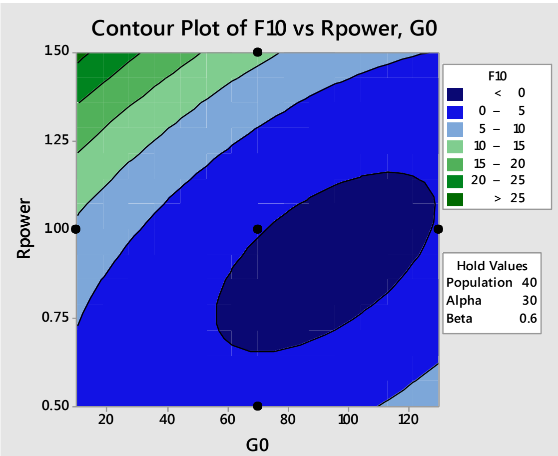

Fig. 8: Counter plot of G0 and Rpower parameter pair for the Ackley function.

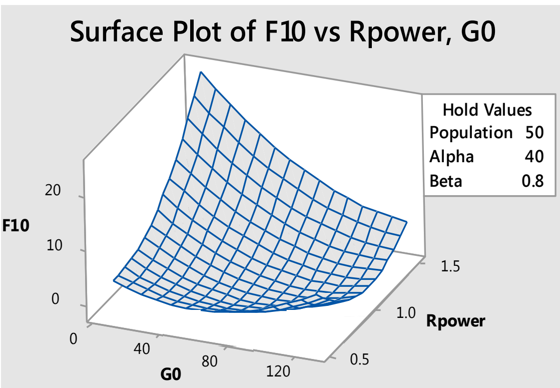

Fig. 9: Surface plot of G0 and Rpower parameter pair for the Ackley function.

Analyzing the Behavior of Gravitational Search Algorithm (GSA)

As previously mentioned, to enhance our understanding of the behavior of GSA, we will examine its performance across 23 standard test functions detailed in Appendix 1 of [13]. These benchmark objective functions serve as the performance evaluation suite for analyzing the gravitational search algorithm. These specific functions were chosen because they were originally introduced and utilized by the authors who developed the gravitational search algorithm (GSA) in their seminal paper [13], [34]. Additionally, these benchmark functions offer a diverse set of complex multi-modal landscapes possessing varied characteristics. These benchmark functions have been widely adopted to characterize, validate, and compare the performance of GSA across subsequent research. All these functions are selected from minimization problems, where n represents the number of dimensions, f_{opt} is the minimum response, and S is a subset of $\mathbb{R}^n$.

These test functions are categorized into three main categories. The first category comprises the unimodal test functions (F1 through F7), each featuring a single local and global optimum. The second category encompasses high-dimensional multimodal functions, F8 through F13, while the third category consists of low-dimensional multimodal functions, F14 through F23. Multimodal functions exhibit multiple local optima but only one global optimum. As the number of dimensions increases, the number of local optima solutions also rises. Successfully navigating through local optima to reach the global optimum is crucial in these problems.

The process previously applied to the Ackley function will now be extended to all the test functions mentioned. Consequently, 23 models will be provided. The impact of parameter tuning on the performance of the GSA will be investigated in the following subsections. This will be followed by a discussion on each parameter's importance, main effect, and interaction effect. Finally, a summarization will rank the levels of parameters, and the optimal value for each factor will be proposed.

A. Presentation of Optimal Solutions

The optimal solution for each function along with the appropriate parameter values are presented in Table 5. The second column of this table ("GSA") displays the solutions obtained from the original GSA [13], averaged over 15 runs. The third column ("Best Tuned GSA with RSM") showcases the best solution among all 780 experiments designed by RSM, with the experiment number identified in the "Expnum" column. The columns labeled "Parameters" showcase the values of parameters associated with the experiment, reflecting the parameter values of the best (optimal) solution. The fourth column presents the average of 15 rounds for this experiment, allowing for comparability with the results of the original GSA.

Table 5: Results for 23 test functions, comparing original GSA with the proposed method

Test Function	GSA	Best Tuned GSA with RSM	Mean Tuned GSA with RSM	Expnum	Parameters				
					Population size	Rpower	Alpha	G0	Beta
F1	7.3×10^{-11}	1.04E-43	**3.21e-24**	295	40	0.5	30	70	0.6
F2	4.03×10^{-5}	2.29E-21	**1.08e-12**	338	50	0.75	40	40	0.8
F3	$0.16\times10^{+3}$	21.61158	52.71	634	40	1	10	70	0.6
F4	3.7×10^{-6}	3.40E-11	**7.25e-11**	40	50	0.75	20	100	0.8
F5	25.16	18.93211	24.42	326	20	1	30	70	0.6
F6	8.3×10^{-11}	0	**1.027e-33**	5	50	0.75	40	100	0.4
F7	0.018	0.011099	0.037	707	50	1.25	20	100	0.8
F8	$-2.8\times10^{+3}$	-4522.85	-2969.49	112	40	1	10	70	0.6
F9	15.32	3.979836	**10.28**	646	40	1	30	10	0.6
F10	6.9×10^{-6}	4.44E-15	**1.178e-14**	141	50	0.75	40	100	0.8
F11	0.29	0.021534	1.500	438	50	0.75	20	100	0.8
F12	0.01	1.57E-32	0.020	754	50	0.75	40	100	0.4
F13	3.2×10^{-32}	1.64E-89	2.659e-32	437	50	0.75	20	40	0.4
F14	3.70	0.998004	**2.805**	368	50	0.75	20	100	0.4
F15	8.0×10^{-3}	0.000581	0.0021	646	40	1	30	10	0.6
F16	-1.0316	-1.03163	-1.031	1	30	0.75	40	100	0.8
F17	0.3979	0.397887	0.397	1	30	0.75	40	100	0.8
F18	3.0	3.0	2.999	216	50	0.75	20	100	0.8
F19	-3.7357	-3.86278	-3.862	9	40	1	30	10	0.6
F20	-2.0569	-3.322	-3.200	272	50	0.75	40	100	0.8
F21	-6.0748	-10.1532	-8.161	4	40	1	30	70	0.6
F22	-9.3399	-10.4029	**-10.402**	7	40	1	30	70	0.6
F23	-9.4548	-10.5364	**-10.536**	419	40	1	50	70	0.6

Overall, the performance of the GSA algorithm has significantly improved across various functions; for instance, F6 has shown noticeable enhancements. As with other metaheuristic approaches, the fine-tuning of parameters plays a crucial role in optimizing the algorithm's performance.

B. Parameter Ranking

In this section, the analysis of results obtained from 23 standard functions is presented, focusing on determining and rating effective parameters based on their effect size for each function. Finally, an assessment of the importance of each parameter is conducted across all 23 test functions.

Effective parameters are identified from the ANOVA table and presented in Table 6 which includes five rows for main parameters, ten rows for two-way interactions between parameters, and 23 columns for test functions.

Parameters with a p-value less than 0.001 are considered to have a significant effect, and these significant parameters are highlighted in the table.

Table 6, reveals that the most effective parameters are Rpower and G0 sequentially. Additionally, when investigating interaction effects, Rpower plays a meaningful role in combination with other parameters, while population has an effective role in only a few functions.

Although the importance of parameters is well represented in Table 6, a quantitative technique is needed for a more thorough analysis. Therefore, Table 7 rates the importance of parameters based on their significant contribution to how many functions. Although the importance of parameters is well represented in Table 6, a quantitative technique is needed for a more thorough analysis.

Therefore, Table 7 rates the importance of parameters based on their significant contribution to how many functions. For instance, population demonstrated a significant effect in 16 test functions, effective in 69.75% of them. According to this table, Rpower and G0 are the most effective parameters, standing in first and second place, respectively, and population, Beta, and Alpha follow in subsequent positions.

In addition to the quantitative criterion, the effect size of parameters is crucial. Effect size is measured with the absolute effect measure, calculated for each parameter in each test function, summarized, ranked, and presented in Table 7. According to this table, Rpower exhibits the highest variance in response values in 15 functions (65.22% of all functions) and is the second most impactful in 3 functions. In five functions, population ranks first, suggesting a considerable role overall. G0 secures the second place in 10 functions, maintaining its position from the previous analysis. Considering both first and second place analyses, it is concluded that Rpower holds the top spot in terms of importance, followed by G0 in the second place.

Table 6: Significance matrix for the 23 benchmark test functions

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20	F21	F22	F23
Population	■	■		■		■	■	■	■	■	■			■			■	■	■	■		■	■
Rpower1	■	■	■	■	■	■	■	■	■	■	■	■	■	■			■	■			■	■	■
Alpha	■		■	■	■	■	■			■	■	■	■		■								
G0	■	■	■	■	■	■	■		■	■	■	■	■	■				■	■	■	■	■	■
Beta	■		■	■	■	■	■		■	■	■	■	■				■	■	■	■			■
Population*Rpower		■					■		■								■						
Population*Alpha										■													
Population*G0		■					■		■								■						
Population*Beta							■		■								■	■		■			
Rpower*Alpha	■		■	■	■	■				■		■	■										
Rpower*G0	■	■	■	■	■	■		■	■	■		■	■	■								■	■
Rpower*Beta	■		■	■	■	■	■		■	■		■	■				■	■					■
Alpha*G0				■	■					■		■	■										
Alpha*Beta		■			■				■	■		■	■										
G0*Beta				■	■		■		■	■		■	■					■				■	

Table 7: The number and percentage of functions (from23 test function) where each parameter showed a significant effect for on the optimization, ranked by effect size

Factor	significance ($p<0.001$)	%	E_i in first stand	%	E_i in second stand	%	First and second Stand
Rpower	19	82.61	15	65.22	3	13.04	78.26
G0	19	82.61	3	13.04	10	43.48	56.52
Population	16	69.57	5	21.74	5	21.74	43.48
Beta	16	69.57	0	0.00	2	8.70	8.70
Alpha	11	47.83	0	0.00	3	13.04	13.04

C. Investigation of Parameter Levels

Table 8 displays distribution of factor levels across test functions. The optimal (best) levels detected for each parameter based on the number of test functions. For instance, Level 2 (equal to 0.75) is identified as the best level for Rpower, with the participation of 12 test functions.

Similarly, the best levels for other parameters are determined. In summary, the optimal values for the population, Rpower, Alpha, G0, and Beta parameters of GSA are 50, 0.75, 30 (or 40), 100, and 0.6, respectively, as presented in Table 9. For a more in-depth analysis, contour, surface, and interaction plots can be generated and investigated.

Table 8: Distribution of Optimal Factor Levels Across Test Functions

	Level 1		Level 2		Level 3		Level 4		Level 5	
Factor	N.	%	N.	%	N.	%	N.	%	N.	%
Population	1	4.35	2	8.70	9	39.13	11	**47.83**	0	0.00
Rpower	1	4.35	12	**52.17**	9	39.13	1	4.35	0	0.00
Alpha	2	8.70	6	26.09	7	**30.43**	7	**30.43**	1	4.35
G0	3	13.04	2	8.70	7	30.43	11	**47.83**	0	0.00
Beta	0	0.00	4	17.39	10	**43.48**	9	39.13	0	0.00

Table 9: Optimal levels for each parameter across 23 benchmark test functions

Factor	Optimal Level	Value
Population	Level 4	50
Rpower1	Level 2	0.75
Alpha	Level 3 ,Level 4	30 , 40
G0	Level 4	100
Beta	Level 3	0.6

Conclusion

The performance of metaheuristic algorithms relies on parameter tuning. While several approaches have been proposed for parameter tuning, a comprehensive analysis of the parameters' influence on the algorithm's behavior is of paramount importance. The approach presented in this paper utilizes Response Surface Methodology (RSM) to systematically design experiments. The model created by RSM offers valuable insights through efficient measures and plots, enabling a deeper understanding of the algorithm's behavior.

This study focused on the Gravitational Search Algorithm (GSA), a valuable metaheuristic algorithm. The proposed approach is applied to 23 single and multimodal test functions, revealing the relative importance of each parameter and analyzing interactions between parameters.

Ultimately, this study provides valuable insights for researchers seeking to enhance the performance of not only the GSA algorithm but also other metaheuristic algorithms using the proposed method. The approach is adaptable and can be applied to fine-tune parameters in various optimization problems.

For future work, this approach can be extended for sensitivity analysis of GSA.

This paper has focused on single and multimodal GSA; however, future work could extend the analysis to include multi-objective versions as well. Additionally, the methodology can be adapted for a more in-depth understanding and fine-tuning of other metaheuristic algorithms.

Since heuristic algorithms share similarities in controlling exploration and exploitation of problem spaces, this method can be applied when a close resemblance exists between parameters. Additionally, designing experiments spanning aligned domains facilitates performance assessments under equivalent conditions. Thereby, equitable evaluations can be conducted for closely related algorithms. Furthermore, in other situations using normalized and standardized metrics maps measures to a common scale. This enables impartial contrasting through statistically analyzing unified data. Consequently, the methodology provides pathways for fair benchmarking even with inherent dissimilarities through mapping to a shared assessment platform.

Author Contributions

M. Amoozegar carried out: Implemented algorithm, Interpreted the results and wrote the manuscript.

S. Golestani: Data analysis, interpreted the results and wrote the manuscript.

Conflict of Interest

The authors declare no potential conflict of interest regarding the publication of this work. In addition, the ethical issues including plagiarism, informed consent, misconduct, data fabrication and, or falsification, double publication and, or submission, and redundancy have been completely witnessed by the authors.

Abbreviations

GSA	Gravitational Search Algorithm
RSM	Response Surface Methodology
DOE	Design of Experiments
GA	Genetic Algorithm
CCD	Central Composite Design
BBD	Box-Behnken Design

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