A Simple and Fast Method for Field Calibration of Triaxial Gyroscope by Using Accelerometer

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Extended Abstract

Background and Objectives: Using field calibration methods without precision laboratory equipment, systematic faults of inertial sensors can be reduced and measurement accuracy can be increased.

Methods: In this paper, a simple and fast method called improved least squares is used to find calibration coefficients of an accelerometer including bias, scale factor and non-orthogonality. In this method, this principal is used that the magnitude of acceleration measured by accelerometer in static condition is equal to the magnitude of gravity vector and a cost function is then defined. Also, in gyroscope field calibration, sensor is rotated manually around all three axes separately and then it is put in the static mode. Changes in the angle obtained from gyroscope at each movement are compared with the ones obtained from the calibrated accelerometer. Calibration coefficients including bias and scale factor are obtained using least squares method.

Results: Simulation results in MATLAB show that the measurement accuracy of accelerometer after calibration has improved by about 60% and the accuracy of the gyroscope has increased by about 40%. Also, comparison with the other methods proves that the proposed method performs well in the accuracy, speed, time required, and the effect of noise changes.

Conclusion: This paper by finding a fast, simple, and low-cost field calibration method to calibrate MEMS accelerometer and gyroscope without using accurate laboratory equipment can help a wide range of industries that use advanced and expensive sensors or use expensive laboratory equipment to calibrate their sensors, to decrease their costs.

Keywords: Accelerometer, Gyroscope, Bias, Scale factor, Non-orthogonality

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Introduction

In order to determine position, speed and status of a device, inertial sensors including accelerometer and gyroscope are used. The outputs of these sensors are not accurate due to faults such as manufacturing problems, installation faults and external factors. In order to compensate systematic errors of these sensors, calibration is used [1, 2].

Calibration can be divided into two laboratory (traditional) and field categories [3, 4]. Calibration in laboratory using laboratory instruments like rate table, centrifuge and temperature control chambers, is called laboratory calibration. Among the laboratory calibration methods, multi-state test, turn table test, centrifuge test, thermal test, hit test, polar axis test, velocity transmission test, magnetic sensitivity test and test through adjusting clamp position can be mentioned [4-8]. It should be noted that laboratory calibration methods are costly and time-consuming [9]. In addition,
after calibrating sensors in laboratory and installing them on devices, factors like displacement of sensors, environmental conditions and time degradation of the output; make them to be calibrated again. Calibrating sensors in environment without accurate laboratory equipment is called field calibration [9]. Field calibration methods are in existence since 1995 when Ferraris first introduced field calibration method [10]. In [10], accelerometer and gyroscope were calibrated without accurate laboratory equipment and their bias and scaling factors were obtained. In 1998, Lötters et.al. calibrated accelerometer using static edge detector without any equipment [11]. In [11], authors employed the rule that norm of a vector measured by the accelerometer in static conditions is equal to the input vector (gravity vector). Using this rule, a cost function was defined for calibration of both the accelerometer and gyroscope, but other relative studies tried to solve this by using different optimization methods, like Newton, Quasi-Newton, Powel, AFSA and other algorithms and hence obtained calibration coefficients of the sensor [12-16]. Auxiliary sensors like magnetometer, GPS, camera and calibrated and accurate IMU sensors can be used to calibrate inertial sensors. Indeed these methods are similar to costly laboratory methods and accuracy depends on the auxiliary sensors being calibrated and accurate [17, 18].

In this paper, calibrating accelerometer using improved least squares method is described first and then calibrating gyroscope sensor through changing angles around sensitive axes is explained. We choose improved least squares method for calibration of accelerometer, because it is a simple and fast field calibration method that does not need any equipment and also bias, scale factor and non-orthogonality can be obtained without using any complicated equations [19]. Furthermore, the proposed method for calibration of gyroscope is simple and fast and it also does not need any equipment. It just needs the gyroscope to be rotated around its sensitive axis. Rest of this paper is organized as follows: Section 2 studies field calibration of the accelerometer, Section 3 presents the proposed method and three other common methods. Section 4 presents simulation results of calibrating the accelerometer and gyroscope in MATLAB. Finally, the paper is concluded in Section 5.

Field Calibration of the Accelerometer

The output of the accelerometer sensor used in this paper is considered as follows [20]:
\[ a_n = Sa + Ma + b + n \] (1)
In which \( a_n \) is the measured acceleration vector, \( a \) is the real acceleration vector, \( S \) is the scale factor matrix, \( b \) is the bias vector and \( M \) is the non-orthogonality matrix considered as \( M = \begin{bmatrix} 0 & 0 & 0 \\ m & 0 & 0 \\ m^2 & m & 0 \end{bmatrix} \) similar to [21] and [22] and \( n \) is a zero-mean Gaussian noise with standard deviation of \( \sigma \).

A. Improved Least Squares Calibration Method

According to [23], when non-orthogonality is not considered in the output model, the least squares method performs well and it is the fastest method which performs well against noise variations and does not depend on the initial conditions. But, when this method is used to find unknown calibration coefficients without considering non-orthogonality, equations change such that the least squares become complex and hence cannot be used. In the improved least squares method, the output equations of the accelerometer are rewritten so that these nonlinear equations can be solved using the least squares method and calibration coefficients including bias, scale factor and non-orthogonality can then be obtained[19].

As it is known, obtained acceleration in static mode is equal to the gravity acceleration plus the error factors. This rule can be written as follows [24]:
\[ |h|^2 = a_x^2 + a_y^2 + a_z^2 = |h|^2 \] (2)

By replacing (1) in (2), and simplify the coefficients which are very small, the following equation is then obtained [19]:
\[ a_n^2 = \left(\frac{1}{k_{11}} 2a_{nx} a_{ny} 2a_{nx} a_{nz} 2a_{ny} a_{nz} 2a_{nx} a_{ny} a_{nz} \right) \] (3)

in which, coefficients \( k_i \) and \( b_i \) are equal to
\[ k_{11} = \frac{1}{s_x} \quad k_{22} = \frac{1}{s_y} \quad k_{33} = \frac{1}{s_z} \quad k_{21} = \frac{m}{s_y} \quad k_{31} = \frac{m}{s_z} \quad k_{32} = \frac{m}{s_z} \] (4)
\[ b_1 = \frac{b_x}{s_x} \quad b_2 = \frac{b_y}{s_y} \quad b_3 = \frac{b_z}{s_z} \]

Now, if measurements are performed \( n \) times under different conditions, equation becomes \( L=AX \) which can be solved to obtain calibration coefficients including bias, scale factor and non-orthogonality.
Field Calibration of the Gyroscope

In this section, three most common methods and the proposed method for field calibration of the gyroscope are described.

B. Output model of the gyroscope

Output model of the gyroscope is considered as follows [9]:

$$\omega_m = S\omega + b + n$$

(5)

In which, $\omega_m$ is the measured angular velocity vector, $\omega$ is the real angular velocity vector, $S$ is the scale factor matrix, $b$ is the bias vector and $n$ is a zero-mean Gaussian noise with standard deviation of $\sigma$. Unlike accelerometer model, non-orthogonality coefficients are not considered in the gyroscope model because these values are very small and make computations complicated and time consuming [9].

C. Using Input Angular Velocity Principal

This method is based on [20] and [22] which employs a single-axis rotating table to locate the gyroscope on it in order to calibrate the gyroscope because cheap sensors like MEMS sensors do not calculate rotation speed of the earth; thus, here, the reference is the rotation speed of the table. This principal is like (2) and is described as follows:

$$\|\omega_m\|^2 = \omega_x^2 + \omega_y^2 + \omega_z^2 = \omega_r^2$$

(6)

in which $\omega_r$ is the rotation speed of the table. Now, by combining (5) and (6), a cost function is obtained which is minimized using optimization methods to obtain error factors of the gyroscopes including bias and scale factors.

D. Using Accelerometer and Euler Angles

First, in order to calibrate the gyroscopes, its bias is estimated and eliminated from the output equation of the gyroscope. For this purpose, the sensor is put into static mode and the gyroscope along the three axes. T, S and C represent Tan, Sin and Cos of the angles. Knowing initial conditions of Euler angles and angular velocity measured by the gyroscope, Euler angles and acceleration (using (9)) can be obtained at each instant. Now, by minimizing the cost function as defined above, the scale factor of the gyroscope can also be obtained.

E. Using Accelerometer and Quaternions

As specified in (9), this equation has one singular point and also using Euler angles is time-consuming. Therefore, Quaternions are used where their differential equation is represented as follows:

$$\begin{bmatrix}
\dot{q}_0 \\
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
-\omega_x & -\omega_y & -\omega_z & q_0 \\
\omega_y & -\omega_z & \omega_x & q_1 \\
\omega_z & \omega_x & -\omega_y & q_2 \\
-\omega_y & \omega_x & \omega_y & q_3
\end{bmatrix} \begin{bmatrix}
q_0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix}$$

(10)

in which $q_j$, $j = 0, 1, 2, 3$ are the quaternions and $\omega_i$, $i = x, y, z$ is the angular velocity measured by the gyroscope along the three axes.

Knowing initial conditions of the quaternions and the angular velocity measured by the gyroscope, the quaternions and acceleration at each instant is obtained using the following conversion matrix:

$$C = \begin{bmatrix}
q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_3q_0) & 2(q_2q_3 + q_0q_1) \\
2(q_1q_2 + q_3q_0) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_1q_3 - q_2q_0) \\
2(q_2q_3 - q_0q_1) & 2(q_1q_3 + q_2q_0) & q_0^2 - q_2^2 - q_1^2 + q_3^2
\end{bmatrix}$$

(11)

F. Proposed Method for Gyroscope Field Calibration Using Calibrated Accelerometer

If, it is assumed that gyroscope is only rotated around the sensitive axis and then it is stopped, the angle variation of the sensor around this axis is obtained as follows [9]:

$$\Delta \alpha_k = \omega_k T_k$$

(12)

in which $\Delta \alpha$ is variation of the sensor angle around ith axis at kth movement. In addition, $T_k$ is the rotation duration of the sensor until it stops and $\omega_k$ is the real velocity of the sensor. Now, by replacing the above equation in (5), the following equation is obtained:
\[ \Delta \alpha_{\text{M}} = S_2 \left( \alpha_{\text{M}} - b \right) \]

Now, the sensor is rotated along each axis several times with a constant velocity and then it is put in to static mode for a short time (about 5 sec). This is done n times. Now, the bias and the scale factor are obtained by solving \( \mathbf{A}\mathbf{x} = \mathbf{d} \) simultaneously using the least squares method in which \( \mathbf{A} \), \( \mathbf{d} \) and \( \mathbf{X} \) are defined as follows:

\[
\begin{bmatrix}
\alpha_{\text{M}} & T_1 & T_1 \\
\alpha_{\text{M}} & T_1 & T_1 \\
\alpha_{\text{M}} & T_1 & T_1 \\
\end{bmatrix}
\begin{bmatrix}
M
M
M
\end{bmatrix}
= 
\begin{bmatrix}
S_a \\
S_b \\
\Delta \alpha_{\text{M}}
\end{bmatrix}, \quad \mathbf{d} = 
\begin{bmatrix}
M
M
M
\end{bmatrix}
\Delta \alpha_{\text{M}}
\]

In the method proposed for gyroscope calibration, calibrated accelerometer is used to obtain variations of the angle around each axis. For example, if it is assumed that the three axes of the sensor are aligned with NED, the variations of the angle around N (first axis of the sensor) and E (second axis of the sensor) are obtained using (14); but, because the third sensor is aligned with D, angle variation around this axis cannot be obtained through rotation around this axis. In order to resolve this problem, the third sensor is aligned with N or E and then it is rotated; now the angle variation can be obtained from accelerometer.

Simulation Results

In this section, calibration methods introduced for accelerometer and gyroscope are simulated using MATLAB.

G. Simulating Field Calibration of the Accelerometer

First, a fixed scenario with a fixed condition is defined for the accelerometer for which roll, pitch and yaw angles vary as a sinusoidal function. Then, the improved least squares field calibration is applied to find the bias, the scale factor and the non-orthogonality. In order to evaluate the performance of this method, it is investigated in terms of accuracy, speed and noise impact and the results are given in Table 1. In order to investigate the accuracy, zero-mean Gaussian noise with standard deviation of 0.1 is applied to the output of the sensor and the estimated mean and the standard deviation are calculated after 10 iterations using Monte-Carlo method. In addition, accuracy improvement after calibration is also calculated. Speed and time required for calculating calibration coefficients are also averaged after 10 iterations. In order to observe the effect of noise changes, three Gaussian noises with zero mean and standard deviations of 0.01, 0.1 and 1 are applied to the accelerometer; after 10 iterations, Mean Square Error (MSE) of bias, non-orthogonality and the scale factor is calculated using Monte-Carlo method. As can be seen in the results, the estimation accuracy of the calibration coefficients is very good. In addition, these coefficients have improved the output of the sensor by 60%. The speed of calculating coefficients is also high and this method is robust against noise variations. In addition, an important advantage of this method is that it is independent of the initial conditions.

H. Simulating Field Calibration of the Gyroscope

In this case, a scenario is designed for gyroscope and output model of the gyroscope is considered as in (5), in which noise is Gaussian with zero mean and standard deviation of 0.1 and real values are given in Table 2. Now, using the four methods introduced in this paper, scale and bias factors are estimated and the values after 10 iterations using Monte-Carlo method are given in this table. Moreover, mean square error of estimated factors of each method are also given in Table 2. In addition, expected time for estimating and calculating of error coefficients for each field calibration method is also in Table 3. As can be seen in Table 2, using accelerometer and Euler angles has the least mean square error in estimating the errors. The proposed method performs better than the other described methods; and also it can be seen from Table 3 that the proposed method is faster than the other methods. Another advantage of the proposed method is that it is independent of the initial conditions in estimating errors. Results show that the proposed method has improved the output of the sensor by about 40%.

Conclusion

Due to different errors, the accuracy of the inertial sensors is decreased significantly and they do not perform well. In order to compensate these faults, laboratory and conventional methods are not suitable for cheap sensors and accurate laboratory instruments are required which cannot be used in the working environment; therefore, field calibration methods are used. In this paper, the improved least squares field method is used for accelerometer because it is simple, fast and low-cost and error coefficients include bias, scale factor and non-orthogonality can be obtained accurately and without using any complicated and nonlinear equations.

Furthermore, for calibration of gyroscope, a method was proposed and described that the sensor should be rotated around the sensitive axes and finally by comparing the outputs of calibrated accelerometer and gyroscope, error coefficients of gyroscope which include the bias and the scale factor were obtained. Also, this method and three other field calibration methods were simulated using MATLAB. The results showed a good performance of the proposed method in terms of accuracy and speed. Another advantage of the proposed method was that it did not depend on the initial conditions. Moreover, by calibrating the sensors, the outputs of the accelerometer and gyroscope are improved by 60% and 40%, respectively which in turn increases the accuracy of the sensors, significantly.
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Table 1: Investigating the effect of noise, accuracy and speed on field calibration of the accelerometer

<table>
<thead>
<tr>
<th>Calculation accuracy of estimated coefficients in Gaussian noise with zero mean and 0.1 standard deviation</th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>bₓ</td>
<td>bᵧ</td>
<td>b₁</td>
<td>sₓ</td>
<td>sᵧ</td>
<td>s₁</td>
<td>m₁</td>
<td>m₂</td>
<td>m₃</td>
<td>0.3</td>
<td>-0.6</td>
<td>0.3</td>
<td>1.1</td>
<td>0.95</td>
</tr>
<tr>
<td>True Values</td>
<td>0.3</td>
<td>-0.6</td>
<td>0.3</td>
<td>1.1</td>
<td>0.95</td>
<td>1.2</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.29×10⁻³</td>
<td>0.75×10⁻³</td>
<td>0.3×10⁻¹</td>
<td>0.5×10⁻²</td>
<td>0.44×10⁻⁵</td>
</tr>
<tr>
<td>Mean of estimated coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60%</td>
<td>0.12</td>
<td>0.151</td>
<td>0.001</td>
<td>0.32</td>
</tr>
<tr>
<td>Variance of estimated coefficients</td>
<td></td>
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</tr>
<tr>
<td>Percentage of output improvements measured by accelerometer</td>
<td>60%</td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Table 2: Investigating the effect of noise, accuracy and speed on field calibration of the gyroscope

<table>
<thead>
<tr>
<th>Effect of Noise</th>
<th>Speed</th>
<th>σ=0.01</th>
<th>MSE = 0.21×10⁻⁵</th>
<th>σ=0.1</th>
<th>MSE = 0.16×10⁻⁵</th>
<th>σ=1</th>
<th>MSE = 0.179</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (second)</td>
<td>5 second</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Comparison of time required for calculating the error coefficient

<table>
<thead>
<tr>
<th>Method</th>
<th>Using input angular velocity principal</th>
<th>Using accelerometer and Euler angles</th>
<th>Using accelerometer and quaternions</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (second)</td>
<td>20</td>
<td>50</td>
<td>38</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Author Contributions

S.Ranjbaran and S.Ebadollahi designed the new methods for field calibration of accelerometer and gyroscope. S.Ranjbaran simulated the methods in MATLAB. S.Ranjbaran, S.Ebadollahi, and A.Roudbari interpreted the results and wrote the manuscript.

Conflict of Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by the authors.

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S. Ranjbaran et al.


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