

Optimum Design of a SRM Using FEM and PSO

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ABSTRACT

Nowadays the use of the Switched Reluctance Motors (SRMs) has been considerably increased in various home and industrial applications. Despite of many advantages of this type of motors, such as simple structure, low cost, and high reliability, the main disadvantage of them is the generation of high torque pulsation. This paper presents a novel method to optimize a typical SRM such that the torque ripple reaches its minimum value. Meanwhile, the torque average and the motor efficiency become maximum. It is shown that the pole width to the pole pitch ratio, for both stator and rotor poles, have a great impact on the torque ripple and torque average. Finite Element Method (FEM) is used to obtain the torque ripple, the torque average and the motor efficiency for a large number of ratios. A functional relationship is developed between the input and the output parameters. Normalized summation of the torque ripple minus the torque average and the efficiency is considered to be the cost function, which must be minimized. Then, the Particle Swarm Optimization (PSO) is used to find the optimum ratio of pole width to pole pitch, for both stator and rotor. The optimum design is verified by FEM.

1. INTRODUCTION

Switched reluctance motors are in the family of the reluctant motors which have very simple structure [1], [2]. This motor has salient poles in the rotor and stator and has coils only on the stator poles. The advantages of these types of motors can be listed as: simplicity of structure, low price of manufacturing, variability of speed, safe power controlling, high reliability, long lifespan, high ratio of torque to volume, and the resistance against overheating [1], [3]. These advantages have caused a widespread usage of such motors in the industry. Despite of these advantages, switched reluctance motors have some disadvantages too. In these motors the stator currents waveforms are complex and also they have torque ripples that cause fuss and acoustic noise [4]-[7]. The cause of these ripples in such motors, at first could be attributed to the working in the saturation and also to discontinuous mechanism of the torque production. There are two general methods for decreasing the

ripples of the torque. The first one is changing the design of the motor from a magnetic point of view [8]-[13]. And the second one, is changing the electronic control of the motor-driven [14]-[16].

The first method is presented in this paper. Considering the conducted studies on this area, it has been observed that such studies are in two general categories. In the first state, there is no fundamental change in the motor structure, instead, the influence of the motor dimensions on the cogging torque were inspected; parameters such as stator pole arc, motor axial length, slot skewing, the ratio of the external to the internal radius and so on. In the second one, a new structure has been suggested for such motors. As a sample, it has been worked on the stator and rotor poles [17]. At first, the angle of the stator and rotor poles has been changed from 15° to 23° and 15° to 30°, respectively. Then, the motor torque and its ripple are obtained using the finite element method (FEM) in the static mode. Finally, using these values, an artificial neural network (ANN) has been trained

for optimizing the dimensions of the stator and the rotor poles in order to have more average torque with less torque ripple. A new structure, including two rotors, has been suggested for the motor in [18]. It has been observed that the average torque increased and the torque ripple decreased. In [19], it has been worked on the stator structure. In order to have less saturation, some changes are applied to the edge of the stator teeth. It was observed that the torque and its ripple reached a better condition.

This paper focuses on the ratio of the width of the teeth to the pitch of the pole. At first, a motor with defined dimensions has been considered. By changing the ratio of the tooth width to the pole pitch for the stator and rotor, and simulating motor by FEM, a lot of data have been obtained that has been arranged in a table. In order to gain the points which are not available in the table of the data, the interpolation method has been used. Then, using the particle swarm optimization (PSO), an optimum value has been achieved for the ratio of the pole width to the pole pitch for both stator and rotor. The cost function has been defined so that the motor would have the maximum average torque, minimum torque ripple and maximum efficiency.

2. THE EQUATION OF THE TORQUE

The equation of the voltage for one phase of the switched reluctance motor is as follow:

$$v = ri + \frac{d\lambda}{dt} \quad (1)$$

where v is the voltage of the phase, r is the resistance of the coil and λ is the flux linkage. Supposing that the core is linear and the rotor rotates with the angular speed of ω , the voltage equation will be as follow:

$$v = ri + L \frac{di}{dt} + i\omega \frac{dL(\theta)}{d\theta} \quad (2)$$

where L is the inductance and θ is the position of the rotor. From (2), the equation of the power in motor will be as follows:

$$vi = ri^2 + Li \frac{di}{dt} + i^2 \omega \frac{dL(\theta)}{d\theta} \quad (3)$$

$$vi = ri^2 + \frac{d}{dt} \left(\frac{1}{2} Li^2 \right) + \frac{1}{2} i^2 \omega \frac{dL(\theta)}{d\theta} \quad (4)$$

The left side of the (4) is the input power. The first term in the right hand side of the equation is the resistive power losses. The second term is the power which is stored in the magnetic field, and the third part is the mechanical power. So, the torque equation can be achieved as:

$$\tau = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} \quad (5)$$

Fig.1 shows a typical torque waveform produced by such motors. This waveform has been obtained from the finite element analysis. This figure shows a lot of ripple in the torque waveform.

3. THE TORQUE RIPPLE

As it is clear in Fig.1, the torque is changing from the minimum to the maximum value. In various papers, different formulas have been used for measuring the ripples of the torque. In this paper, the ripple of the torque is defined as follows:

$$\Delta\tau = \frac{\tau_{\max} - \tau_{\text{ave}}}{\tau_{\max}} \quad (6)$$

where $\Delta\tau$ is the ripple of the torque, τ_{\max} is the maximum torque, and τ_{ave} is the average torque.

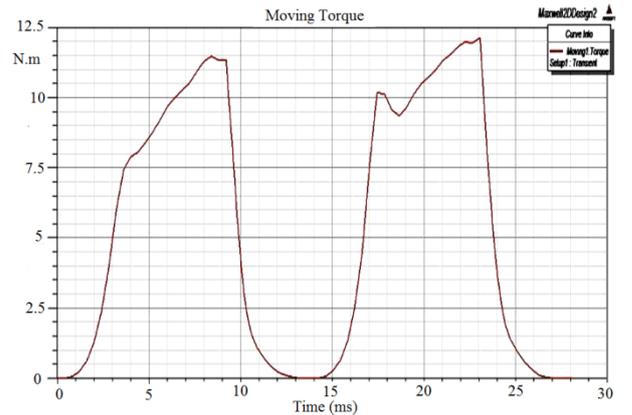


Figure 1: A typical waveform of the moving torque produced by SRM.

4. FINITE ELEMENT ANALYSIS

The governing equation for 2D magneto static problem can be represented as follow:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu J_s \quad (7)$$

where μ is magnetic permeability, A is the vector magnetic potential and J_s is the current density in the coil area. The above differential equation is converted to algebraic equation through the Galerkin method:

$$[S]_{n \times n} [A]_{n \times 1} = J_s [D]_{n \times 1} \quad (8)$$

in which, $[S]$ is the coefficient matrix, $[A]$ is the vector of potentials, $[D]$ is a fixed vector and n is the number

of the nodes. Matrix [S] and vector [D] are constant and they depend on the geometry of the 2D space and the way of meshing [19]. The voltage equation for one phase is as follows:

$$v_k = r_k i_k + \frac{d\lambda_k}{dt} \quad (9)$$

where v_k is the phase voltage source, i_k is the phase current, r_k is the resistance of the phase and λ_k is the flux linkage of the phase winding. The flux linkage, λ_k , is achievable through the following equation:

$$\lambda_k = l_z [D]^T [A] \quad (10)$$

In that l_z is the axial length. The phase current, i_k , is in relationship with the current density in the coil area, J_s , as follow:

$$i_k = \frac{S_j}{n_w} J_s \quad (11)$$

where S_j is the coil's cross-sectional area and n_w is the number of turns in the that area. Equations (8) to (11) have been indicated in the form of matrix as follow:

$$\begin{bmatrix} [S] & [D] \\ [D]^T & -\frac{S_j}{n_w} \frac{r\Delta t}{l_z} \end{bmatrix} \times \begin{bmatrix} [A(t+\Delta t)] \\ -J_s(t+\Delta t) \end{bmatrix} = \begin{bmatrix} 0_{n \times n} & 0 \\ [D]^T & 0 \end{bmatrix} \quad (12)$$

$$\times \begin{bmatrix} [A(t)] \\ -J_s(t) \end{bmatrix} + \begin{bmatrix} 0_{n \times 1} \\ \frac{\Delta t}{l_z} v_k(t+\Delta t) \end{bmatrix}$$

After solving (12) and determining magnetic potential, the flux density can be achieved from:

$$\bar{B} = \nabla \times \bar{A} \quad (13)$$

where \bar{B} is the flux density vector and \bar{A} is the magnetic potential vector. According to Maxwell stress formulation, the electromagnetic torque can be represented as follow [20].

$$\tau = \frac{l_z}{\mu_0} \int_0^{2\pi} r^2 B_r B_\theta d\theta \quad (14)$$

where B_r and B_θ are the radial and tangential components of vector \bar{B} respectively and τ is the motor torque.

5. DETERMINING THE MOTOR EFFICIENCY

Efficiency is one of the important factors which is

affected by the design of the motor. To achieve efficiency of a motor, the below equation could be used:

$$\eta = \frac{P_{out}}{P_{in}} \times 100 \quad (15)$$

In this equation, η is efficiency, P_{in} is the input power and P_{out} is the output power of the motor. To gain the input power, (16) must be used.

$$P_{in} = \frac{1}{T} \int_0^T v i dt \quad (16)$$

And also the output power is achieved through the below equation.

$$P_{out} = \tau_{ave} \omega \quad (17)$$

6. PARTICLE SWARM OPTIMIZATION (PSO)

Particle Swarm Optimization was designed by Kennedy and Eberhart for the first time [21]. The primary idea of this algorithm has been deduced from the flight of the birds. In the PSO algorithm, there are some elements known as particles, which are expanded in the search-space. Each particle can be an N-dimensional vector. The goal is to find the best particle (or best position) which has the minimum value of the objective function. At first, the particles are initialized with random values. Then, the objective function is calculated for each particle. After that, from current position and velocity of the particle and the best position of it (from beginning up to now) and the position of the best particle, the next position and the next velocity of each particle can be found out. This process is done for all particles in each iteration. After some iteration (depends on the required accuracy), the particle which has the minimum value of the objective function is available. The following equations indicate updating process of the velocity and position for each particle.

$$v_{i+1}(p) = w.v_i(p) + c_1.rand_1.(best_i(p) - x_i(p)) + d_i.rand_2.(best_i(X_{pop}) - x_i(p)) \quad (18)$$

$$x_{i+1}(p) = x_i(p) + v_{i+1}(p) \quad (19)$$

where $v_i(p)$ and $x_i(p)$ are the velocity and the position of each particle respectively, c_1 is the local learning coefficient and d_i is the global learning coefficient. They are in the range of 0 to 2 and they effect on the convergent speed [21]. The inertia weight, w , is very important in the convergence of PSO. Increasing w increases the convergence speed and it means less

exploration property. To choose a suitable w , the following equation can be used:

$$w(k) = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{\text{iter}_{\max}} \right) \times \text{iter} \quad (20)$$

Where, iter_{\max} is the maximum number of iterations, iter is the number of iteration, w_{\max} is the primary weight, and w_{\min} is the final weight. The primary value for w is around 1 and it will be decreased towards zero gradually.

7. SELECTING THE APPROPRIATE PARAMETERS FOR THE OPTIMIZATION

Four different parameters are defined in the SRM, These four parameters are:

$$\gamma_s = \frac{\beta_s}{\alpha_s}, \gamma_r = \frac{\beta_r}{\alpha_r}, p_r = \frac{R_{ir}}{R_{or}}, p_s = \frac{R_{is}}{R_{os}} \quad (21)$$

where α_s is the stator pole pitch, β_s is the stator pole width, α_r is the rotor pole pitch and β_r is the rotor pole width and are shown in Fig.5. Moreover R_{ir} is the rotor inner radius, R_{or} is the rotor outer radius, R_{is} is the stator inner radius and R_{os} is the stator outer radius. γ_s and γ_r are the ratio of the pole width to the pole pitch for stator and rotor respectively and P_s and P_r are the ratio of inner radius to outer radius for stator and rotor, respectively. These four parameters, γ_s , γ_r , P_s and P_r , were changed in the range of 0.35 to 0.65. The other specifications of the motor are listed in Table1. The average torque and the torque ripple were calculated at each point. Results are shown in Fig. 2 and Fig.3, respectively. It is seen that the role of γ_s and γ_r on the average torque and the torque ripple is more significant than the P_s and P_r . So γ_s and γ_r were chosen as the optimization parameters.

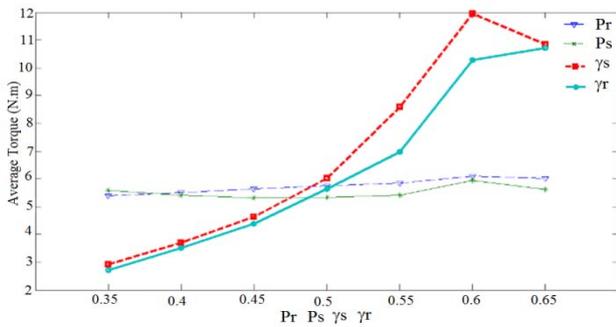


Figure 2: The changes of the average torque versus design parameters, including P_s, P_r, γ_s and γ_r .

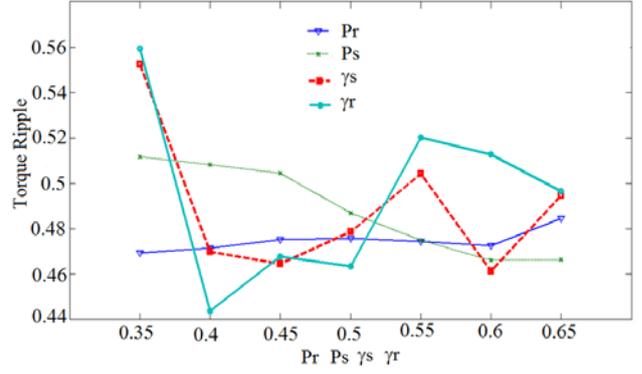


Figure 3: The changes of the torque ripple versus design parameters, including P_s, P_r, γ_s and γ_r .

TABLE 1: SPECIFICATIONS OF A TYPICAL MOTOR.

Number of stator poles	12
Number of rotor poles	8
Number of phases	3
Stator pole pitch angle	30°
Rotor pole pitch angle	45°
Stator pole arc angle	15°
Rotor pole arc angle	22.5°
External diameter of the stator	140 mm
Internal diameter of the stator	84 mm
Depth of the stator slot	14.46 mm
Minimum air gap	1 mm
External diameter of the rotor	83 mm
Internal diameter of the rotor	49.8 mm
Motor length	65 mm
Winding turns	142

8. OPTIMIZATION OF THE TYPICAL MOTOR

The parameters of the studied switched reluctance motor are listed in Table1. This motor has been modeled by the finite element software (Maxwell-13). Fig.4 shows the contour-plot of the magnetic flux in a typical condition of the motor analysis. As mentioned before, γ_s and γ_r are the important factors in the optimization process. γ_s was changed in the range of 0.53 to 0.73 and γ_r was changed in the range of 0.33 to 0.5. These ranges are selected so that the torque average maintain in an acceptable range. As it can be seen in Fig. 2, less than 0.35, the average torque decreases to an unacceptable value and above the 0.65 the torque average starts to decrease from its maximum value.

The step of the changes was considered to be 0.01. It means that $\gamma_s=0.53:0.01:0.73$ and $\gamma_r=0.33:0.01:0.50$. In each case, the average torque, the torque ripple and the efficiency are calculated using finite element analysis. In this way 504 output data were obtained. Some of them are shown in Table2 to Table4. The graphs of the results are shown in Fig. 6 to Fig.8.

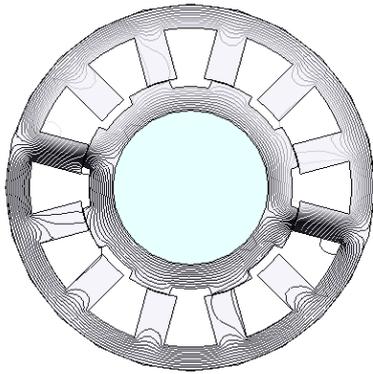


Figure 4: Contour plot of magnetic flux obtained by finite element analysis.

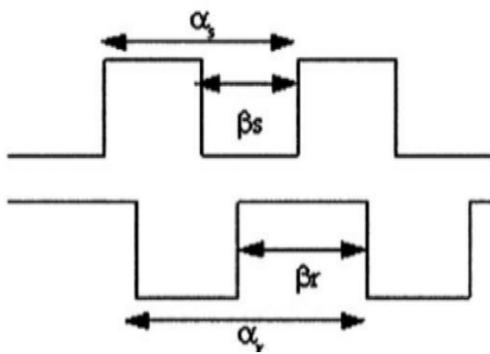


Figure 5: Pole pitch, α_s and α_r , and pole width, β_s and β_r , for both stator and rotor.

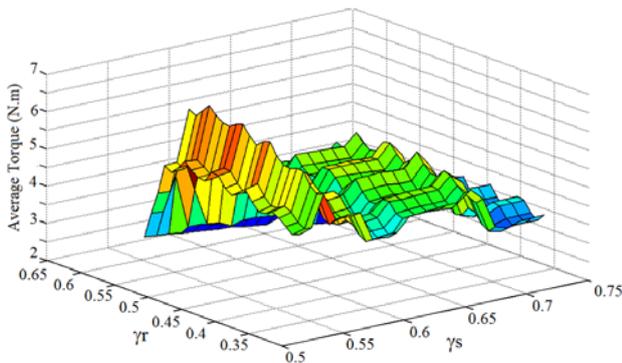


Figure 6: The changes of average torque in the range of $0.53 \leq \gamma_s \leq 0.73$ and $0.33 \leq \gamma_r \leq 0.53$.

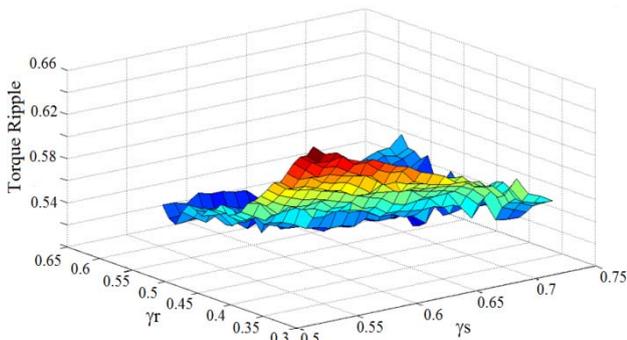


Figure 7: The changes of the torque ripple in the range of $0.53 \leq \gamma_s \leq 0.73$ and $0.33 \leq \gamma_r \leq 0.53$.

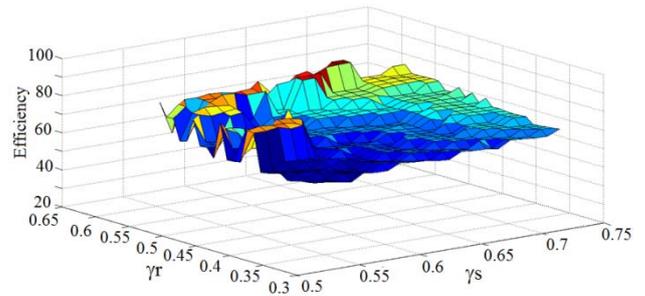


Figure 8: The changes of the efficiency in the range of $0.53 \leq \gamma_s \leq 0.73$ and $0.33 \leq \gamma_r \leq 0.53$.

TABLE 2: AN EXAMPLE OF THE AVERAGE TORQUE VARIATION IN TERMS OF γ_s AND γ_r .

$\gamma_r=0.54$	$\gamma_r=0.53$	$\gamma_r=0.52$	τ_{ave} (N.m)
5.3128	5.1292	5.0097	$\gamma_s=0.54$
5.2030	6.4770	6.3657	$\gamma_s=0.55$
5.1289	5.0258	4.8930	$\gamma_s=0.56$

TABLE 3: AN EXAMPLE OF THE TORQUE RIPPLE VARIATION IN TERMS OF γ_s AND γ_r .

$\gamma_r=0.54$	$\gamma_r=0.53$	$\gamma_r=0.52$	$\Delta\tau$ (N.m)
0.5427	0.5428	0.5532	$\gamma_s=0.54$
0.5446	0.5362	0.5508	$\gamma_s=0.55$
0.5350	0.5413	0.5507	$\gamma_s=0.56$

TABLE 4: AN EXAMPLE OF THE EFFICIENCY VARIATION IN TERMS OF γ_s AND γ_r .

$\gamma_r=0.54$	$\gamma_r=0.53$	$\gamma_r=0.52$	$\Delta\tau$ (N.m)
0.5427	0.5428	0.5532	$\gamma_s=0.54$
0.5446	0.5362	0.5508	$\gamma_s=0.55$
0.5350	0.5413	0.5507	$\gamma_s=0.56$

Using finite element analysis, a discrete space with the high accuracy and high resolution points is available. So, by means of the linear interpolation, it is possible to represent a continuous space, including γ_s , γ_r and related torque or efficiency.

The final goal is to find the optimum γ_s and γ_r with the maximum average torque and efficiency and the minimum torque ripple. It can be done using the PSO. The following equation is defined as cost function and can be used for optimization of γ_s and γ_r .

$$OPT = \left(\frac{\Delta\tau}{\tau_{max}}\right) - \left(\frac{\eta}{\eta_{max}}\right) - \left(\frac{\tau_{ave}}{\tau_{ave,max}}\right) \quad (22)$$

The PSO was converged in 13 iterations, as it is

clear in Fig.9. The objective function's variations are shown in Fig.10. The optimal value for γ_s is 0.55 and for γ_r is 0.53. In this case the average torque is 6.45 N.m, the ripple torque is 0.54, and the efficiency is 80.63%. These values are the best choice for γ_s and γ_r .

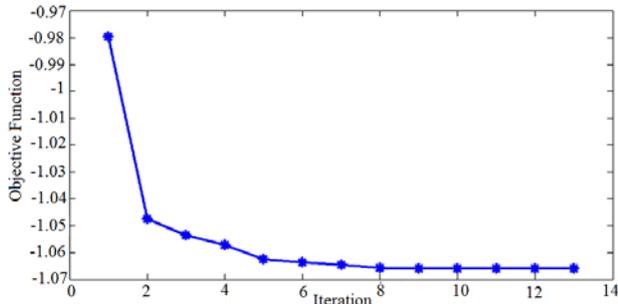


Figure 9: Decreasing of cost function versus iterations.

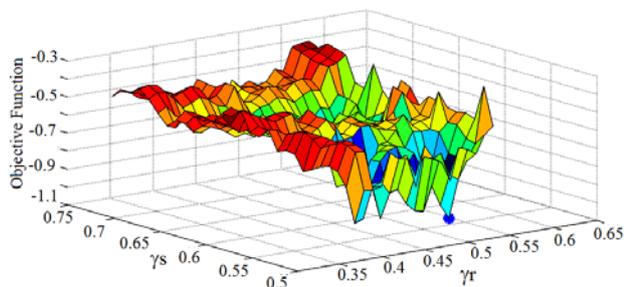


Figure 10: The changes of the cost function in terms of γ_s and γ_r ; the point with the minimum cost function is shown with *

9. CONCLUSION

According to the widespread usage of the Switch Reluctance Motors in home and industrial applications, study of these motors is now became a great concern. One of the problems of this motor is the torque ripple which causes noise. In this paper it is shown that the most effective factors on the torque ripple are the ratio of the pole width to the pole pitch for stator and rotor (γ_s and γ_r). On the other hand, decreasing the torque ripple in some cases, may cause a decreasing in the average torque. Therefore, each optimization procedure must obtain the minimum of the ripple with the maximum of the average. Moreover, the motor efficiency is important too therefore, the optimization method must be performed in a way that it reaches its maximum value. To achieve this multi-purpose optimization, the PSO has been used. At first, values of the average torque, torque ripple, and the efficiency have been gained for a lot of γ_s and γ_r , using the finite element software. By means of interpolation, average torque, torque ripple, and efficiency have been represented as continuous functions of γ_s and γ_r . Finally, using PSO, the best value of γ_s and γ_r has been achieved. The conducted analysis

by finite element method, validates the achieved optimum points.

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