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Analysis and Comparison of PAPR Reduction Techniques in OFDM Systems

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ABSTRACT

The destructive impact of fading environments and also bandwidth limitations are two main challenges which communication is dealing with them. These challenges can affect on the growth of wireless communication and even cause reliable communications and high data rate to be prevented. Thus, OFDM (Orthogonal Frequency Division Multiplexing) modulation by using of fast calculation hardwares such as FFT, high ability for combating multipath fading and appropriate spectral efficiency has taken into consideration. However, we should know that OFDM systems potentially have high Peak to Average Power Ratio (PAPR). This drawback drives the power amplifier into saturation leading to higher distortions and also degrades BER performance. Since increasing the dynamic range of power amplifier is not affordable, reduction of the PAPR is so important. In this paper, we investigate the PAPR and its reduction methods by using the theoretical and numerical analysis. These techniques can be classified into two main categories, signal distortion techniques, multiple signaling and probabilistic techniques. The advantages and disadvantages of each technique are derived from different prospectives. Moreover, we compare the numerical results of the techniques in the first classification from BER prospective which demonstrates that for changing the parameters corresponding to each technique, its performance can be changed greatly. Hence, we are sure that a technique can not outperform the other ones in all cases. Finally, the computational complexity of the techniques in the second classification are compared to each other which their results show that TR and TI techniques are much more complex than the other ones.

1. Introduction

Because of the increasing growth of data communication networks and also the requirement for transmitting high-rate information, many progressive technologies have been widely introduced. However, wireless communications has faced challenges; therefore, the tendencies to the techniques that effectively are able to solve these challenges and provide a simple way for wireless communication, are growing fast. For instance, OFDM (Orthogonal Frequency Division Multiplexing) can be used to resolve these issues effectively [1]-[2]. By breaking wideband channel into several narrow-band sub-channels and transferring parallel information, the OFDM technique obtains an appropriate performance in frequency selective fading channels.

The main shortcoming of multi-carrier systems such as OFDM is their highpeaks in the time domain signal. Since an OFDM signal consists of several independently modulated subcarriers, when the subcarrier signals are combined, the resulted signal has high peaks in its amplitude. These peaks in OFDM signal result in saturating the amplifiers and the following RF stages. Consequently, the intermodulation distortion due to the saturated stages is the reason for increasing the error probability at the receiver. In order to avoid the amplifier being saturated, high dynamic range amplifiers may be used which costs very expensive. On the other hand, these peaks make digital to analog converters (D/A) and analog to digital converters (A/D) more complicated [3]. The high peaks in an OFDM signal are mostly evaluated by peak-to-average-power-ratio (PAPR). Reduction of PAPR is concerned highly in this work. Techniques for reduction of PAPR have been categorized into three main methods. In the following, two of them- distortion based methods and multiple signaling and probabilistic methods- will be discussed.

2. PAPR

An OFDM signal in an interval symbol ($m\,T_u \leq t < (m+1)T_u$) can be viewed as [4]

$$x(t) = \sum_{k=0}^{N_c-1} a_k^m e^{j2k\pi\Delta f t} \qquad (1)$$

where, N_c is the number of subcarriers, a_k^m is the modulated signal to kth subcarrier, Δf is the frequency spacing between adjacent subcarriers and T_u represents the interval of one OFDM symbol. It is noteworthy that (1) offers m^{th} OFDM symbol definition. If the number of subcarriers is large enough, based on central limit theorem, the resulting signal $x(t)$ will be approximated as a complex Gaussian process. Therefore, the real and imaginary parts of an OFDM signal are Gaussian distributed and its envelope and power follows Rayleigh and exponential distributions, respectively [5]. Besides, the PAPR for continuous time signal ($x(t)$) is termed as maximum power to its average ratio. For mth symbol of OFDM, we have [6]

$$PAPR = \frac{\max\limits_{mT_u \leq t \leq (m+1)T_u} |x(t)|^2}{\frac{1}{T_u}\int_{mT_u}^{(m+1)T_u} |x(t)|^2\,dx} \qquad (2)$$

Since IFFT is used to generate an OFDM signal, we are dealing with discrete time samples of the OFDM. As addressed in [7], the maximum values of continuous time signal are not well aligned with the maximum values of the sampled signal. In other words, the PAPR for discrete time OFDM is different from the PAPR for continuous time OFDM. To overcome this problem, oversampling by a factor of greater than one is used. It is found that the PAPR of the oversampled discrete-time signal offers an accurate approximation of the PAPR of the continuous-time OFDM signal, if the oversampling factor is at least 4 [8]. With the above given information, the PAPR of the discrete-time OFDM signal is expressed as [6]

$$PAPR = \frac{\max\limits_{0 \leq n \leq N-1} |x(n)|^2}{E\{|x(n)|^2\}}\,, \qquad N = LN_c \qquad (3)$$

where, N_c is the number of OFDM subcarriers and L is oversampling factor.

A. PAPR Distribution Function

Cumulative distribution function (CDF) is one of the most useful methods for evaluating PAPR reduction techniques. In the literature, complementary CDF (CCDF) is usually replaced with CDF. CCDF of an OFDM signal is defined as the probability of OFDM signal envelope being greater than a threshold value [5]:

$$CCDF\big[PAPR\big(x^n(t)\big)\big] = prob\big[PAPR\big(x^n(t)\big) > \delta\big] \quad (4)$$

where, $PAPR\big(x^n(t)\big)$ is the PAPR of nth OFDM symbol and δ is the same as $PAPR$ threshold. As mentioned earlier, based on the central limit theorem, the OFDM signal power has exponential distribution with CDF as [6]

$$CDF(\delta) = \left(1 - e^{-\delta}\right) \qquad (5)$$

The probability that the PAPR of an OFDM signal with N samples to be smaller than a threshold value is equal to the probability that all N samples of an OFDM signal are below at hreshold value. Therefore, with the assumption of OFDM samples are mutually independent, probability function is calculated as [5]

$$CDF\big[PAPR\big(x^n(t)\big)\big] = \left(1 - e^{-\delta}\right)^N \qquad (6)$$

This probability function is an approximation for distribution of sampled OFDM signal PAPR that is valid for the peak OFDM samples but not necessarily for the maximum peak of the continuous OFDM signal. Oversampling with adding zero samples to the original sampled OFDM signal is a solution for this problem. However, for the oversampled OFDM, the assumption that OFDM samples are mutually independent is no longer valid. In an empirical approximation the distribution of the oversampled OFDM signal of N subcarriers is approximated by the distribution of OFDM signal of αN subcarriers without oversamplingas follows $\mathrm{CDF}\big[\mathrm{PAPR}\big(\mathrm{x^n(t)}\big)\big] =$

$(1-e^{-\delta})^{\alpha N}$ [5], where α is a parameter determined by computer simulation. Fig. 1 shows CCDF approximation for α=1 and PAPR for five different values of N(64,128,256,512,1024). As can be observed from the figure, as N increases, CCDF and hence PAPR increases. Therefore, at high transmission rate of OFDM systems which forces large N, PAPR matters most.

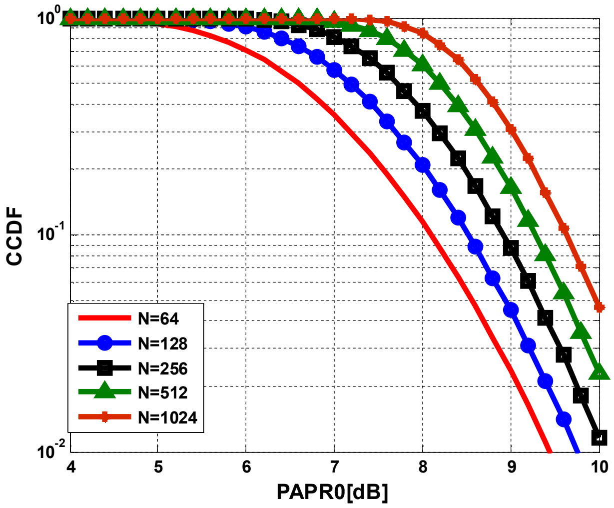

Figure 1: CCDF for α=1 and different values of N [5]

3. Distortion Based Methods

A. Clipping and Filtering

One of the simplest ways to reduce the PAPR is to clip the high peaks of the OFDM signal, prior to passing it through an amplifier. Employing a clipper, limits the signal envelope to a pre-determined value (called clipper level), if the signal exceeds that value or level [4]. The clipper function is given as [5]

$$T(x(n)) = \begin{cases} x(n) & |x(n)| \le CL \\ CLe^{j\measuredangle x(n)} & |x(n)| > CL \end{cases} \tag{7}$$

where x(n) is the OFDM signal, CL is the clipping level, and ∡x(n) is the signal phase.

Clipping is a non-linear function, and the resulting noise will cause in-band and out-of-band distortions. Out-of-band distortion results in spectral spreading of the signal which can be compensated by filtering. While the in-band distortion which degrades BER performance, cannot be compensated by filtering. Although, by increasing sampling rate and utilizing longer IFFT in which the in-band distortion effect is reshaped outside of the signal band that can be eliminated later by filtering [5].

B. Peak Windowing

In this method, high peaks are corrected multiplying by weighted function. The weighted function is called window function. Many window functions have been introduced for reducing high peaks of OFDM signal. Among them, Hamming, Kaiser, Hanning, and Cosine are the most well-known ones. Window function should be designed in a way, so that it is aligned with the signal samples. Window function valley is multiplied by the signal peaks, and higher amplitudes of the window function are multiplied by lower amplitudes of the signal [5]. The width of window in time domain must also be as low as possible, so that, it cannot affect on much samples of the signal. Since, in this method, the function is aligned with the samples, it has lower distortion and it is called soft clipping.

C. Companding Transforms

Companding transforms are mostly used to optimize the number of bits in every sample of speech signal. The OFDM and speech signal, in terms of high peaks occurring, behave the same; i.e. high peaks rarely occur in both signals. Therefore, one can utilize companding transform to reduce PAPR of an OFDM signal [9]-[10]. These transforms compress higher amplitudes and amplify lower amplitudes of the input signal [5]. This method, has lower complexity, is independent from the number of subcarriers. Side information (Information that should be transmitted to the receiver to allow the recovery of original symbol sequence at the receiver) is not needed; thus, this method will not cause BER degradation. Companding transforms are generally classified into four types: linear symmetrical transform (LST), linear asymmetrical transform (LAST), nonlinear symmetrical transform (NLST), and nonlinear asymmetrical transform (NLAST). In this research, LST and LAST are considered for simulation and comparitive study. Hence, details of these two methods are introduced. The LST companding transform, CLST, is given by [5]

$$C_{LST}(x(n)) = ax(n) + b \tag{8}$$

The LAST companding transform, CLAST, is given by [5]

$$C_{LAST}(x(n)) = \begin{cases} \frac{1}{u}x(n) & |x(n)| \le v \\ ux(n) & |x(n)| > v \end{cases} \tag{9}$$

where $0 < v <$ max|x(n)| is the threshold value and u is the piecewise slope parameter.

D. Peak Cancellation

What we wish to perform in all the distortion based methods, is to reduce the amplitude of the signal samples that exceed a predefined threshold. A reference shifted and scaled function is used in this method. The reference function is subtracted from the original signal in such a way that every reference function reduces high peak of at least one signal sample. Selecting appropriate reference function with the same bandwidth with original signal, PAPR reduction is not accompanied with out-of-band radiation [3].

Sinc function can be selected as an appropriate reference [3]. It is not applicable, because Sinc function is non-casual. In practice, Sinc function which is multiplied by an appropriate window function such as raised cosine is used [3].

4. Multiple Signaling and Probabilistic Methods

A. Selective Mapping

Selective mapping (SLM) is a relatively simple method to reduce PAPR. A set of different OFDM symbols $x_{(m)}$, $0 \le m \le M-1$, which represent the same information as the original symbol. Then, it transmits the symbol with minimum PAPR. Thereby, the transmitted OFDM symbol can be written as [11]

$$\tilde{x} = \underset{0 \le m \le M-1}{\operatorname{argmin}} \left[PAPR\left(x_{(m)}\right)\right] \tag{10}$$

To generate different symbols set, original data block $\mathrm{X} = [\mathrm{X}_1\mathrm{X}_2 \ldots \mathrm{X}_\mathrm{N}]$ can be multiplied by M different phase sequences $\mathrm{p_m}$ element-by-element. This is performed prior to IDFT. This phase sequences can be described as [12]

$$P_m = [e^{j\varphi_{m,1}} e^{j\varphi_{m,2}} \ldots e^{j\varphi_{m,N}}], \quad 0 \le m \le M-1 \tag{11}$$

where $\varphi_{m,k} \in [0,2\pi)$, for $k = 1,2,\ldots,N$. Therefore, the modified OFDM symbol is the IDFT of the element-by-element multiplication of X and p_m:

$$x_{(m)} = IDFT[X_1 e^{j\varphi_{m,1}} X_2 e^{j\varphi_{m,2}} \ldots X_N e^{j\varphi_{m,N}}] \tag{12}$$

The amount of PAPR reduction by using SLM method depends on the number of generated phase sequences and the design of the sequences [6].

In order to detect the transmitted signal, information from selected phase sequences must be transmitted to the receiver as side information.

If the size of the OFDM blocks is large and the number of phase sequences (M) is increased, optimizing the process of selecting the best OFDM signal is going to be impossible.

B. Partial Transmit Sequence

In partial transmit sequence (PTS) method, data block of length N is divided into several disjointsub-blocks.

The IDFT is performed for every single sub-block separately, and they are then weighted by a phase factor. The phase factor is chosen in a way that the combined signal of all the sub-blocks gets the minimum PAPR [13].

The data block of $X = [X_1 X_2 \ldots X_N]$ is partitioned into M disjoint sub-blocks of $X_m = [X_{m,1} X_{m,2} \ldots X_{m,N}]$, $1 \le m \le M$, and X is the combination of all the M sub-blocks [13]:

$$X = \sum_{m=1}^{M} X_m \tag{13}$$

The IDFT of each sub-block $\mathrm{x_m}$, $1 \le \mathrm{m} \le \mathrm{M}$, is then calculated. In the process of selecting the optimum phase factors, searching is usually limited to a few phase factors to reduce the complexity.

The complexity of search increases as M increases exponentially [6].

C. Interleaving

Operating on a data block of N symbols, an interleaver generates interleaving. In other words, data block of X=$[X_1 X_2 \ldots X_N]$ is converted into $\acute{X} = [X_{\pi(1)} X_{\pi(2)} \ldots X_{\pi(N)}]$, where $\{n\} \to \{\pi(n)\}$. The permutation can be performed on both bits and symbols.

In order to reduce PAPR, several interleavers are used to generate different permutations [14]. The IDFT is separately performed on each permutation to generate different OFDM signals.

The OFDM signal with minimum PAPR is then selected to be transmitted.

To generate M different OFDM signals, M-1 blocks of interleaver blocks and M blocks of IDFT is needed. For side information bits, $[\log_2 M]$ is also needed [5].

D. Tone Injection

The idea behind tone injection (TI) is to expand the complex in a way that each points of the original complex plane is able to be mapped onto the expanded complex [14].

Since each symbol in data block can be permuted on several points in the expanded complex, this degree of freedom can be used to reduce the PAPR [6]. If a square QAM constellation with M points is considered as original complex, the space between each point in the original complex and the equivalent point in the expanded complex must be $D = \rho d\sqrt{M}$. $\rho \ge 1$ is a constant and *d is* the space between each point in the original complex [6].

The k^{th} symbol of QAM with only one subcarrier and several points in the complex is given as [15]

$$\bar{X}_k = X_k + p_k D + \mathrm{j}\, q_k D \tag{14}$$

where X_k is the k^{th} symbol for the original complex, QAM, p_k and q_k are integer numbers which used to change the real and imaginary parts of X_k, respectively. Tone injection can considerably reduce PAPR at the expense of increase in average signal power [5].

E. Tone Reservation

In tone reservation (TR), the transmitter does not send information on a small set of subcarriers. These subcarriers are used to reduce PAPR [5].

The purpose of this method is to find a time domain signal which has to be added to the original signal to reduce PAPR. The new OFDM signal can now be written as [16]

$$\bar{x} = x + c = IDFT(X + C) \quad (15)$$

where c is the time domain added signal vector, $C = [C_0 C_1 \ldots C_{N-1}]$ is its frequency domain equivalent and $X = [X_0 X_1 \ldots X_{N-1}]$ is the OFDM signal at the frequency domain.

The problem is finding c that minimizes the maximum of the new OFDM signal peak. In fact [5]:

$$\min_{c} \|x + c\|_{\infty} = \min_{C} \|x + IDFT(C)\|_{\infty} \quad (16)$$

Reduction of the PAPR in the TR method depends on the number of the reserved tones, their position on the vector C and the complexity of the optimization problem.

Moreover, the receiver should know the positions of the peak reduction carriers, thus the information which is named overhead information should be sent to the receiver that causes the transmitting rate to decrease [5], [16].

TABLE 1
SIMULATION PARAMETERS

Carrier freq.	Number of OFDM blocks (iteration)	Sampling frequency	Oversampling factor
2×10^6	10000	1×10^6	8

5. SIMULATION RESULTS

In this section, the CCDF diagrams of all aforementioned techniques and also the BER ones for comparison of distortion based techniques are depicted over PAPR in Rayleigh fading wireless environments.

It is worth noting that simulations are derived under the assumption of perfect channel estimation where receiving noises are modeled as additive white Gaussian noise.

In addition, the parameters which are considered the same in all simulations are presented in table.1, such as number of OFDM blocks, etc.

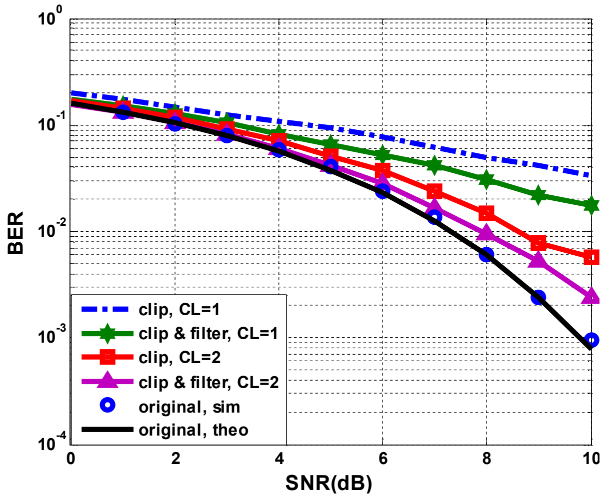

Figure 2: BER for three modes of normal, clipping, and both clipping-filtering with two different clipping level, 1 and 2

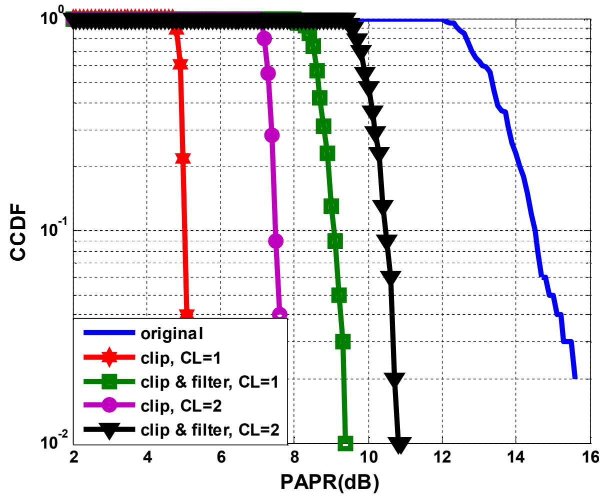

Figure 3: CCDF for three modes of normal, clipping, and both clipping-filtering with two different clipping level, 1 and 2

However, the other ones which include different values for each technique such as modulation type and FFT size are presented in the following when each figure is described. The effect of clipping on BER performance is shown in Fig. 2. In this simulation, OFDM symbols are used with 128 subcarriers. QPSK modulation was used and the information is transmitted without coding. It is also assumed that the receiver is not able to estimate the channel. Three modes of simulations are performed: normal, clipping, and both clipping-filtering with two different clipping level, 1 and 2. According to Fig. 2 and Fig. 3, with increasing the clipping level, the OFDM signal will be clipped by a larger threshold and less data will be lost, henceBER improves, while this will result in the CCDF to be degraded. Besides, both clipping and filtering provide better BER performance and worse CCDF performance rather than only clipping. Fig. 4 shows the CCDF plot for peak windowing method in comparison to normal mode for 128 subcarriers using 16-QAM modulation and cosine window. We can see the improvement in this method. CCDF diagram for three modes of simulations, normal, LST conversion, and LAST conversion with 128 subcarriers for QPSK modulation is shown in Fig. 5.

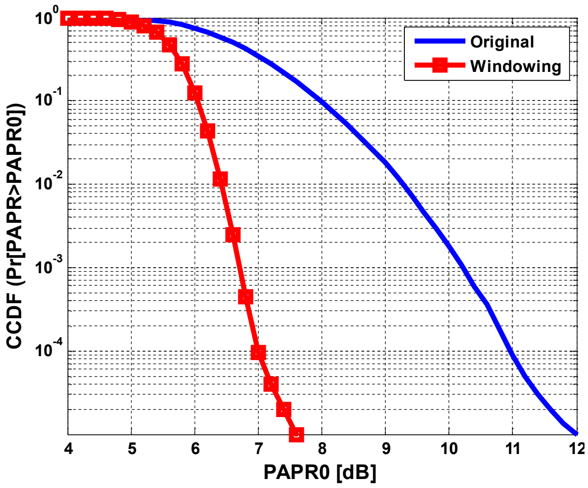

Figure 4: CCDF with and without peak windowing

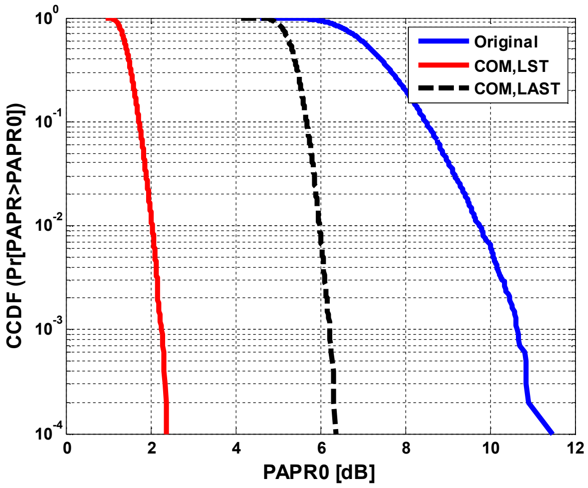

Figure 5: CCDF with and without companding

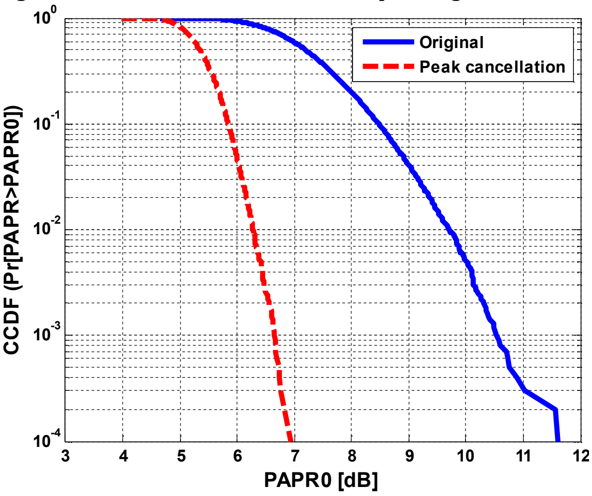

Figure 6: CCDF with and without peak cancelation

As it is seen, the performance is improved by the use of conversions. Fig. 6 shows the two modes of utilizing and not utilizing peak cancellation method with QPSK modulation, 128 subcarriers.

Fig. 7 shows the simulation results in two cases, with and without using SLM method. As we can see, by use of SLM, the PAPR reduces and also increasing the number of transmitted signal sub-blocks (N) causes the performance improves from the PAPR point of view.

It should be mentioned that the signals are transmitted via QPSK modulation and the number of subcarriers is equal to 128.

Fig. 8 shows the CCDF plot for different OFDM signal sub-blocks (M) using 16-QAM modulation and 256 subcarriers. As it is seen, with increasing the number of transmitted blocks, lower CCDF for the same PAPR is achieved. In Fig. 9, the CCDF plots vs. PAPR are compared for two different values of number of signal sub-blocks (M) using QPSK modulation and 64 subcarriers.

It is clear that for greater M, more reduction in PAPR is archived.

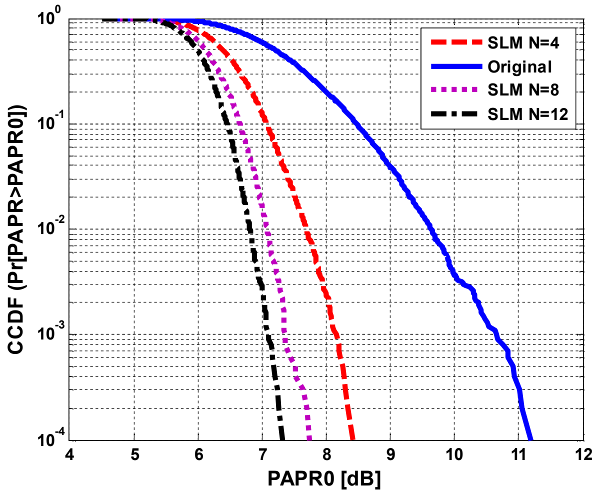

Figure 7: Comparison of CCDF with and without using SLM

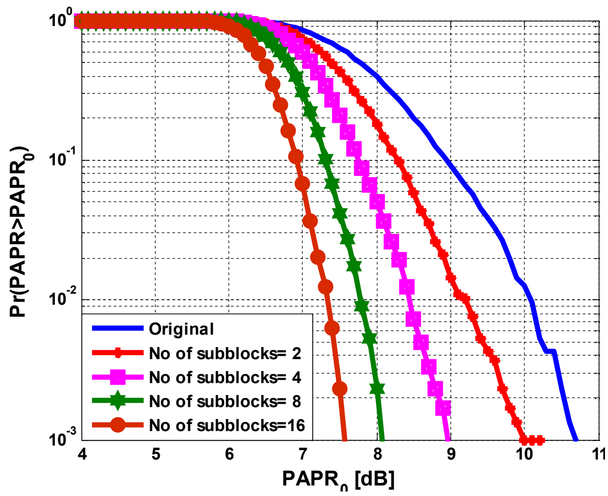

Figure 8: CCDF with and without employing PTS (M = 2, 4, 8, 16)

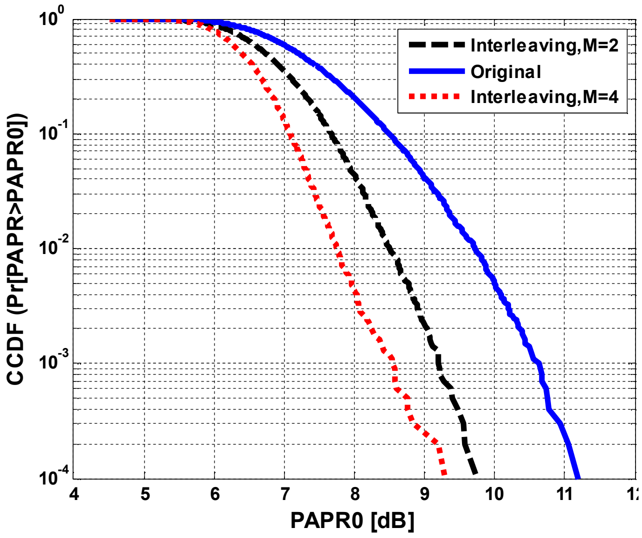

Figure 9: CCDF with and without employing Interleaving method (M = 2, 4)

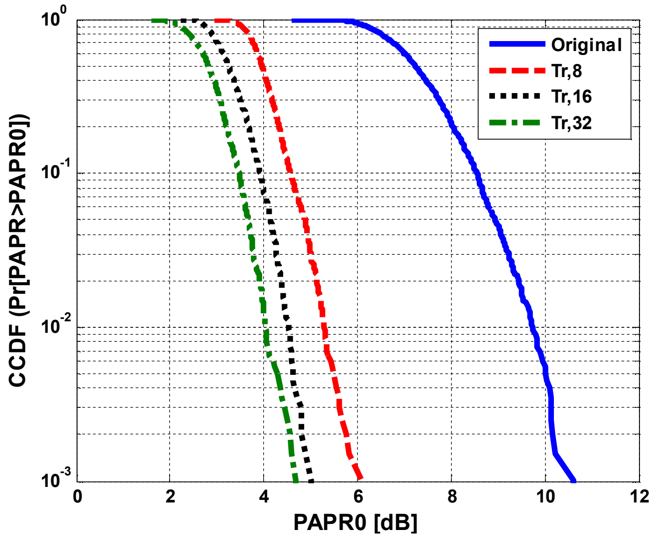

Figure 11: CCDF versus PAPR with different numbers of the reserved tones

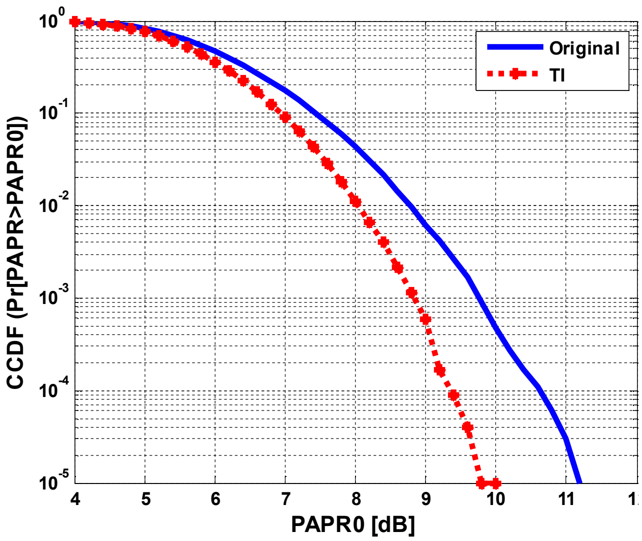

Figure 10: CCDF versus PAPR with and without using TI

In the Fig. 10, the diagram of the CCDF versus PAPR is depicted for 64 subcarriers, $\rho = 1$, d=2 and the mapping based on 16-QAM modulation. As shown, TI improves the PAPR, however it is possible to increase the performance more by choosing optimum injection.

Fig. 11 corresponds the using of TR method for 128 subcarriers and QPSK modulation while this method is evaluated for 4 different numbers of reserved tones. It is obvious that increasing this number causes the PAPR to reduce more, however this case also increase the transmitted signals power.

6. The Criteria for Selecting the Best Method to Reduce PAPR

A. *The Ability of Reducing PAPR*

This is the most important factor to select the right method. While selecting the method, all the effects such as in-band and out-of-band distortion when clipping should be considered.

B. *Power of the Transmitted Signal*

In order to reduce PAPR, some methods need more power for sending a signal. For example, TR technique requires more power because some parts of the power are used for PRCs. TI technique, due to the usage of equivalent points to points in original constellation which have more energy, causes this technique requires more power.

C. *Increasing BER*

This criterion has a strong relation with the power of transmitted signal. Some methods of reducing PAPR degrade BER at the receiver. To stop degrading BER after implementing the PAPR reduction method, power of the transmitted signal should be increased. For example, after performing TI, if power of the transmitted signal is fixed, BER increases. On the other hand, in some methods such as PTS, SLM, and Interleaving, in case of occurring error at side information, the whole data block might corrupt which makes BER increases.

D. *Reduction of Transmission Rate*

The use of coding for PAPR reduction or transmitting side information in PTS, SLM, and Interleaving are accompanied with reduction of transmission rate. In other words, the methods that need side information will result in remarkable reduction of data transmission rate.

7. BER Performance of the First Category Techniques

As we said, the techniques described in the first category reduce the PAPR by increasing the BER.

Hence, we should compare them from BER point of view to select the technique with the best performance for transmission. On the other hand, the techniques such as companding and clipping and filtering include some parameters which the performance of each technique can be changed for different values of these parameters. Thus, we cannot select a technique which outperforms the other ones in all cases. For these reasons, the performance of the first category techniques is depicted for two different values of parameters susing 4-QAM modulation and 128 subcarriers. In the Fig. 12, the BER performance is demonstrated for constant and slope parameters as 0.2 and 0.9 in LST companding technique, respectively, and also for clipping level as 4 in clipping and filtering technique. As it was shown, the clipping and filtering technique outperforms the other ones in this special case, where peak cancellation and peak windowing achieve the same results and LST companding obtain the worst BER performance. It is noteworthy that at SNR=10dB clipping and filtering outperforms the peak cancellation, peak windowing and companding techniques with a gain of nearly 10dB, 10dB and 20dB, respectively. But, in the Fig. 13 which employs constant and slope parameters as 0.06 and 0.55 in LST companding technique, respectively, and also clipping level as 0.1 in clipping and filtering technique, peak windowing and peak cancellation techniques have better BER performance than the other ones, where at SNR=12 dB they outperform the clipping and filtering and LST companding with a gain of nearly 13 dB and 8dB, respectively.

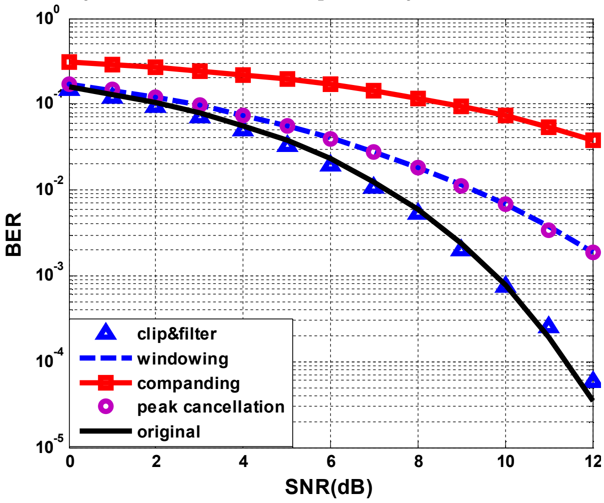

Figure 12: Comparison of the BER performance of the signal distortion techniques for a=0.9, b=0.2, CL=4

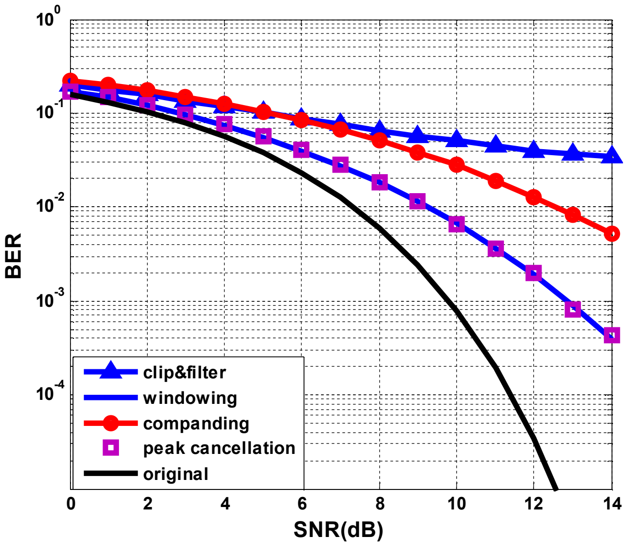

Figure 13: Comparison of the BER performance of the signal distortion techniques for a=0.55, b=0.06, CL=0.1

8. Computational Complexity of Multiple Signaling and Probabilistic Techniques

In this section, we investigate the implementation complexity of the second category techniques which can be observed in table 2.

As can be observed these techniques reduce the PAPR with the price of more complexity. Thus, comparison of multiple signaling techniques in terms of implementation complexity leads us to select the better technique for transmission which reduces the PAPR, meanwhile is simpler to implement. As shown in the table 2, TR and TI techniques are much more complex than the other techniques. On the other hand, among three other techniques which are simpler and also have approximately the same complexity, the interleaving includes less number of real additions and multiplications than PTS and SLM techniques.

9. Conclusion

Common methods for reduction of PAPR were introduced in this paper. Each method has its own advantages and disadvantages.

Thus, it is impossible to select a specific method for all the subcarrier systems. The designer has to find an appropriate method based on type of the system they are working with.

Various factors for selecting the right method to reduce of PAPR can be specified. For example, the ability of reducing PAPR, the power of transmitted signal, BER, transmission rate, computational complexity, and etc.

TABLE2.
COMPUTATIONAL COMPLEXITY OF MULTIPLE SIGNALING TECHNIQUES

Technique	Computational complexity
TR	$O\ (LN^2)$ (L is the number of peak reduction carriers)
TI	$(NS)^K$ (S is the number of candidate constellation points and K is the number of dimensions needed to be D-shifted)
PTS	Real multiplications: $2MN\log_2 N + 2N + 1$ Real additions: $3MN(\log_2 N) + (M-1)[2N(M+1)-1]$ (M is the number of sub-blocks)
SLM	Real multiplications: $2MN(1+\log_2 N) + M$ Real additions: $3MN(1+\log_2 N) + M(N-1) - 1$ (M is the number of generated symbols)
Interleaving	Real multiplications: $2MN\log_2 N$ Real additions: $3MN\log_2 N$ (M is the number of similar generated symbols)

When the data rate is high, the methods that reduce transmission rate such as SLM, PTS, TR and interleaving are not recommended. It is worth noting that TI is the only method from the second aforementioned classification which requires no side information at all and there is no loss of bit rate. In the view of complexity and cost, multiple signaling and probabilistic methods cannot be good candidates, while signal distortion methods are suggested when we need to transmit with more simple or high data methods. TR and TI are two methods which reduce the PAPR at the expense of some increase in signal power, hence are not appropriate when the transmitted power is limited. It should be mentioned that the methods such as clipping, and filtering and companding or generally distortion based methods degrade the BER performance while the other ones don't increase BER sensibly.

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