Journal of Electrical and Computer Engineering Innovations (JECEI)

Journal homepage: http://www.jecei.sru.ac.ir

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Research paper

Noise Folding Compensation in Compressed Sensing based Matched-Filter Receiver

M. Kalantari *

Faculty of Computer Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran.

Article Info

Article History:
Received 18 May 2024
Reviewed 15 July 2024
Revised 14 August 2024
Accepted 31 August 2024

Keywords:
Compressed sensing
Matched-filter
Noise folding

*Corresponding Author's Email Address: *mkalantari@sru.ac.ir*

Abstract

Background and Objectives: Compressed sensing (CS) of analog signals in shift-invariant spaces can be used to reduce the complexity of the matched-filter (MF) receiver, in which we can be approached the standard MF performance with fewer filters. But, with a small number of filters the performance degrades quite rapidly as a function of SNR. In fact, the CS matrix aliases all the noise components, therefore the noise increases in the compressed measurements. This effect is referred to as noise folding. In this paper, an approach for compensating the noise folding effect is proposed.
Methods: An approach for compensating of this effect is to use a sufficient number of filters. In this paper the aim is to reach the better performance with the same number of filter as in the previous work. This, can be approached using a weighting function embedded in the analog signal compressed sensing structure. In fact, using this weighting function we can remedy the effect of CS matrix on the noise variance.
Results: Comparing with the approach based on using the sufficient number of filters to counterbalance the noise increase, experimental results show that with the same numbers of filters, in terms of probability of correct detection, the proposed approach remarkably outperforms the rival's.
Conclusion: Noise folding formation is the main factor in CS-based matched-filter receiver. The method previously presented to reduce this effect demanded using the sufficient number of filters which comes at a cost. In this paper we propose a new method based on using the weighting function embedded in the analog signal compressed sensing structure to achieve better performance.

Introduction

In many emerging and important applications, the Nyquist sampling rate, a rate equals to twice the highest frequency, is so high that we encounter with far too many samples that must be processed and stored in high-capacity memory. Meanwhile, in applications in which inputs are wideband signals, it is so costly and mostly physically impossible to build analog to digital convertor capable of acquiring samples at Nyquist rate [1], [2]. So, in the past few years the vast interest in the area of compressed sensing (CS) caused the sampling theory has again been revived [3]-[5]. Compressed sensing is a framework for sensing and compression of finite-dimensional vectors simultaneously [3], [4], [6]. The main idea in CS is that, instead of sampling at a high rate and then compressing the samples, we want to have a way to measure the data in a compressed form directly [7]. For this, the finite-dimensional signal must have a sparse or compressible representation in a known basis [3]. In this way, we capture only the essential information embedded in a signal. Formerly, the CS was a mathematical theory for measuring finite-dimensional vector. In fact, CS was a framework for sampling of discrete-time signals and reconstruction from a finite number of samples. Despite the widespread literature in the area of extending the ideas of CS to analog domain, it remains a difficult challenge [8]-[11]. Eldar extended the CS to consider sub-Nyquist sampling of continuous-time signals in shift-invariant spaces via combining ideas from compressed sensing with analog sampling results [7]. A shift-invariant (SI) subspace is a space in which signals can be represented as a linear combination of shifts of a set of generators [12]-[16]. The subspace of bandlimited signals, multiband signals, the spline functions, the communication transmission such as PAM (pulse amplitude modulation) and QAM (quadratic amplitude modulation) are some important examples of SI subspace [17]-[24]. So, the compressed sensing of analog signals in SI spaces leads to low-rate (sub-Nyquist) sampling of a broad set of analog signals. This sub-Nyquist samples can be processed directly without having to upsample them back to the Nyquist rate, leading to low-rate processing as well.

The idea of analog CS can be used to standard detection problem, concerned in communication systems, for reducing the receiver complexity [7], [25], [26]. In fact, analog CS enables us to convey more information over the channel with the same receiver. It is a well-known result that the MF receiver which consists the same number of filters (correlators) as the number of transmitted signals, say $\boldsymbol{N}$, maximize the probability of correct detection. Nevertheless, it can be shown that, using the idea of analog CS and in a noise-free environment, regardless of the number of signals for transmission, only two filters is required to detect the transmitted signal exactly [7]. In the presence of noise, in order to achieve good performance, the number of correlators must be increased. In fact, when noise is present, for strictly maximizing the probability of correct detection we require $\boldsymbol{N}$ filters. However we can get very good performance with fewer correlators. But with a smaller number of filters the performance degrades quite rapidly as a function of SNR. In fact, the CS matrix aliases all the noise components, therefore the noise increases in the compressed measurement. This effect is referred to as noise folding [27]. An approach for compensating this effect is to use sufficient number of filters. It is shown that approximately $\boldsymbol{\log N}$ filters are needed to countervail this increase in noise [28].

In this paper the aim is to reach the better performance with the same number of filter as in the previous work in [28], [7]. This, can be approached using a weighting function embedded in the analog signal compressed sensing structure. In fact, using this weighting function we can remedy the effect of CS matrix on the noise variance.

This paper is organized as follows. The second section reviews the fundamentals of analog signal compressed sensing. The third section presents the proposed method for amending the noise folding effect. Comparisons with the rival method are presented in the fourth section, and the end section concludes the paper.

Compressed Sensing of Analog Signals and CS based Matched-filter Receiver

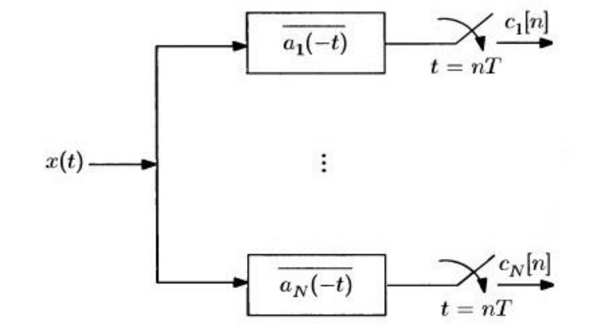
This section shortly explains the theory of analog signal compressed sensing [7], [29]. The formulation provided in this section will be utilized in the third section to develop the proposed method.

A. Sampling and Reconstruction in SI Spaces

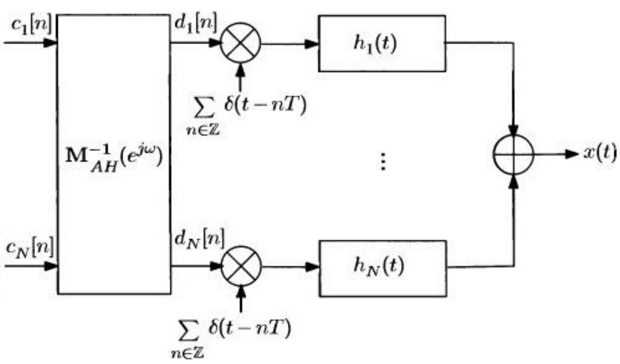
A mentioned in the previous section, SI signals are specified by a set of generators $\{h_\ell(t), 1 \le \ell \le N\}$, where N may be finite or infinite. So, any signal in SI space can be written as

$$x(t) = \sum_{\ell=1}^{N} \sum_{n\in\mathbb{Z}} d_\ell[n] h_\ell(t - nT) \tag{1}$$

where $d_\ell[n] \in \ell_2, 1 \le \ell \le N$, ℓ_2 is the space of discrete-time finite-energy signals and T is the period. It is well known that such signals can be recovered from sample at a rate of N/T. Sampling and reconstruction of signals in SI space have been depicted in Fig. 1.



(a)



(b)

Fig. 1: (a) Sampling, (b) Reconstruction in shift- invariant spaces [7].

In this sampling method, $x(t)$ is passed through a bank of N filters, each with almost arbitrary impulse response

of $\overline{a_\ell(-t)}$. Then, the outputs are sampled with period T unformly, resulting the sample sequnces $c_\ell[n]$. The vector containing the DTFTs of $c_\ell[n], 1 \le \ell \le N$, denoted by $\mathbf{c}(e^{j\omega})$, and the vector collecting the DTFTs of $d_\ell[n], 1 \le \ell \le N$, denoted by $\mathbf{d}(e^{j\omega})$.
It can be shown that

$$\mathbf{d}(e^{j\omega}) = \mathbf{M}_{AH}^{-1}(e^{j\omega})\mathbf{c}(e^{j\omega}) \tag{2}$$

where $\mathbf{M}_{AH}(e^{j\omega})$ is an $N \times N$ matrix, with entries

$$[\mathbf{M}_{AH}(e^{j\omega})]_{i\ell} = \frac{1}{T}\sum_{k\in\mathbb{Z}} \overline{A_i\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)} H_\ell\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right) \tag{3}$$

$A_i(\omega)$ and $H_\ell(\omega)$ are the CTFTs of $a_i(t)$ and $h_\ell(t)$ respectively, and $\mathbf{M}_{AH}^{-1}(e^{j\omega})$ is the inverse of $\mathbf{M}_{AH}(e^{j\omega})$. The reconstruction of $x(t)$ is accomplished via modulating each output sequence $d_\ell[n]$ by a periodic impulse train $\sum_{n\in\mathbb{Z}} \delta(t - nT)$ with period T, followed by filtering with analog filter $h_\ell(t)$. Similar to finite interpolation in the Shannon-Nyquist theorem, if $h_\ell(t)$ decay fast enough, interpolation with finitely many samples leads to sufficiently accurate reconstruction.

For signals that can be represented by k generator, $k < N$, chosen from a finite set of N functions, we have

$$x(t) = \sum_{|\ell|=k}\sum_{n\in\mathbb{Z}} d_\ell[n]h_\ell(t - nT) \tag{4}$$

If we know the k active generators then we can uniformly sample the output of k appropriate filters with sampling period of T as in Fig. 1, resulting a sampling rate of k/T. On the other hand, if we know that only k out of N generators are active but don't know in advance which one, then the minimal sampling rate is at least $2k/T$. Thus the lack of knowledge about the exact subspace to which $x(t)$ belongs, leads to an increase of at least a factor 2 in the minimal sampling rate [7]. By combining ideas from analog sampling and CS, this minimal rate has been achieved [29].

B. Compressed Sensing in Sparse Unions

Suppose that $\mathbf{d}[n]$ is a vector whose ℓth component is given by $d_\ell[n]$, in which only k out of the N sequences $d_\ell[n]$ are nonzero. Compressively measuring the vector sequence $\mathbf{d}[n]$ can be accomplished by a $p \times N$ sensing matrix $\mathbf{A}$, $p < N$, that allows recovery of k-sparse vectors. The choice $p < N$, reduces the sampling rate below the Nyquist rate. In fact, a compressive sampling system produces a vector of low-rate samples $\mathbf{y}[n] = \left[y_1[n], \dots, y_p[n]\right]^T$ satisfying the relation

$$\mathbf{y}[n] = \mathbf{A}\mathbf{d}[n], \quad \|\mathbf{d}[n]\|_0 \le k \tag{5}$$

where $\|\mathbf{d}[n]\|_0$ is the number of nonzero elements of $\mathbf{d}[n]$. For each n,

$$\mathbf{y}[n] = \mathbf{A}\mathbf{d}[n], \quad n \in \mathbb{Z} \tag{6}$$

Equation (6) is an infinite measurement vector (IMV) problem and for each n, $\mathbf{d}[n]$ is k-spares [7], [29]. Meanwhile, the infinite set of vectors $\{\mathbf{d}[n], n \in \mathbb{Z}\}$ shares a joint sparsity pattern: at most k of the sequences $d_\ell[n]$ are nonzero. These set of equations can be transformed into an equivalent multiple measurement vector (MMV) problem using the continuous to finite (CTF) block technique [7]. The perfect recovery of $\mathbf{d}[n]$ (or recovery with high probability) is guaranteed because $\mathbf{A}$ was designed to enable CS techniques.

The frequency-domain counterpart of (6) is as follows,

$$\mathbf{y}(e^{j\omega}) = \mathbf{A}\mathbf{d}(e^{j\omega}), \quad 0 \le \omega \le 2\pi \tag{7}$$

where $\mathbf{y}(e^{j\omega})$, $\mathbf{d}(e^{j\omega})$ are the vectors containing the DTFTs $Y_\ell(e^{j\omega}), D_\ell(e^{j\omega})$ respectively. $\mathbf{d}[n]$ may also be recovered from

$$\mathbf{y}(e^{j\omega}) = \mathbf{W}(e^{j\omega})\mathbf{A}\mathbf{d}(e^{j\omega}), \quad 0 \le \omega \le 2\pi \tag{8}$$

where $\mathbf{W}(e^{j\omega})$ is any invertible $p \times p$ matrix with elements $W_{i\ell}(e^{j\omega})$. In this way we can generalize the class of sensing operators. This extra freedom can be used in the proposed method for noise folding compensation as we will see in the following sections. The analog compressed sampling can be seen in Fig. 2.

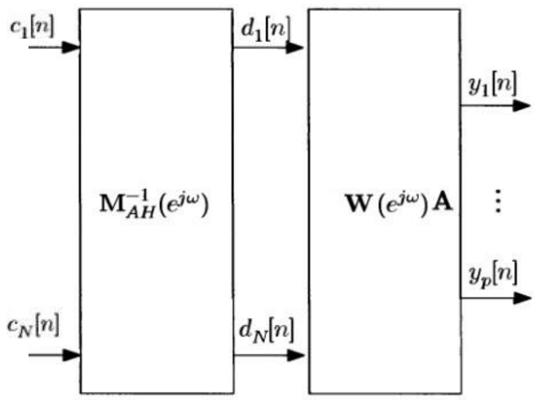


Fig. 2: Analog compressed sampling [7].

The inputs in Fig. 2 come from Fig. 1(a). Though the sampling method of Fig. 2 leads to compressed measurements $\{y_\ell[n]\}$, the sampling rate is still N/T. Eldar reduced this rate to p/T where $2k \le p < N$ via proving the following theorem [29],

Theorem 1. $\{y_\ell[n]\}, 1 \le \ell \le p$, in Fig. 2 can be obtained by filtering $x(t)$ in (4) with p filters $\overline{\{s_\ell(-t)\}}$ and sampling the outputs at rate $1/T$, where

$$\mathbf{s}(\omega) = \overline{\mathbf{W}(e^{j\omega T})\mathbf{A}}\mathbf{v}(\omega) = \overline{\mathbf{W}(e^{j\omega T})\mathbf{A}\mathbf{M}_{AH}^{-1}(e^{j\omega T})}\mathbf{a}(\omega) \tag{9}$$

where $\mathbf{s}(\omega), \mathbf{a}(\omega)$ are the vectors with ℓth elements $S_\ell(\omega), A_\ell(\omega)$ respectively, and $V_\ell(\omega)$, the components of $\mathbf{v}(\omega)$, are Fourier transform of generators $v_\ell(t)$ such that

$\{v_\ell(t-nT)\}$ are biorthogonal to $\{h_\ell(t-nT)\}$. In the time domain we have,

$$s_i(t) = \sum_{\ell=1}^{N} \sum_{r=1}^{p} \sum_{n\in\mathbb{Z}} \overline{w_{\iota r}[-n]\mathbf{A}_{r\ell}}\, v_\ell(t-nT) \tag{10}$$

in which $w_{ir}[n]$ is the inverse DTFT of $W_{ir}(e^{j\omega})$, the elements of matrix $\mathbf{W}(e^{j\omega})$, and

$$v_i(t) = \sum_{\ell=1}^{N} \sum_{n\in\mathbb{Z}} \overline{\varphi_{\iota\ell}[-n]}\, a_\ell(t-nT) \tag{11}$$

here $\varphi_{i\ell}[n]$ is the inverse DTFT of $[\mathbf{M}_{AH}^{-1}(e^{j\omega T})]_{i\ell}$. When $\mathbf{W}(e^{j\omega}) = \mathbf{I}$,

$$s_i(t) = \sum_{\ell=1}^{N} \overline{\mathbf{A}_{\iota\ell}}\, v_\ell(t). \tag{12}$$

Fig. 3 shows the compressed sensing of analog signals with sampling rate of p/T.

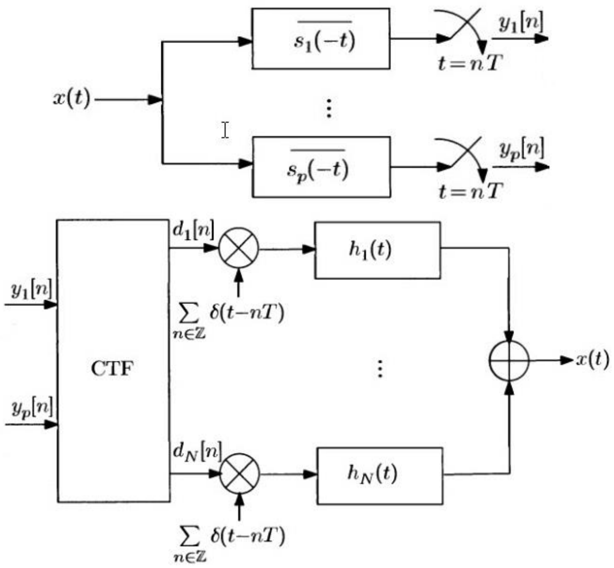


Fig. 3: Compressed sensing of analog signal [7].

C. Compressed-Sensing Based Matched- Filter Receiver

Suppose that a basic communication system transmits digital data to a receiver by sending one of a set of N linearly independent known signals $\{h_i(t), 1 \le i \le N\}$ over a symbol duration of T. The channel add a zero-mean white Gaussian noise $n(t)$ with variance σ^2 to the signal, so the received signal is as

$$y(t) = h_\ell(t) + n(t) \tag{13}$$

for some index ℓ. The goal is to determine the index ℓ in order to decode the transmitted symbol. Demodulator for a typical matched-filter receiver is shown in Fig. 4 in which $\overline{s_\ell(-t)} = \overline{h_\ell(-t)}$.

It is well known that a MF receiver is a sufficient statistic for detection; that is, the optimal detector can be computed based on the MF output providing the noise is Gaussian. The maximum-likelihood detector for MF receiver is as

$$\ell = \arg\max_i \mathcal{R}\{y_i\} \tag{14}$$

where $\mathcal{R}\{y_i\}$ is the real part of y_i.

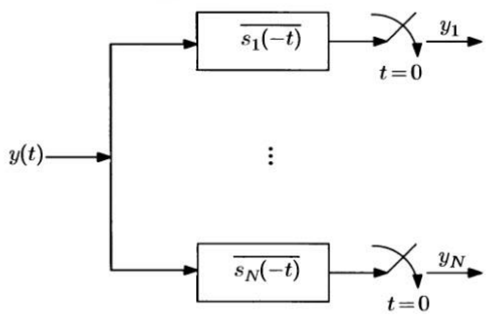


Fig. 4: Demodulator for a matched-filter receiver [7].

By exploiting the ideas of analog CS, we can reduce the number of filters in demodulator part of the MF receiver. Reformulation of the detection problem as a CS recovery problem is as follows. Any signal $h_\ell(t)$ can be written in the form $h_\ell(t) = H\mathbf{x}$, where $H: \mathbb{R}^N \to L_2$ is the set transformation corresponding to $\{h_i(t)\}$ and $\mathbf{x}$ is a vector containing a 1 in the ℓth position. Thus, in term of the basis defined by the transformation H, the transmitted signal is sparse. This scenario is a special case of (4) in which $k = 1$, where we consider only one symbol interval. So, we can recover $\mathbf{x}$ using $p < N$ filters chosen according to Theorem 1. For that, let $\mathbf{A}$ be an arbitrary $p \times N$ CS matrix for a 1-spares vector. Then, according to Theorem 1 the demodulator consists of filters $\{\overline{s_\ell(-t)}, 1 \le \ell \le p\}$,

$$s_\ell(t) = \sum_{m=1}^{N} \overline{\mathbf{A}_{\ell m}}\, v_m(t) \tag{15}$$

where $\{v_m(t)\}$ are the biorthogonal functions defined as

$$v_m(t) = \sum_{i=1}^{N} \emptyset_{mi} h_i(t) \tag{16}$$

with $\mathbf{\Phi} = (H^*H)^{-1}$. In operator notation, $S = V\mathbf{A}^* = H(H^*H)^{-1}\mathbf{A}^*$ and $V^*H = I$ where S, V are set transformation corresponding to $\{s_i(t)\}$ and $\{v_i(t)\}$ respectively.

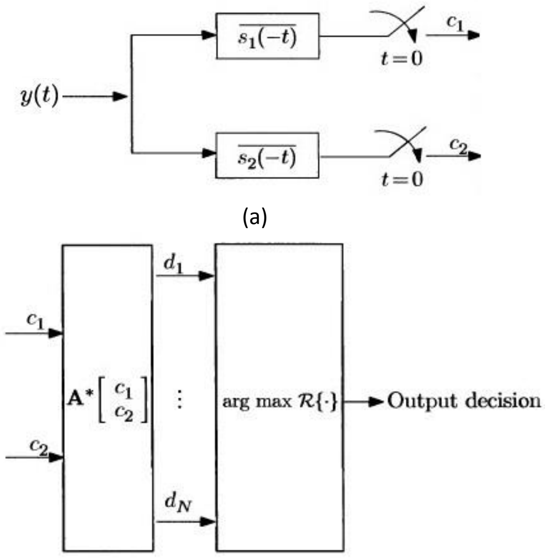
Suppose a noise-free case in which $y(t) = h_\ell(t) = H\mathbf{x}$ for some index ℓ. After applying the p filters on $y(t)$, the output vector is as follows

$$\mathbf{c} = S^* y(t) = \mathbf{A}(H^*H)^{-1}H^* y(t) = \mathbf{A}\mathbf{x} \tag{17}$$

So, the problem reduces to recovery of a 1-sparse vector $\mathbf{x}$ from compressed measurement $\mathbf{c}$. From the theory of uniqueness sparse recovery, we know that for recovery of a k-sparse vector the spark of the sensing matrix must be greater than $2k$. In particular, for uniqueness we must have that $p \geq 2k$. So, $\mathbf{A}$ may be a $2 \times N$ matrix in which no two columns are multiple of each other. The support of $\mathbf{x}$ can be recovered by choosing

$$\ell = \arg\max_{i} \mathcal{R}\{\langle \boldsymbol{a}_i, \mathbf{c} \rangle\} \tag{18}$$

where $\boldsymbol{a}_i$ is the ith column of $\mathbf{A}$. So, only two correlators are required to detect the transmitted signal exactly. The overall receiver is depicted in Fig. 5.



(a)

(b)

Fig. 5: A noise-free detector based on analog compressed sensing [7].

But, in noisy environment the number of correlator must be increased in order to achieve good performance. In fact we can get very good performance with $2 < p < N$ correlators. It can be shown that the selection rule is identical to the one in the noise-free case as expressed in (14).

The Proposed Method

In noisy environment, strictly maximizing the probability of correct detection will require N correlators, but as mentioned before, by using $p < N$ filters, we can get pretty good performance. With a small number of filters the performance degrades quite rapidly as a function of SNR. In fact, the CS matrix aliases all the noise components, therefore the noise increases in the compressed measurements. This effect is referred to as noise folding which is the main problem in this type of receiver. The Noise folding compensation can be accomplished by employing a sufficient number of filters. It is shown that approximately $\log N$ filters are needed to countervail this increase in noise [28]. In this paper we propose a new method based on using the weighting function embedded in the analog signal compressed sensing structure, $\mathbf{W}(e^{j\omega})$, to achieve better performance. According to Theorem 1 and in operator notation we have

$$S = V\mathbf{D}^* \tag{19}$$

where S, V are set transformation corresponding to $\{s_i(t)\}$ and $\{v_i(t)\}$ respectively and $\mathbf{D}$ is a $p \times N$ matrix with elements

$$d_{i\ell} = \text{IDFT}\left(B_{i\ell}(e^{j\omega})\right) \tag{20}$$

in which $B_{i\ell}(e^{j\omega})$ are the elements of the matrix

$$\mathbf{B}(e^{j\omega}) = \overline{\mathbf{W}(e^{j\omega})\mathbf{A}} \tag{21}$$

From (9), we have that

$$\mathbf{s}(\omega) = \mathbf{B}(e^{j\omega T})\mathbf{v}(\omega) \tag{22}$$

In the time domain and in terms of $d_{i\ell}$ we can write

$$s_i(t) = \sum_{\ell=1}^{N} \sum_{n \in \mathbb{Z}} d_{i\ell}[n]\, v_\ell(t - nT) \tag{23}$$

So, we have $S = V\mathbf{D}^*$. Using this and the fact that $V^*H = I$ we have

$$\begin{aligned} \mathbf{c} = S^*y(t) &= \mathbf{D}V^*h_\ell(t) + \mathbf{D}V^*n(t) = \\ &\mathbf{D}V^*H\mathbf{x} + \mathbf{D}V^*n(t) = \mathbf{D}\mathbf{x} + \mathbf{w} \end{aligned} \tag{24}$$

where $\mathbf{w} = \mathbf{D}V^*n(t)$ is the noise component. We can write

$$\begin{aligned} &\mathrm{E}\left[\langle v_j(t), n(t)\rangle \langle v_i(t), n(t)\rangle\right] = \\ &\iint v_j(t)v_i(\tau)E[n(t)n(\tau)] = \sigma^2 \langle v_j(t), v_i(t)\rangle \end{aligned} \tag{25}$$

Therefore,

$$\begin{aligned} \mathbf{R}_w = E[\mathbf{w}\mathbf{w}^*] &= \sigma^2 \mathbf{D}V^*V\mathbf{D}^* \\ &= \sigma^2 \mathbf{D}(H^*H)^{-1}\mathbf{D}^* \end{aligned} \tag{26}$$

In general, the noise is not white, but if the signals $h_\ell(t)$ are orthonormal and the rows of $\mathbf{D}$ are orthogonal then $\mathbf{R}_w = \kappa\sigma^2\mathbf{I}$ where κ is the squared-norm of rows of $\mathbf{D}$.

The problem is reduced to recovery of 1-spares vector $\mathbf{x}$ from noisy measurements $= \mathbf{D}\mathbf{x} + \mathbf{w}$. The standard CS algorithms can be used for solving this problem provided the $B_{i\ell}(e^{j\omega})$ are constant functions or equivalently, $d_{i\ell} = d_{i\ell}[n]\delta[n]$. Otherwise, we should develop a MAP detector [26], [30]. Assume that the $P(\ell|\mathbf{c})$ be the probability that x_ℓ, the ℓ^{th} elements of $\mathbf{x}$, is nonzero, given $\mathbf{c}$. The goal is to choose the ℓ that maximize this probability. Using the Bayes rule and the fact that $P(\ell) = 1/N$, the problem will be the maximizing $P(\mathbf{c}|\ell)$. The vector $\mathbf{c}$ is a Gaussian vector with mean $\mathbf{D}^*\mathbf{x}$ and covariance $\mathbf{R}_w = \sigma^2\mathbf{D}(H^*H)^{-1}\mathbf{D}^*$. We have

$$\begin{aligned} &\ln P(\mathbf{c}|\ell) = \\ &\quad -\Upsilon(\mathbf{c} - \mathbf{d}_\ell)^*(\mathbf{D}(H^*H)^{-1}\mathbf{D}^*)^{-1}(\mathbf{c} - \mathbf{d}_\ell) \end{aligned} \tag{27}$$

where Υ is a constant and $\mathbf{d}_\ell$ is the ℓ^{th} column of $\mathbf{D}$. Maximizing $\ln P(\mathbf{c}|\ell)$ is equivalent to minimizing the following function

$$\Lambda(\ell) = (\mathbf{c} - \mathbf{d}_\ell)^*(\mathbf{D}(H^*H)^{-1}\mathbf{D}^*)^{-1}(\mathbf{c} - \mathbf{d}_\ell) \quad (28)$$

In the special case in which $\mathbf{R}_w = \kappa\sigma^2\mathbf{I}$, the minimization of $\Lambda(\ell)$ leads to the following selection rule

$$\ell = \arg\max_i \mathcal{R}\{\langle \boldsymbol{d}_i, \mathbf{c}\rangle\} \quad (29)$$

which is similar to the noise free case.

The covariance of the noise in the proposed method is similar to the one in the rival method [7], [28]. In fact, the covariance in the rival method is as

$$\mathbf{R}_w = E[\mathbf{w}\mathbf{w}^*] = \sigma^2\mathbf{A}V^*V\mathbf{A}^* = \sigma^2\mathbf{A}(H^*H)^{-1}\mathbf{A}^* \quad (30)$$

So, the only change is the substitution of matrix $\mathbf{A}$ in the previous method with the matrix $\mathbf{D}$. But, note that this substitution has a great impact on the variance of the noise in measurements $\mathbf{c}$. Unlike the matrix $\mathbf{A}$, the elements of matrix $\mathbf{D}$ are sequences, not scalars.

According to (23) the weighting function $\mathbf{W}(e^{j\omega})$ introduces extra freedom when designing the corresponding analog sampling filters. Meanwhile, according to (20), (21), (26), $\mathbf{W}(e^{j\omega})$ has great impact on the variance of the noise. We show this impact with a simple example.

Suppose that the signals $\{h_i(t)\}$ are orthonormal, so we have $H^*H = I$ and from (30), $\mathbf{R}_w = \sigma^2\mathbf{A}\mathbf{A}^*$. Also, suppose that $\mathbf{A}$ is chosen as random rows of a Fourier matrix. This means that $A_{i\ell} = (1/\sqrt{p})\exp\{-j2\pi s_i\ell/N\}$ where s_i is the ith row chosen. In this case, $\mathbf{A}\mathbf{A}^* = (N/p)\mathbf{I}$. So, the noise variance is increased by a factor of N/p. In this example we see simply that choosing $\mathbf{W}(e^{j\omega}) = (p/N)\mathbf{I}$, results in $\mathbf{R}_w = \sigma^2$ according to (26).

But, the cases are not as simple as the previous example, and choosing the appropriate $\mathbf{W}(e^{j\omega})$ is a difficult problem. But, as we can see in the next section, we can simply bypass this problem.

The Experimental Results

In this section we will examine the ability of the proposed method in compensation of the noise folding problem. For that, we demonstrate the effect of additive noise on the proposed method and the rival method as expressed by (18) [28], [7]. There are no comparable works since 2015.

We consider a receiver with $N = 100$ different transmitted signals given by

$$h_\ell(t) = \begin{cases} 1, & (\ell-1) \le t \le \ell \\ 0, & otherwise \end{cases} \qquad \ell = 1,2,\ldots,N \quad (31)$$

and p correlators. For the rival method, the sensing matrix $\mathbf{A}$ is chosen to be equal to p random rows of the $N \times N$ Fourier matrix in which the columns normalized to have unit norm. For the proposed method we need a matrix

$$\mathbf{D} = \mathrm{IDFT}\left(\overline{\mathbf{W}(e^{j\omega})\mathbf{A}}\right) \quad (32)$$

with orthogonal rows. We do not involve ourselves with the problem of selecting appropriate $\mathbf{W}(e^{j\omega})$ and $\mathbf{A}$ matrix. Instead, we simply generate synthetically a $p \times N$ matrix as the matrix $\mathbf{D}$. As mentioned before, the main difference between matrix $\mathbf{A}$ and matrix $\mathbf{D}$ is that in the former the elements of the matrix are scalars and in the latter are sequences.

We saw that when noise is present, for strictly maximizing the probability of correct detection we require $N = 100$ coorelators. Although with fewer filters we can get very good performance, the performance degrades quite rapidly as a function of SNR. This is due to noise folding phenomenon. In fact, the use of the CS matrix reduces the SNR, i.e. the CS matrix aliases all the noise component in the compressed measurement, even those corresponding to zero element in sparse vector $\mathbf{x}$, leading to a noise increase in the compressed measurements.

Fig. 6 to Fig.12 show the probability of correct detection in both methods as a function of the number of correlators for different value of the SNR. The probability is estimated by running 1000 Monte Carlo simulations. In each iteration, the transmitted signal and the noise are chosen randomly. As we can see, as long as the SNR is high enough, perfect detection is achieved in both methods using a much smaller filters compared to the MF receiver consists of 100 correlators and the proposed method needs smaller filters compared to the rival method to achieve the same performance. This in turn shows that the proposed method can remedy the noise folding problem with fewer filters than the rival method.

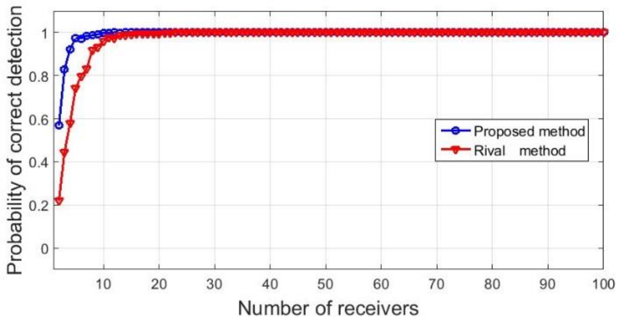


Fig. 6: Probability of correct detection as a function of the number of correlators for SNR=40.

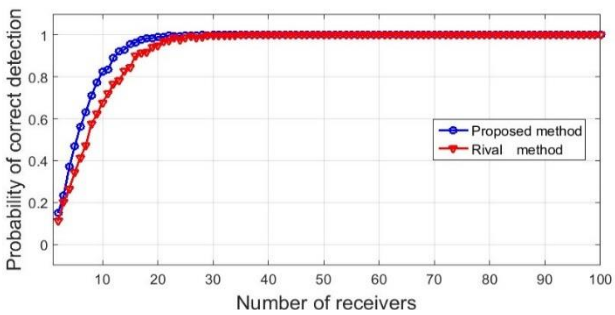


Fig. 7: Probability of correct detection as a function of the number of correlators for SNR=30.

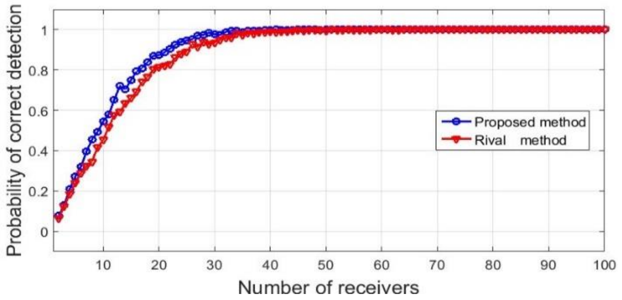


Fig. 8: Probability of correct detection as a function of the number of correlators for SNR=25.

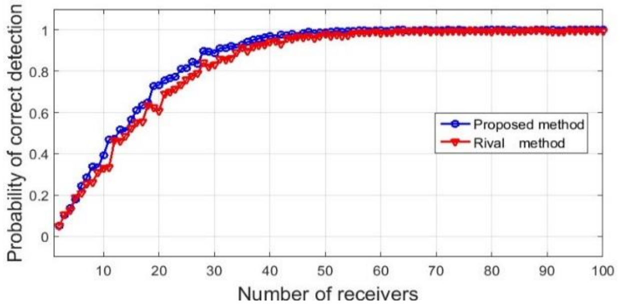


Fig. 9: Probability of correct detection as a function of the number of correlators for SNR=20.

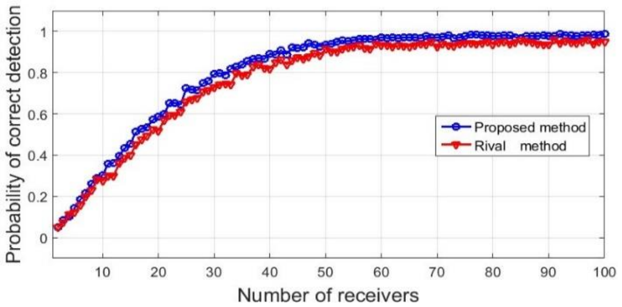


Fig. 10: Probability of correct detection as a function of the number of correlators for SNR=15.

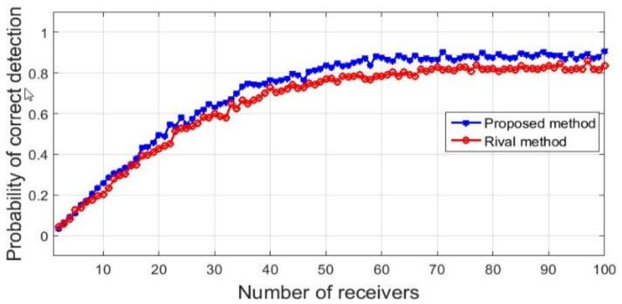


Fig. 11: Probability of correct detection as a function of the number of correlators for SNR=10.

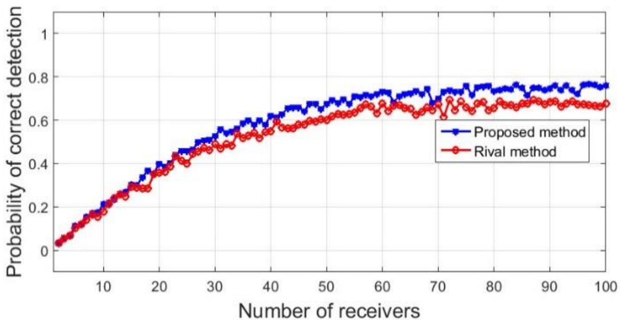


Fig. 12: Probability of correct detection as a function of the number of correlators for SNR=5.

Results and Discussion

The simulation results show that the proposed method has the ability to compensate the noise folding problem more effectively than the rival method. i.e. with fewer correlators. This is due to the fact that the elements of matrix $\mathbf{D}$ are sequences rather than scalars.

Conclusion

Noise folding is the main problem in compressed sensing based MF receiver. An approach for compensating this effect is to use sufficient number of coorelators. The proposed method achieves better performance with the same number of filters as in the previous work. This goal is achieved through the use of weighting function embedded in the analog signal compressed sensing structure. This weighting function can remedy the effect of CS matrix on the noise variance.

As stated in the previous sections, choosing the appropriate weighting functions is a difficult problem. In this paper we bypass this problem via generating synthetically a $p \times N$ matix as the matrix $\mathbf{D}$ with orthogonal rows. In this way, there is no notable difference between the proposed method and the rival's method from the point of time and space complexity. Systematically computing of matrix $\mathbf{D}$ can be suggested for future work in which the time and space complexity is a major concern because of existence of long sequences as elements of the matrix $\mathbf{D}$.

Author Contributions

M. Kalantari has written the whole paper without participation of anybody. All parts of this work have been accomplished by the author as the single author and the corresponding author of the paper.

Acknowledgments

This work was supported by Shahid Rajaee Teacher Training University under contract number 11980.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this manuscript.

Abbreviations

CS	Compressed sensing
MF	Matched filter
SNR	Signal to noise ratio
PAM	Pulse amplitude modulation
QAM	Quadratic amplitude modulation
SI	Shift invariant
DTFT	Discrete time Fourier transform
MMV	Multiple Measurement vector
CTF	Continuous to finite block
$\mathbf{A}$	Sensing matrix
$\mathcal{R}\{.\}$	Real part of argument
$\mathbf{W}(e^{j\omega})$	Weighting matrix

$n(t)$	Noise signal
$\mathbf{R}_w$	Noise covariance matrix
MAP	Maximum a posteriori
$\mathbf{c}$	Noisy measurement vector
$\mathbf{a}_i$	ith column of $\mathbf{A}$
$x(t)$	A SI signal
$\overline{x(t)}$	Complex conjugate of $x(t)$
$h_\ell(t)$	SI generator or a known transmitted signal
$y(t)$	Received signal
$\mathbf{x}$	A sparse vector
IDFT	Inverse discrete Fourier transform

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Biography

Mohammad Kalantari received B.Sc. degree in Computer Engineering from Iran University of Science and Technology (IUST), Tehran, Iran and M.Sc. and Ph.D. in Computer Engineering from Amirkabir University of Technology (AUT), Tehran, Iran in 2001 and 2009 respectively. He is currently working as Assistant Professor at Signal Processing Laboratory in Computer Engineering Department at Shahid Rajaee Teacher Training University (SRTTU), Tehran, Iran. His area of interest includes, statistical signal processing, spherical array processing, sampling theory and compressed sensing.

- Email: mkalantari@sru.ac.ir
- ORCID: 0000-0002-6852-9344
- Web of Science Researcher ID: HZJ-9229-2023
- Scopus Author ID: 55893680300
- Homepage: https://www.sru.ac.ir/kalantari/

How to cite this paper:
M. Kalantari, "Noise folding compensation in compressed sensing based matched-filter receiver," J. Electr. Comput. Eng. Innovations, 13(1): 57-64, 2025.

DOI: 10.22061/jecei.2024.10933.749

URL: https://jecei.sru.ac.ir/article_2197.html